The Random Walks of a Mathematician: Reflections on a Career

Lotfi Hermi
Florida International University
Math Education Seminar
December 4, 2020
Outline

- Mathematics for Business Decision
  - Description of Materials
  - Meet John Sanders... (Loan workout project)
- Learning Technologies and Mathematics Middle East Conference
- Arizona Teacher Institute
- Voting Theory
  - John Sanders goes to the voting booth...
The electronic texts, *Mathematics for Business Decisions* are published by the Mathematical Association of America. You can learn more about them.

Development and dissemination of the electronic texts has been partially supported by a grant from the National Science Foundation. © 2012 Arizona Board of Regents for the University of Arizona. All rights reserved.

Tell me about *Business Mathematics I & II*.

[Business Mathematics I](#): COURSE INFORMATION

[Business Mathematics II](#): COURSE INFORMATION

View or download a two-page COLOR BROCHURE
The Mathematics of Decision Making: A Model for Collaboration Between Mathematics and Other Fields

McCallum, William  Thompson, Richard  Lamoureux, Christopher  Hughes-Hallett, Deborah
University of Arizona, Tucson, AZ, United States

Abstract

Each year, hundreds of thousands of prospective students in business and public administration (BPA) degree programs take required mathematics courses because they need to learn to use mathematics effectively in making decisions. However, this need is often not addressed by the traditionally required courses. The main goal of this project is to produce materials that convince students that mathematics and modern computer technology are valuable tools in solving realistic problems and to teach them to utilize these tools effectively. We plan to develop, site-test, and nationally disseminate an archive of substantial, technology-based case studies (a realistic scenario which can be used to introduce the mathematical content) in the mathematics of decision-making. These will take between a few days and a few weeks to teach, and can be used instead of or in addition to a text. Each case study will be developed by teams consisting of faculty from mathematics and BPA, to be chosen from around the county. The co-PIs Thompson and Lamoureux, in a partnership between the University's Department of Mathematics and the College of Business and Public Administration, have produced case studies on whether to foreclose a commercial loan or work out an arrangement with the creditor, and on how to price a stock option. Hughes Hallett, in collaboration with E. Connally of Wellesley College and faculty at Harvard University's Kennedy School of Government, has produced case studies on the Austrian state pension system, population projection using US census data and data from the CIA World-book, and water flow in the Charles River using data from the US Geological Survey.

For Information Contact:
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Richard B. Thompson
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ICTM3

BUSINESS MATHEMATICS WORKSHOP
The University of Arizona Model for Integrating Technology into Learning

Saturday, July 1st, 2006
4:30 to 6:30 p.m.

Anadolu Room
Marmara Hotel
Istanbul, Turkey

Lotfi Hermi, Department of Mathematics, University of Arizona
Learning Technologies and Mathematics Middle East Conference

BUSINESS MATHEMATICS WORKSHOP
The University of Arizona Model for Integrating Technology into Learning

Saturday, March 31, 2007
4:30 to 6:00 p.m.

Sultan Qaboos University
Muscat, Oman

Lotfi Hermi, Department of Mathematics, University of Arizona
Department of Mathematics
University of Arizona

Business Mathematics I

Math 115a

Fall Semester 2005

The Course Homepage
Textbook Information
Math 115a Files
Interactive Tutorial
Course Policies
Syllabus: MWF Sections
TuTh Sections
Homework

To Accompany
Mathematics for Business Decisions Part 1
Alternative Edition, Release 1.5a, 2005
Most of your course information for *Math 115a, Business Mathematics I*, will come from the *Course Homepage*. You can access this web site at

http://business.math.arizona.edu/~busmath

or by clicking on any *Course Homepage* box.

What is on the *Course Homepage*? Here is a partial list.

- General *information* about *Business Mathematics I* and *II*.
- *Readiness exercises* to let you know if you have the necessary computer skills for the course.
- Current course *Announcements*. Be in the know! Check this regularly.

(material continues)
- **Study guides** and their solutions.
- **Homework solutions**.
- **Policies, team assignment, and project information**; provided by the instructor of your individual section of *Math 115a*.

The last item is of particular importance. Each section of *Business Mathematics I* has its own page on the course web site. Your instructor will use this to keep you informed about activities in your own section.

Explore the *Course Homepage* now and check it regularly during the semester.
MATHEMATICS FOR BUSINESS DECISIONS PART 1

Math 115a uses the Alternative Edition, Release 1.5a, of the electronic text Mathematics for Business Decisions Part 1, by Richard Thompson, Christopher Lamoureux, and Pamela Slaten. This is copyrighted by the Arizona Board of Regents and is published by the Mathematical Association of America.

