GESTURE, STRUGGLE, AND PROGRESS: EXAMPLES FROM THE UNDERGRADUATE TOPOLOGY CLASSROOM

Keith Gallagher

University of Nebraska Omaha
Department of Mathematics

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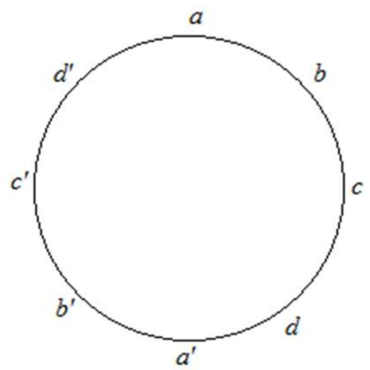
University of Nebraska Omaha

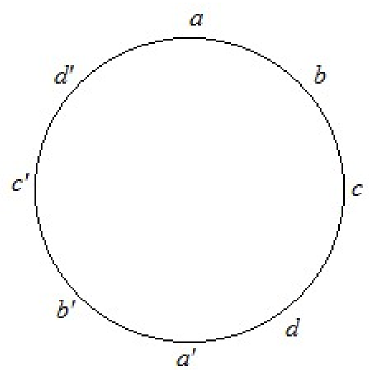
Picture a circle. Any two points on the circle connected by any diameter are called “antipodal points.”



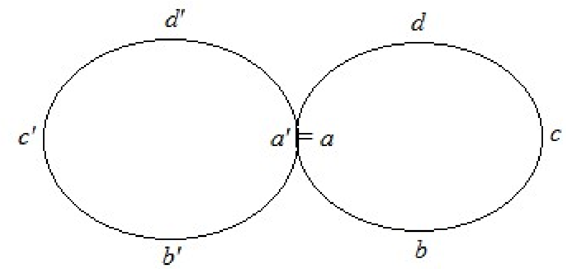
What shape is obtained by gluing every point on the circle to its antipodal point?

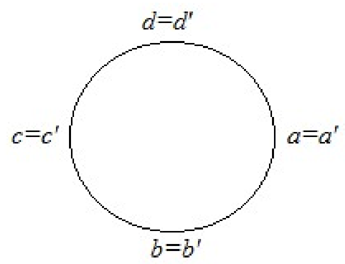
What do you think?

Think of the circle as a rubber band.



1. Twist the rubber band in the middle and glue the cross point (“identify” a=a’).





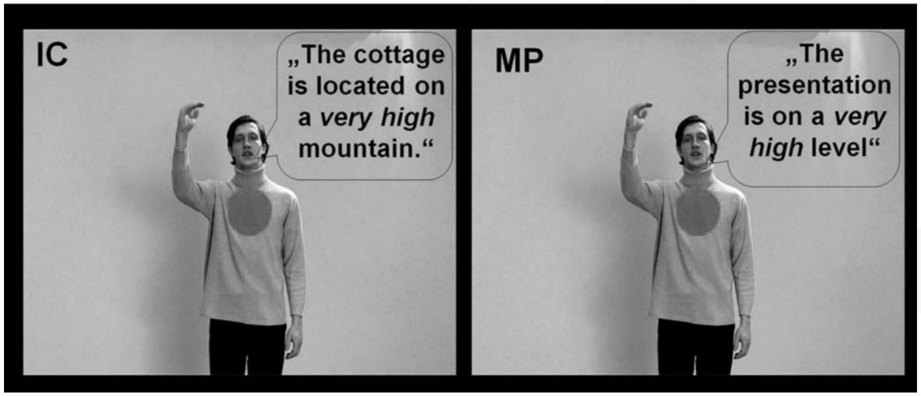
2. Fold the resulting circles onto one another and glue!

Why study gestures?