Copies of the e-text can be purchased from the University of Arizona bookstore. The material consists of a set of PowerPoint, Excel and Word files, along with many links to streaming video and internet sites. It is packaged as boxed software, with an installation CD and a Student Notebook, containing screen captures of all of the PowerPoint slides. The CD can be used to install all of the e-text files onto a computer hard drive or as a way to run the files directly from the CD drive on any computer.
SYSTEM REQUIREMENTS

*Mathematics For Business Decisions Part 1* will run on PC's with Windows 98 or higher; and *Excel, PowerPoint, and Word* from *Microsoft Office Suite 2000* or higher. It is *not* designed to run on the *Macintosh* platform. Some people have found that the course files run acceptably on *Macintosh* computers, others have had major problems with that operating system. Use the texts on *Macintosh* systems at your own risk.

**USING THE CD-ROM**

**Running From The CD**

1. Place the course CD into the CD drive and allow time for the computer to recognize the CD.

2. Left click on the *My Computer* desktop icon, or expand the *My Computer* folder in *Windows Explorer*.

3. Right click on the icon for the drive containing the course CD, and then left click on *Open* in the pull down menu.
4. Double left click on the subfolder **MBD Part 1a**, and then double left click on its subfolder **Course Files Alt**.

5. Select and open the **PowerPoint**, **Excel**, or **Word** file that you wish to view. If you are using **PowerPoint** from **Microsoft Office 97** and will be following links to **Excel** or **Word**, open these applications before starting a **PowerPoint** file.

**Installing On A Hard Drive**

1. Place the course CD into the CD drive and allow time for the computer to recognize the CD.

2. If the CD does not open automatically, click on **Start**, then on **Run**. Select the file **start.exe** on the CD drive, then click on **OK**.

3. When the splash page of the CD appears, click on **Install to Hard Drive**, and follow the on-screen instructions.

4. Read the **License Agreement** and, if you are willing to accept its terms, continue.
5. Accept the default installation folder \texttt{C:\MBD\Part\1a}, or click \texttt{Change} to install to a different folder.

6. Use the \texttt{My Computer} desktop icon or \texttt{Windows Explorer} to open the subfolder \texttt{Course Files} in the folder that you have just installed. All of the files for your e-text are in this folder.

Complete instructions for \texttt{restoring} or \texttt{uninstalling} the files are given the file \texttt{Exploring MBD Part 1}. This is available in either \texttt{Word} or \texttt{PDF} form in the folder \texttt{Read First}.

The main content of \textit{Mathematics For Business Decisions, Part 1a} is in three \texttt{PowerPoint} files \texttt{MBD\ Part\ 1a.ppt}, \texttt{MBD\ 1\ Proj\ 1a.ppt}, and \texttt{MBD\ 1\ Proj\ 2a.ppt}. These, and all other instructional files, are located in the folder \texttt{Course\ Files\ Alt}. \texttt{MBD\ Part\ 1a.ppt} is the starting point for study of the material. It contains information about the course and its project structure. There is also an extensive section on how to use the electronic text effectively, and a self-grading test over the necessary prerequisite computer skills. All parts of the file may be accessed from links in its \texttt{Table\ of\ Contents}, or located with its electronic \texttt{Index}.
COPYRIGHT & REGISTRATION

The text *Mathematics for Business Decisions Part 1* is copyrighted by the Arizona Board of Regents for the University of Arizona. *Any copying of the installation CD, the individual e-text files, or the Student Notebook for use by an individual other than the purchaser is a violation of this copyright.*

Since the Mathematical Association of America is a not for profit organization, the e-texts are sold at lower prices than would have resulted from commercial publication. Any royalties paid to Professor Thompson on sales of the material at the University of Arizona will be transferred to the University for further development of the project to benefit business students.

For more information on the e-text visit its web site

In addition to the e-text, *Mathematics for Business Decisions Part 1*, you will use a set of files that apply specifically to *Math 115a* at the University of Arizona. Your individual instructor may also create course files for you to download or copy. These will contain information that applies only to your section of the course.

To download the *Math 115a Files*, go to the [Course Homepage](#) and click on *Business Mathematics I* in "Business Mathematics I: COURSE INFORMATION", then click on *FILES* in "Download FILES for Math 115a at the University of Arizona." The download will be a self-extracting Zipped file, *M115a.exe*. Save this in a folder or on the Desktop, then double click on the file to extract the full set of *Math 115a* files.

The *Math 115a Files* and other course material can also be viewed or copied onto a ZIP Disk or CD from a computer in the [Information Commons](#) in the Integrated Learning Center (*ILC*). For directions, click on *ILC Bus Math Files.ppt*. 

(material continues)
The following files are needed for *Math 115a* at the University of Arizona.

**PowerPoint**
- UA Bus Math I.ppt
- ILC Bus Math Files.ppt

**Word**
- Study Guide 1.doc
- Study Guide 1 Solutions.doc
- Study Guide 2.doc
- Study Guide 2 Solutions.doc
- Team Contract.doc

(material ends)
An interactive, basic level tutorial program is available on computers in the Integrated Learning Center, ILC. This is a PowerPoint presentation, using animations, links to Excel, and voice narration to help you get started using your computer text. The tutorial shows how to get the most benefit from your course material, by the effective use of both the running and non-running modes of PowerPoint.

Topics are described with both audio and visual presentations, and are then illustrated with animations. The presentation pauses while you practice the new operations. A small amount of time spent with the tutorial may help you avoid some of the problems that can occur with the use of unfamiliar software. All of the material is presented at a very basic level, assuming no prior experience with PowerPoint or Excel.

For directions on running the Interactive Tutorial, click on ILC Bus Math Files.ppt.
FALL SEMESTER, 2005

The following items apply to all sections of Math 115a at the University of Arizona. Office hours, assignments, final examination locations, and other matters that apply to individual sections of the course are listed under instructors’ pages in the Course Homepage.

Kick-Off. The Eller College of Management is holding a Kick-off for this course on Wednesday, September 7 at 5:00 p.m. in ILC 120, with a make-up session on Saturday, September 10 at 9:00 am in MCLND 133. Participation in this event will give credit for Homework 1.
Presentation Competition. The Eller College of Management is also organizing a presentation competition. All teams will give their oral reports on Project 1 as shown in the Syllabus. For competition purposes, the presentations will be judged by members of the Eller Undergraduate Programs Team and students from the Eller Business Support Lab. The winners of the first round will advance to the final round, which will be held on Saturday, September 22, 2005. The final round will be judged by members of the local business community, and the first and second place teams will receive prizes. Your instructor will receive the evaluations of your presentation from the judges. However, for course grade purposes, the score for your presentation will be determined by your instructor.

Prerequisites. Successful completion of Introduction to Computing (MIS 111) and Business Problem Solving and Analysis (BAD 112L), and either successful completion of College Algebra with Data Analysis (Math 109), College Algebra (Math 110), or College Algebra Accelerated (Math 112) or an acceptable score on the Mathematics Readiness Test.
Text. *Mathematics for Business Decisions, Part 1*, Release 1.5a, 2005, by Thompson and Lamoureux, Mathematical Association of America. Your instructor may require you to bring your CD to class on selected dates.

Course Materials Needed. (i) *Textbook*. (ii) *Math 115a Files*. (iii) Blank CD-R's, CD-RW’s, Zip Disks, or 3-1/2" Floppy Disks; as needed for reports and homework.

Turnitin. You will be required to have or create a *Turnitin* user profile and to join the *Turnitin* class created for your section of *Business Mathematics I*. Your instructor will provide you with the class ID and enrollment password.

Examinations. *Two midterm examinations* will be given as shown in the *Syllabus*. *Final examinations in all sections of Math 115a will be given at a common time, Tuesday, December 13, 2005, from 5-7 p.m.* Your instructor will notify you about the location of the examination. *See the Course Homepage, for each instructor’s policy on missed examinations.*
Daily Homework. The Syllabus lists the topics that are to be studied during each class meeting. The text contains exercises, all of which should be completed while studying the material.

Graded Homework. Exercise sets will be provided by your instructor and collected on a regular basis. The due dates for these assignments will be announced in class. Your instructor will establish a policy on whether graded homework may be submitted by individuals or by teams.

Reports. Teams will give preliminary and both oral and written final reports on the projects. All written reports must be printout of either word processing or Excel workbooks. Any member of a team may be called upon to explain material during the oral report. Your instructor will inform you of the report grading policy in your section.

Additional Work. Your instructor may give credit for announced or unannounced quizzes, and for various types of additional work, including Excel assignments.
Course Grades. Each midterm test will be worth 100 points, the homework will be worth 100 points, the two team projects will be worth 150 points each, and the final examination will be worth 200 points. Other work, if any, may earn additional points. Your course grade will be based upon the fraction of the total possible points that you earn. If you earn at least 55% of the total points for individual work and at least 55% of the total points for team work, the following grading scale will be used.

- A: 90% of total points
- B: 80% of total points
- C: 70% of total points
- D: 60% of total points

Otherwise, you will receive a failing grade for the course. For course grade purposes, individual work is defined to include the midterm examinations, the final examination, and the quizzes, if any. Team work is defined to include the preliminary and final reports on the projects and the homework.
**Academic Integrity.** You are assumed to be familiar with, and to abide by, all University of Arizona policies and procedures, particularly the *Code of Academic Integrity* and the *Student Code of Conduct*. A discussion of these issues in the context of *Business Mathematics* is given in the *Business Mathematics Academic Integrity Policy*. Your instructor may provide additional information about the expectations for your section. Students found to be in violation of any of these policies will be appropriately sanctioned.

**Objectives and Structure.** *Business Mathematics I* is specifically designed to prepare students for subsequent work in their business majors and for their future careers in the business community. To read a full statement of the course objectives and mandatory team and report structure, click on *Objectives and Structure*. 