Identifying gestures

Types of gesture:

- Beat
- Deictic
- Iconic (IC)
- Metaphoric (MP)



Static
Or
Dynamic

Results from neurology

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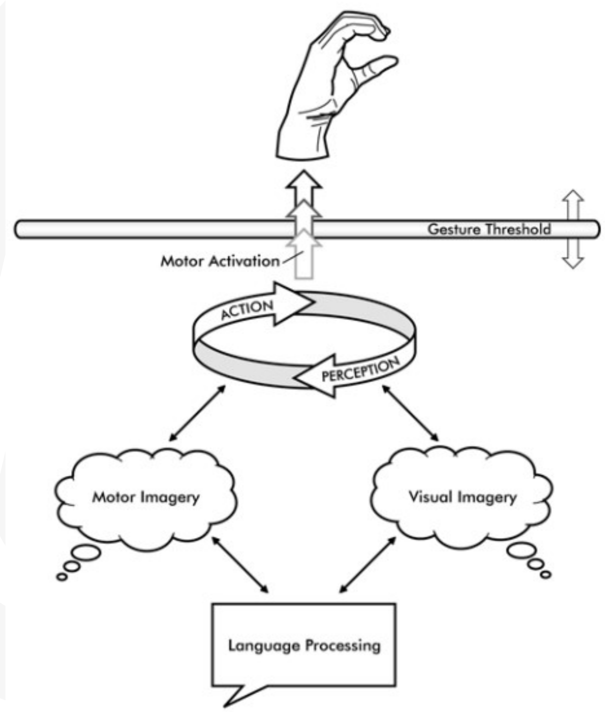
  - Gesture processing: perception of iconic gestures activates a subset of the regions activated by metaphoric gestures
- Bernard, Millman, & Vital (2015) –
  - Gesture production: beat gestures associated with brain volume in discrete motor timing regions, metaphoric gestures associated with brain volume in metaphoric processing regions

Results from cognitive science

- Hostetter & Alibali (2008) –
  - Gesture-as-Simulated-Action Framework: "gestures emerge from the perceptual and motor simulations that underlie embodied language and mental imagery." (p. 502)
  - Recall: gesture processing and production activate the same brain regions



Results from cognitive science

- Goldin-Meadow & Beilock (2010) –
  - Gestures not only reflect thinking, but feed back and *alter* thinking
  - Actions and gestures influence thinking
  - Gestures contain perceptual-motor information

Gesture

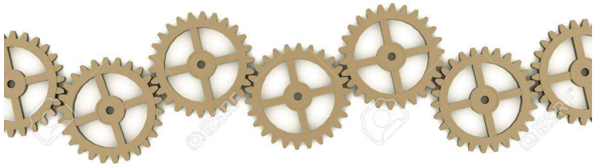
\+

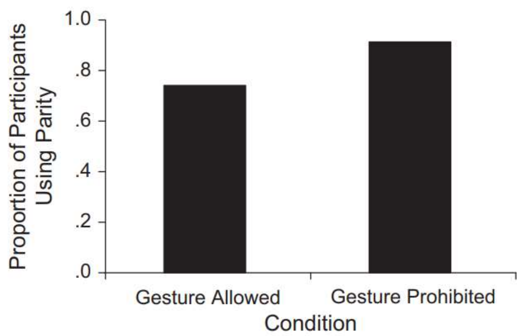
Mental representation

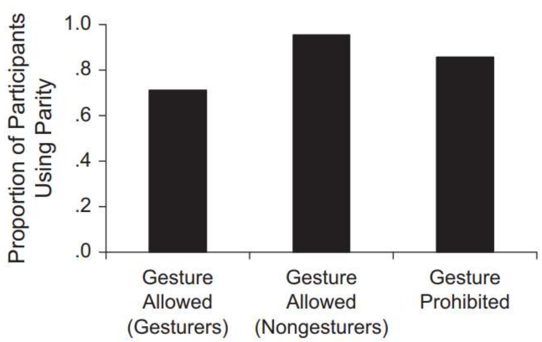
→ **New mental representation** → **Behavior**

Results from cognitive science

- Alibali et al. (2011) –
  - “Spontaneous gestures influence strategy choice in problem solving”
  - Participants solved six problems involving gears, e.g., “Imagine [4, 7, 9, 5, 8, 6] gears are arranged in a horizontal line. If you turn the gear on the left clockwise, what would the gear on the right do?”