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**Course Homepage**

(material ends)
August 22 (M): Introduction to Course
August 24 (W): Using the Computer Text (Teams Formed)
August 26 (F): Introduction to Project 1
August 29 (M): Basic Probability
August 31 (W): Basic Probability
September 2 (F): Basic Probability
September 5 (M): Labor Day – no classes
September 7 (W): Word Processing Mathematics
   Business Mathematics Kick-Off
   5:00-6:45 p.m., ILC Room 120.
September 9 (F): Summation Notation
September 12 (M): Expected Value
September 14 (W): Expected Value
(material continues)
September 16 (F): Preliminary Reports on Project 1
September 19 (M): Database Functions and Filtering
September 21 (W): Database Functions and Filtering
September 23 (F): Conditional Probability
September 26 (M): Conditional Probability
September 28 (W): Bayes’ Theorem
September 30 (F): Bayes’ Theorem
October 3 (M): Management Example
October 5 (W): Test 1, material through September 30
October 7 (F): Discussion of Project 1

(material continues)
October 10 (M): Reports on Project 1
October 12 (W): Reports on Project 1
October 14 (F): Reports on Project 1
October 17 (M): Last day to drop the course.
October 19 (W): Introduction to Project 2
October 21 (F): Histograms
October 22 (Sa): Presentation Competition, Final Round
October 24 (M): Preliminary Reports on Project 2
October 26 (W): Exponential Growth
October 28 (F): Exponential Growth
October 31 (M): Probability Distributions
November 2 (W): Probability Distributions

(material continues)
November 4 (F): Probability Distributions
November 7 (M): Random Sampling
November 9 (W): Random Sampling
November 11 (F): Veteran’s Day – no classes
November 14 (M): Random Sampling
November 16 (W): Monte Carlo Methods
November 18 (F): Monte Carlo Methods
November 21 (M): Monte Carlo Methods
November 23 (W): Wholesale Ordering Example
November 25 (F): Thanksgiving Recess – no classes
November 28 (M): Test 2, material from October 17 through November 21
November 30 (W): Discussion of Project 2

(material continues)
December 2 (F): Reports on *Project 2*

December 5 (M): Reports on *Project 2*

December 7 (W): Reports on *Project 2*

December 13 (Tu): **Final Examination**, All course material

*Final examinations in all sections of Math 115a will be given at a common time, Tuesday, December 13, 2005, from 5-7 p.m.* Your instructor will notify you about the location of the examination.
August 23 (Tu): Introduction to Course Using the Computer Text

August 25 (Th): Introduction to Project 1
(Teams Formed)

August 30 (Tu): Basic Probability

September 1 (Th): Basic Probability

September 6 (Tu): Word Processing Mathematics

September 7 (W): Business Mathematics Kick-Off
5:30-7:30 p.m., ILC Room 120.

September 8 (Th): Summation Notation
Expected Value

September 13 (Tu): Expected Value

September 15 (Th): Preliminary Reports on Project 1
Database Functions and Filtering

(material continues)
September 20 (Tu): Database Functions and Filtering
September 22 (Th): Conditional Probability
September 27 (Tu): Conditional Probability
                   Bayes’ Theorem
September 29 (Th): Bayes’ Theorem

October 4 (Tu): Management Example

October 6 (Th): \textbf{Test 1}, material through September 29
                Discussion of \textit{Project 1}

October 11 (Tu): Reports on \textit{Project 1}

October 13 (Th): Reports on \textit{Project 1}

October 14 (F): \textbf{Last day to drop the course.}

October 18 (Tu): Introduction to \textit{Project 2}
                   Histograms

(material continues)
October 20 (Th): Histograms
October 22 (Sa): *Presentation Competition, Final Round*
October 25 (Tu): Preliminary Reports on *Project 2* Exponential Growth
October 27 (Th): Exponential Growth
November 1 (Tu): Probability Distributions
November 3 (Th): Probability Distributions
November 8 (Tu): Random Sampling
November 10 (Th): Random Sampling
November 15 (Tu): Monte Carlo Methods
November 17 (Th): Monte Carlo Methods
November 22 (Tu): Wholesale Ordering Example
November 24 (Th): Thanksgiving Recess – no classes
*material continues*
November 29 (Tu): **Test 2**, material from October 18 through November 17
Discussion of *Project 2*

December 1 (Th): Reports on *Project 2*

December 6 (Tu): Reports on *Project 2*

December 13 (Tu): **Final Examination**, All course material

*Final examinations in all sections of Math 115a will be given at a common time, Tuesday, December 13, 2005, from 5-7 p.m.* Your instructor will notify you about the location of the examination.
Graded homework assignments are to be handed in at the start of class on the announced due dates.

These exercises will be listed in Word files, which will be posted on your instructor’s web page. To complete an assignment, solve the problems, open the appropriate file in Word, and enter the required information on the cover page. Word process your solutions, then save and print the file. On some assignments, you will also need to include material copied from Excel files that you have created.

Your instructor will indicate whether these assignments are to be completed individually, or by more than one member of a currently operating project team. Credit for the assignment will be given to those students whose signatures appear on the cover page. Work done by members of two or more project teams is not acceptable and can be viewed as plagiarism.
Meet John Sanders...
Business Mathematics I

Math 115a

Spring Semester 2007

Loan Workout Class Project
Project 1. **Loan Work Outs**

**Video** introduction by

Prof. Christopher Lamoureux
Department of Finance
University of Arizona.

1. **Project Description**
   - A. Business Background
   - B. Class Project

2. **Mathematical Tools**
   - A. Basic Probability
   - B. Summation Notation
   - C. Expected Value
   - D. Conditional Probability
   - E. Bayes’ Theorem
   - F. Management Example

3. **Computer Tools**
   - A. Word Processing Mathematics
   - B. Database Functions and Filtering
Acadia Bank has records of 8,226 business loans in which work out agreements were made. These come from three banks; BR, Cajun, and DuPont, whose merger formed Acadia. The three former banks each kept records on different aspects of the loans.

<table>
<thead>
<tr>
<th>Bank Information</th>
<th>Borrower</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer Number</td>
<td>Former Bank</td>
<td>Years In Business</td>
</tr>
<tr>
<td>1</td>
<td>Cajun</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>BR</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>BR</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>Cajun</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>DuPont</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>BR</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>BR</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>DuPont</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>BR</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>BR</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>DuPont</td>
<td>12</td>
</tr>
</tbody>
</table>
Meet John Sanders...

<table>
<thead>
<tr>
<th>Years in Business</th>
<th>Education</th>
<th>Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 years</td>
<td>Bachelor’s</td>
<td>Normal</td>
</tr>
</tbody>
</table>

Loan Focus.xls
**Loan Details**

<table>
<thead>
<tr>
<th></th>
<th>Full Value</th>
<th>Foreclosure Value</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$ 4,000,000</strong></td>
<td>$2,100,000</td>
<td>$ 250,000</td>
<td></td>
</tr>
</tbody>
</table>
Problem:

Armed with this data, should Acadia Bank enter into a work out agreement with John, or should it foreclose on the loan?
How can probability help us with the decision on whether or not to attempt a loan work out?

Let $S$ be the event that an attempted work out is successful and let $F$ be the event that it fails. In the sheet Basic Prob of the Excel file Loan Focus.xls we use the COUNTIF function to find the fraction of past work out arrangements which were successful. This fraction is our estimate for $P(S)$. Likewise, we find the fraction of attempts that failed and use this as our estimate for $P(F)$. 

Loan Focus.xls  Class Project  (material continues)
# Overall Records

<table>
<thead>
<tr>
<th>Customer Number</th>
<th>Former Bank</th>
<th>Years In Business</th>
<th>Education Level</th>
<th>State Of Economy</th>
<th>Loan Paid Back?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Records</td>
<td>8226</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paid Loan</td>
<td>3818</td>
<td></td>
<td>P(S)</td>
<td></td>
<td>46%</td>
</tr>
<tr>
<td>Did not pay</td>
<td>4408</td>
<td></td>
<td>P(F)</td>
<td></td>
<td>54%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Exercise 10  As in the exercises in Database Functions, use \( Y \), \( T \), and \( C \) as the events that a randomly selected borrower from Acadia Bank has the same number of years experience, the same educational level as your team’s specific borrower, and that the economic times are the same as in your team’s loan, respectively.

Let \( Z_Y \) and \( Z_T \) be the random variables giving the amounts of money, in dollars, that Acadia Bank receives from a future loan work out attempt to a borrower with the same number of years experience or educational level, respectively, as your team’s borrower. Let \( Z_C \) be the random variable giving the amounts of money, in dollars, that Acadia Bank receives from a future loan work out attempt to a borrower in the same economic times as your team’s loan.

(i) Use the bank records to estimate \( P(S|Y) \), \( P(F|Y) \), \( P(S|T) \), \( P(F|T) \), \( P(S|C) \), and \( P(F|C) \). (ii) Compute \( E(Z_Y) \), \( E(Z_T) \), and \( E(Z_C) \). See the Focus On The Project section for the corresponding work with the class project.
### Years in the Business

<table>
<thead>
<tr>
<th>Total Records</th>
<th>239</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer Number</td>
<td>Former Bank</td>
</tr>
<tr>
<td>Paid Loan</td>
<td>105</td>
</tr>
<tr>
<td>Did not pay</td>
<td>134</td>
</tr>
</tbody>
</table>
## Educational Training

<table>
<thead>
<tr>
<th>Customer Number</th>
<th>Former Bank</th>
<th>Years In Business</th>
<th>Education Level</th>
<th>State Of Economy</th>
<th>Loan Paid Back?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paid Loan</td>
<td>510</td>
<td>P(S</td>
<td>T)</td>
<td>Bachelor</td>
<td></td>
</tr>
<tr>
<td>Did not pay</td>
<td>644</td>
<td>P(F</td>
<td>T)</td>
<td>Bachelor</td>
<td></td>
</tr>
</tbody>
</table>

- **Total Records**: 1154
- **Paid Loan**: 510, 44%
- **Did not pay**: 644, 56%
### Economic Conditions

<table>
<thead>
<tr>
<th>Total Records</th>
<th>1547</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Customer Number</strong></td>
<td><strong>Former Bank</strong></td>
</tr>
<tr>
<td>Paid Loan</td>
<td>807</td>
</tr>
<tr>
<td>Did not pay</td>
<td>740</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Overall Records</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Success</td>
<td>46%</td>
</tr>
<tr>
<td>Failure</td>
<td>54%</td>
</tr>
</tbody>
</table>
What is next?