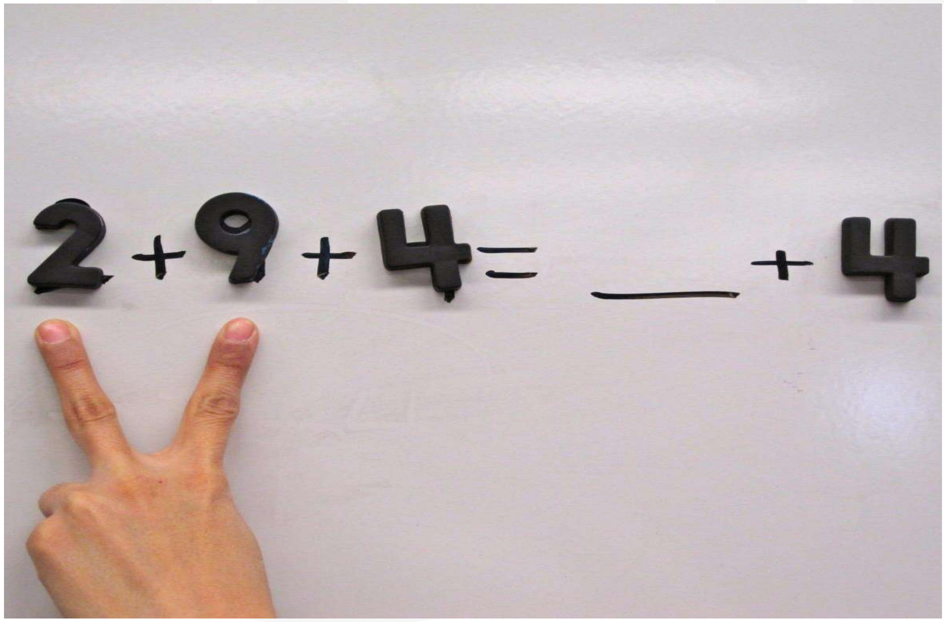




What does this have to do with math?

- Gestures can give insight into students' thinking and attention

- Gesture processing and production can alter students' thinking

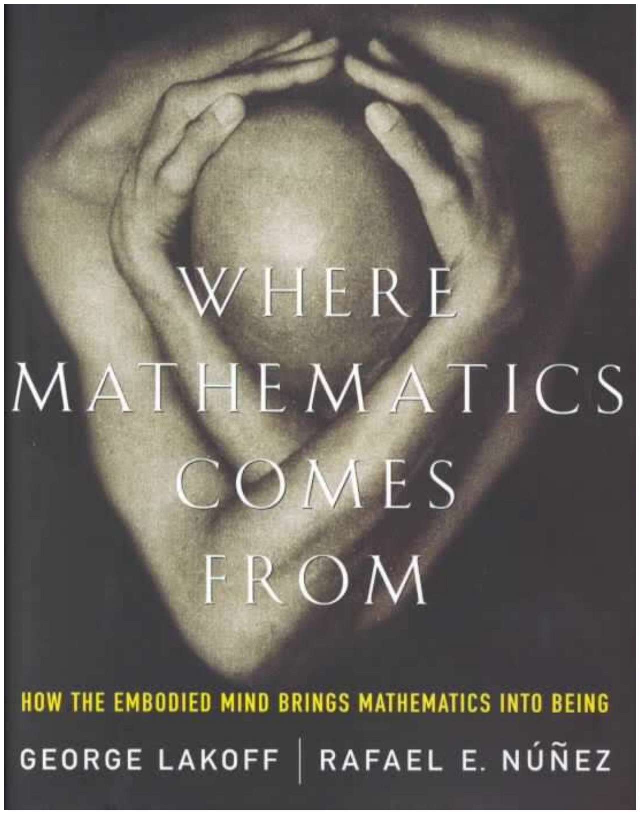


Two handy conceptual frameworks

Embodied Cognition

Commognition

What is embodied cognition?



"Many features of cognition are embodied in that they are deeply dependent upon characteristics of the physical body of an agent, such that the agent's beyond-the-brain body plays a significant causal role, or a physically constitutive role, in that agent's cognitive processing."

(Stanford Encyclopedia of Philosophy, 2017)

What is embodied cognition?

Three roles for the body:

- **Body as Constraint:** an agent's body functions to significantly constrain the nature and content of the representations processed by that agent's cognitive system.