- How to combine the information?

\[ P(S \mid Y \cap T \cap C) \quad P(F \mid Y \cap T \cap C) \]

- How to use the loan values (Full Value, Foreclosure Value, Default Value)

\[ E(Z) = P(S \mid Y \cap T \cap C) \times FV + P(F \mid Y \cap T \cap C) \times DV \]

- Business Decision:

\[
\text{Foreclosure} \geq E(Z) \quad \text{Foreclose}
\]

\[
\text{Foreclosure} < E(Z) \quad \text{Workout}
\]
These numbers are not accessible

\[ P(S \mid Y \cap T \cap C) \quad P(F \mid Y \cap T \cap C) \]

Bayes’ Theorem

\[
P(S \mid Y \cap T \cap C) = \frac{P(Y \cap T \cap C \mid S) \cdot P(S)}{P(Y \cap T \cap C \mid S) \cdot P(S) + P(Y \cap T \cap C \mid F) \cdot P(F)}
\]

\[
P(F \mid Y \cap T \cap C) = 1 - P(S \mid Y \cap T \cap C)
\]
Assumptions

❖ Events are independent

\[ P(Y \cap T \cap C \mid S) = P(Y \mid S) \times P(T \mid S) \times P(Y \mid S) \]

\[ P(Y \cap T \cap C \mid F) = P(Y \mid F) \times P(T \mid F) \times P(Y \mid F) \]

❖ Database merger problem: partial info represents what happens in Acadia Bank

\[ P(Y \mid S) \approx P(Y_{BR} \mid S_{BR}) = \frac{P(Y \cap S_{BR})}{P(S_{BR})} \]
What would lead “bank analysts” to reverse their decision?

❖ Sensitivity analysis: “Range of Years”: within 1 and 2 years

\[
P(S \mid Y' \cap T \cap C) \quad P(F \mid Y' \cap T \cap C)
\]

❖ What is the least default value that would lead to a reversal of decision?

\[
\text{Foreclose} \geq P(S \mid Y \cap T \cap C) \times FV + P(F \mid Y \cap T \cap C) \times DV
\]

\[
DV \geq \frac{\text{Foreclose} - P(S \mid Y \cap T \cap C) \times FV}{P(F \mid Y \cap T \cap C)}
\]
SULTAN QABOOS UNIVERSITY
OMAN
MARCH 31-APRIL 2, 2007

LEARNING TECHNOLOGIES
AND MATHEMATICS
MIDDLE EAST CONFERENCE
If you build it, they will come....

https://www.math.arizona.edu/~atp-mena/conference/presentations/invited.html
De Carthage vers le monde
Le problème isopérimétrique de la reine Dido
et ses ramifications mathématiques

From Carthage to the World
The Isoperimetric Problem of Queen Dido
and its Mathematical Ramifications

Carthage, Tunisia
May 24-29, 2010
CALL FOR APPLICATIONS

Summer 2021 African Diaspora Joint Mathematics Workshop (ADJOINT) in Berkeley, California

The Mathematical Sciences Research Institute invites applications for its 2021 ADJOINT workshop taking place June 21 - July 2 in Berkeley, California.
John Sanders Votes: Democracy and its Discontents ...
Why Stop at Business Math?

A Mathematical Exploration of Apportionment Procedures Around the World

Lotfi Hermi; Deborah Hughes Hallett; William G. McCallum
CHAPTER I
DEMOCRACY AND ITS MATHEMATICAL DISCONTENTS

1.1 Choosing an Electoral System
1.2 Apportionment Schemes
1.3 The Mathematical Problem
1.4 Assumptions
1.5 A Brief History of Democracy

EXCEL FILES

- World Voting Systems
- Filtering Data (Use of the "Filter" and "Countif" commands in Excel)
- Women in World Parliaments, April 2004
- South Africa Elections (April 2004)
- injustice.xls

LINKS

- The International IDEA Handbook of Electoral System Design (PDF)
  - The Electoral Systems of Independent States (IDEA, 2002)
  - Basic Kit from IDEA
  - Electoral System Families Flowchart (IDEA)
- The Global Distribution of Electoral Systems (ACE Project)
- IFES democracy/large (International Foundation for Election Systems)
- electionguide.org
- Independent Electoral Commission of South Africa
- Women in National Parliaments (April 2004)
- "Ladies First": Read the reasons why Rwanda has the highest women in parliament rate
- Psephos Election Archive
- News Coverage: Lexis-Nexis Service
A Mathematical Exploration of Apportionment Procedures Around the World

COMPLETE LISTING OF EXCEL SUPPLEMENTS

Methods

- largest-remainder.xls (handles up to 300 parties, unlimited number of seats, 703 KB)
- divisor-methods5p700s.xls
  (Eight most commonly used divisor methods for an apportionment of up to 5 parties, 700 seats, 5.6 MB)
- divisor-methods5p1000s.xls
  (Eight most commonly used divisor methods for an apportionment of up to 5 parties, 1000 seats, 7.6 MB)
- dhondt50parties700seats.xls
  (d'Hondt method for an apportionment of up to 50 parties, 700 seats, 6 MB)
- dhondt50parties1000seats.xls
  (d'Hondt method for an apportionment of up to 50 parties, 1000 seats, 8.4 MB)
- sainte-lagugue50parties700seats.xls
  (Sainte-Lague method for an apportionment of up to 50 parties, 700 seats, 6 MB)
- sainte-lagugue50parties1000seats.xls
  (Sainte-Lague method for an apportionment of up to 50 parties, 1000 seats, 8.4 MB)
- huntington50parties500seats.xls
  (Huntington method for an apportionment of up to 50 parties, 500 seats, 5.6 MB)
- huntington50parties700seats.xls
  (Huntington method for an apportionment of up to 50 parties, 700 seats, 8.1 MB)
- huntington50parties1000seats.xls
  (Huntington method for an apportionment of up to 50 parties, 1000 seats, 11.6 MB)
- ngpp-disproportionality.xls (effective number of political parties, Index of Disproportionality)

Extras

- army-divisions.xls
- imf.xls
- india2004parties.xls
- india2004states.xls
- mozambique1999.xls
- netherlands2003.xls
- south-africa.xls
Democracy and its Mathematical Discontents
Biden v Trump
OUTLINE

- Winning Elections with the Least Popular Vote
- The Electoral College Priority List
- The Huntington Method of Apportionment
- Proportional Methods in the World
  - Divisor Methods
  - Largest Remainders’ Methods
- The Alabama Paradox
- How to Measure Injustice?
The 17 Camel Conundrum

17 Camel Conundrum - An inheritance riddle. Divide 17 camels such that eldest son gets 1/2, middle son gets 1/3 and youngest gets 1/9.

THE MAN WHO COUNTED
A Collection of Mathematical Adventures
Malba Tahan

EL HOMBRE QUE CALCULABA
Malba Tahan

<table>
<thead>
<tr>
<th>Party</th>
<th>Votes</th>
<th>Hare</th>
<th>Droop</th>
<th>Imperiali</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.500</td>
<td>9</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>0.333</td>
<td>6</td>
<td>6</td>
<td>6</td>
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<tr>
<td>3</td>
<td>0.111</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
What if the father left 24 camels? Solution using Voting Theory and Excel

Exercise: What if the father left 35 camels?
Solution in “The Man who counted” By the Brazilian author Malba Tahan, 1895 - 1974

http://math.arizona.edu/~voting-theory
What is the least fraction of the popular vote that will elect either candidate to office?
Winning Fraction

Electoral Votes: 538
Half of Electoral Votes: 269
Needed: 270 EVs to win.

$T$: Total number of votes.
$W$: Votes carried by the winning candidate.
$r_i$: Number of Representatives of state $i$.
$r_i + 2$: Number of electoral votes for state $i$.

Winner needs

$$(r_1 + 2) + (r_2 + 2) + \ldots + (r_s + 2) \geq 270$$
Number of votes cast is proportional to number of representatives

\[ \nu_i = Kr_i \]

\[ K = \frac{T}{435} = \frac{101,463,220}{435} \approx 233,250 \]

To win \((r+2)\) ev’s of a given state, a candidate needs at least \((rK/2) + 1\) votes

\[ W \geq \left(\frac{1}{2}Kr_1 + 1\right) + \left(\frac{1}{2}Kr_2 + 1\right) + \ldots + \left(\frac{1}{2}Kr_s + 1\right) \]

\[ \frac{W}{T} \geq \frac{270 - 2s}{870} + \frac{s}{435K} \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_i)</td>
<td>Number of Reps for state (i)</td>
</tr>
<tr>
<td>(v_i)</td>
<td>Number of votes in state (i)</td>
</tr>
<tr>
<td>(s)</td>
<td>Number of states to win 270 EV’s</td>
</tr>
<tr>
<td>(T)</td>
<td>Total votes</td>
</tr>
<tr>
<td>(W)</td>
<td>Winner's votes</td>
</tr>
</tbody>
</table>
## Largest States