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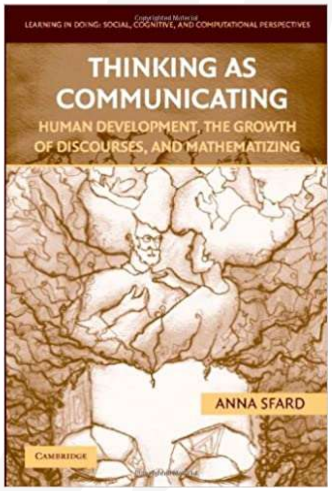
- **Body as Distributor:** an agent's body functions to distribute computational and representational load between neural and non-neural structures.

- **Body as Regulator:** an agent's body functions to regulate cognitive activity over space and time, ensuring that cognition and action are tightly coordinated.

What is commognition?

- Thinking is the individualized form of the activity of communicating.
- Thinking is *not* a self-sustained process separate from communication but is an act of communication with oneself.
- This self-communication does not have to be audible or visible and does not have to be in words.
- Cognitive processes and interpersonal communication processes are different manifestations of basically the same phenomenon. (Sfard, 2008, pp. 82-83)



Gestures in proof writing

- How do undergraduates use gestures and diagrams to support their proof construction in topology?

- **Semiotic bundle**
  - "The novelty of the semiotic bundle…is that it allows us to describe the multimodal semiotic activity of subjects in a holistic way as a dynamic production and transformation of various signs and of their relationships. In particular, it properly frames the role of gestures in mathematical activities." (Arzarello, Paoloa, Robutti, and Sabena, 2009, p. 100)

Gestures in proof writing

- **Key ideas** in proof
  - **Heuristic idea** – based on informal understandings and provides a sense of understanding (convincing yourself)
  - **Procedural idea –** based on logic and formal manipulations to provide a sense of conviction (convincing others)

  - "A **key idea** is an heuristic idea which one can map to a formal proof with appropriate sense of rigor." (Raman, 2003, p. 323) (convincing yourself and others)

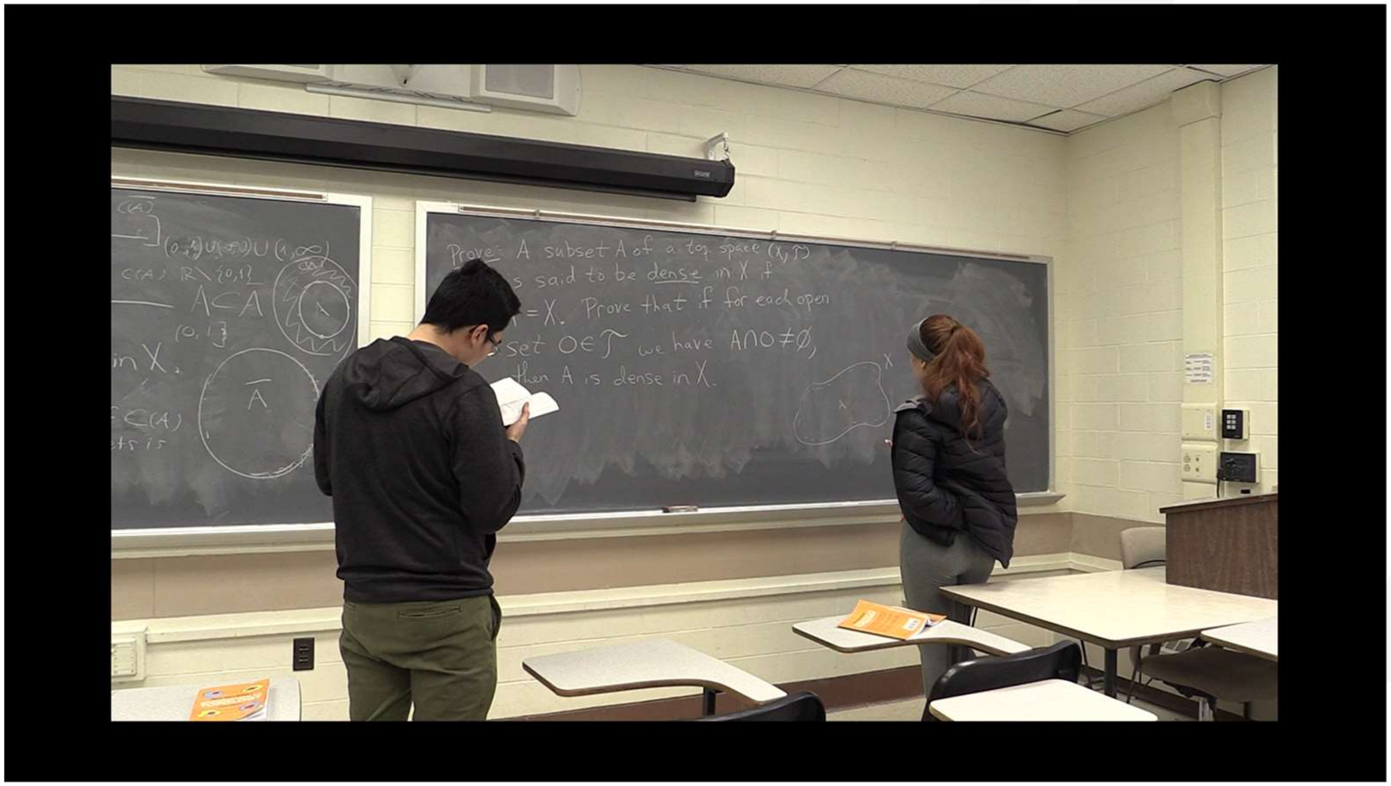
Gestures in proof writing

- **The task**

  - <u>Definition:</u> Given a topological space $(X, \mathcal{T})$, a subset A of X is *dense* in X if the (topological) closure of A is equal to X.

  - <u>Prove:</u> Let $(X, \mathcal{T})$ be a topological space, and let A be a subset of X. If for each open set $O \in \mathcal{T}$ we have $A \cap O \neq \emptyset$, then A is dense in X.

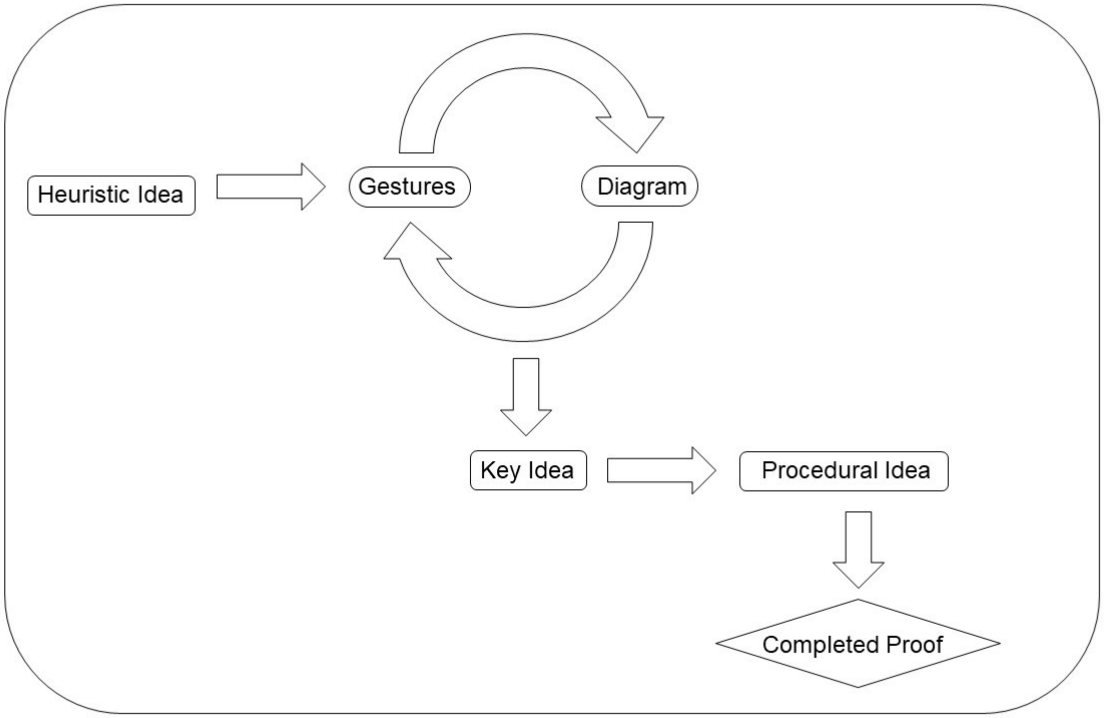
Gestures in proof writing



Gestures in proof writing

[Draws diagram] I can't really show it with a picture because I can't draw a dashed line over a … solid line, but we have X [static point to the label X] on the outside [tracing point along the boundary of X] and then we have the set A [static point to the label A] which is represented by the dashed [tracing point along the boundary of A], which I wish I could get closer to this [static point to the border of X], but I can't. So, if we had the closure of A [static point to the label A], then it would just be the same as that solid line [tracing point along the border of X]. So then if you take any open set [drawing circles on her diagram] anywhere, there has to be some kind of intersection with A [static point to one of her open sets]. So if it wasn't … if the intersection could be … the empty set [static point to $\emptyset$ in the problem statement] – [draws new diagram] You've got X here, and A here, and you could have an open set here, and their intersection would be the empty set [recognizes key idea]. But then this closure [static point to the boundary of A] wouldn't be equal to X [static point to the boundary of X]. I get it conceptually I think, but I'm not sure how to prove it.

Gestures in proof writing



Gestures and struggle

- **Productive Struggle**
  - "[E]ffort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, 2007, p. 387)

- **Unproductive Struggle**
  - "[N]eedless frustration or extreme levels of challenge created by nonsensical or overly difficult problems... [or] the feelings of despair that some students can experience when little of the material makes sense" (ibid.)

Gestures and struggle

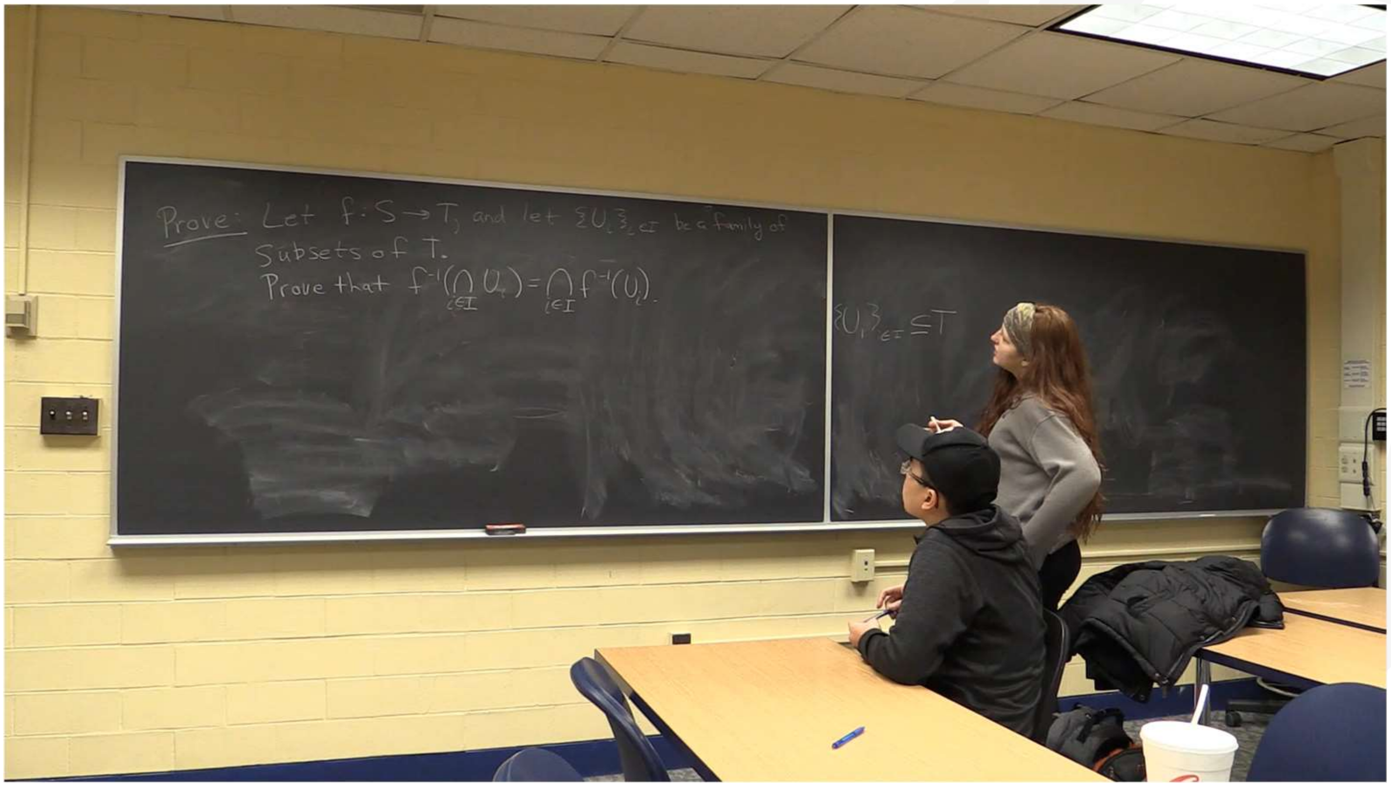
- **Zone of proximal development**
  - “[T]he distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86)

Gestures and struggle

- **The task**
  - Prove: Let $f: S \to T$ be a function, and let $\{U_i\}_{i \in I}$ be a family of subsets of T. Prove that
    - $f^{-1}(\bigcap_{i \in I} U_i) = \bigcap_{i \in I} f^{-1}(U_i)$.

- Started proof by element chasing
  - Immediate confusion over notation, where x should lie
  - I recommended drawing a diagram

Gestures and struggle



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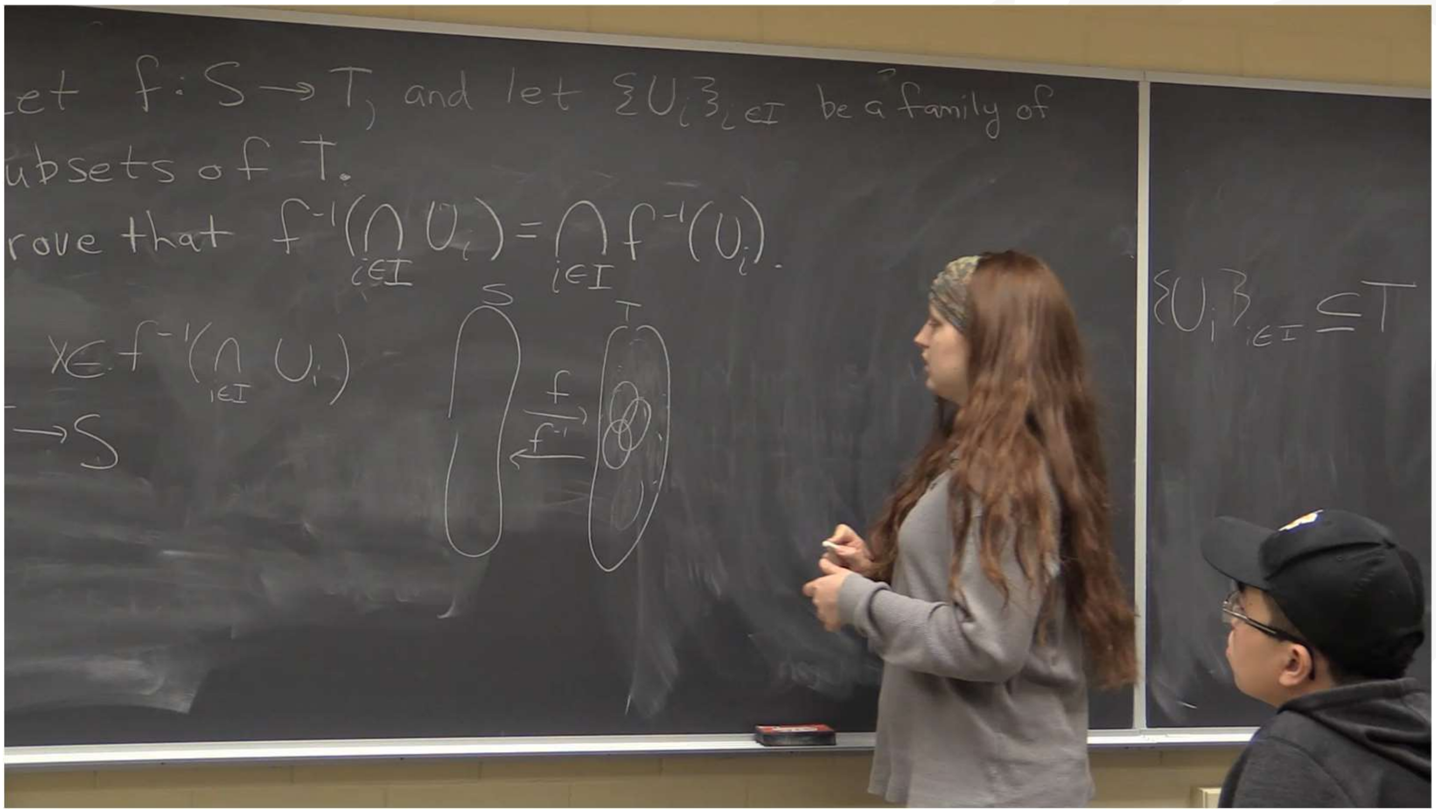
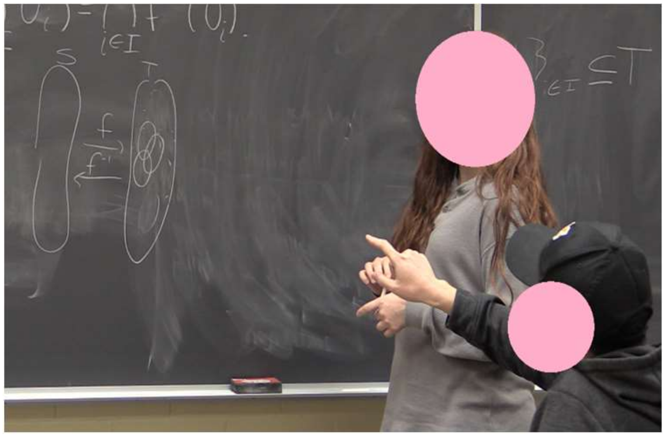
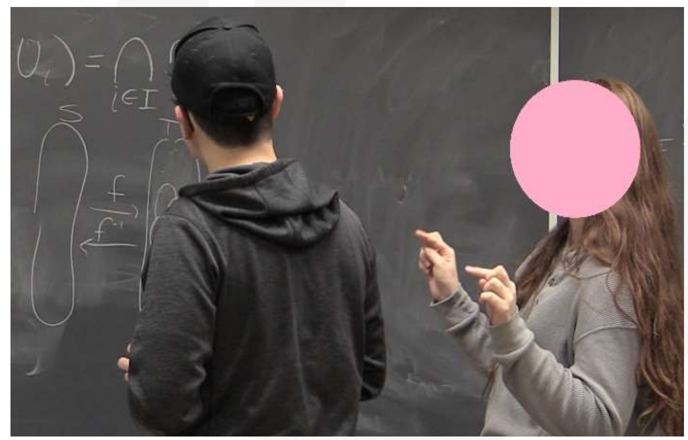


Gestures and struggle



Gestures accompanied ***progress***



No gestures during *unproductive struggle*

Some additional reading

- Kokushkin, V. (2020). The role of gestures in teaching and learning proof by mathematical induction*. Proceedings of The 23rd Annual Conference on Research in Undergraduate Mathematics Education, Boston, MA.*
- Pier, E. L., Walkington, C., Clinton, V., Boncoddo, R., Williams-Pierce, C., Alibali, M. W., & Nathan, M. J. (2019). Embodied truths: How dynamic gestures and speech contribute to mathematical proof practices. *Contemporary Educational Psychology, 58, 44-57.*
- Williams-Pierce, C., Pier, E. L., Walkington, C., Boncoddo, R., Clinton, V., Alibali, M. W., & Nathan, M. J. (2017). What we say and how we do: Action, gesture, and language in proving. *Journal for Research in Mathematics Education, 48(3), 248-260.*

Thank you!

Keith Gallagher
keithgallagher@unomaha.edu