### 2000 Elections

<table>
<thead>
<tr>
<th>State</th>
<th>EV</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>54</td>
</tr>
<tr>
<td>New York</td>
<td>33</td>
</tr>
<tr>
<td>Texas</td>
<td>32</td>
</tr>
<tr>
<td>Florida</td>
<td>25</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>23</td>
</tr>
<tr>
<td>Illinois</td>
<td>22</td>
</tr>
<tr>
<td>Ohio</td>
<td>21</td>
</tr>
<tr>
<td>Michigan</td>
<td>18</td>
</tr>
<tr>
<td>New Jersey</td>
<td>15</td>
</tr>
<tr>
<td>North Carolina</td>
<td>14</td>
</tr>
<tr>
<td>Georgia</td>
<td>13</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>270</strong></td>
</tr>
</tbody>
</table>

\[
\frac{W}{T} \geq \frac{248}{870} \approx 28.81\% 
\]

### 2020 Elections

<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
<th>EV</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>37,253,956</td>
<td>53</td>
</tr>
<tr>
<td>Texas</td>
<td>25,145,561</td>
<td>36</td>
</tr>
<tr>
<td>New York</td>
<td>19,378,102</td>
<td>27</td>
</tr>
<tr>
<td>Florida</td>
<td>18,801,310</td>
<td>27</td>
</tr>
<tr>
<td>Illinois</td>
<td>12,830,632</td>
<td>18</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>12,702,379</td>
<td>18</td>
</tr>
<tr>
<td>Ohio</td>
<td>11,536,504</td>
<td>16</td>
</tr>
<tr>
<td>Michigan</td>
<td>9,883,640</td>
<td>14</td>
</tr>
<tr>
<td>Georgia</td>
<td>9,687,653</td>
<td>14</td>
</tr>
<tr>
<td>North Carolina</td>
<td>9,535,483</td>
<td>13</td>
</tr>
<tr>
<td>New Jersey</td>
<td>8,791,894</td>
<td>12</td>
</tr>
<tr>
<td>Virginia</td>
<td>8,001,024</td>
<td>11</td>
</tr>
<tr>
<td>Washington</td>
<td>6,724,540</td>
<td>10</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>6,547,629</td>
<td>9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>278</strong></td>
</tr>
</tbody>
</table>

\[
\frac{W}{T} \geq 28.05\% 
\]
### Smallest States

<table>
<thead>
<tr>
<th>EV</th>
<th>States</th>
<th>EV*States</th>
<th>sum</th>
<th>sum of ev's</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
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<td>24</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
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<td>48</td>
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<tr>
<td>5</td>
<td>4</td>
<td>20</td>
<td>18</td>
<td>68</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>12</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>21</td>
<td>23</td>
<td>101</td>
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<td>2</td>
<td>20</td>
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<td>11</td>
<td>4</td>
<td>44</td>
<td>37</td>
<td>231</td>
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<tr>
<td>12</td>
<td>2</td>
<td>24</td>
<td>39</td>
<td>255</td>
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<tr>
<td>13</td>
<td>2</td>
<td>26</td>
<td>41</td>
<td>281</td>
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</tbody>
</table>

\[
\frac{W}{T} \geq \frac{190}{870} \approx 21.69\%
\]

### Elections 2020

<table>
<thead>
<tr>
<th>EV</th>
<th>States</th>
<th>EV*States</th>
<th>sum of states</th>
<th>sum of ev</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>8</td>
<td>8</td>
<td>8</td>
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<tr>
<td>2</td>
<td>5</td>
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<td>13</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>16</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>24</td>
<td>22</td>
<td>51</td>
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<td>5</td>
<td>3</td>
<td>15</td>
<td>25</td>
<td>66</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>12</td>
<td>27</td>
<td>78</td>
</tr>
<tr>
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<td>167</td>
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<tr>
<td>10</td>
<td>1</td>
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<td>39</td>
<td>177</td>
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<tr>
<td>11</td>
<td>1</td>
<td>11</td>
<td>40</td>
<td>188</td>
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<td>2</td>
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<td>1</td>
<td>16</td>
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<tr>
<td>18</td>
<td>2</td>
<td>36</td>
<td>47</td>
<td>293</td>
</tr>
</tbody>
</table>

\[
\frac{W}{T} \geq 22.76\%
\]
Number of votes cast is proportional to number of representatives

To win \((r+2)\) ev’s of a given state, a candidate needs at least \((r \frac{K}{2}) + 1\) votes

\[
W \geq \left(\frac{1}{2} Kr_1 + 1\right) + \left(\frac{1}{2} Kr_2 + 1\right) + \ldots + \left(\frac{1}{2} Kr_s + 1\right)
\]

\[
\frac{W}{T} \geq \frac{270 - 2s}{870} + \frac{s}{435K}
\]
Electoral Votes: 537

<table>
<thead>
<tr>
<th>PRESIDENT</th>
<th>EV</th>
<th>STATES</th>
<th>%</th>
<th>VOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>KENNEDY</td>
<td>303</td>
<td>23</td>
<td>49.72</td>
<td>50,456,169</td>
</tr>
<tr>
<td>NIXON</td>
<td>219</td>
<td>26</td>
<td>49.55</td>
<td>50,996,116</td>
</tr>
</tbody>
</table>

Largest States:

\[ \frac{W}{T} \geq 28.03\% \]

Smallest States:

\[ \frac{W}{T} \geq 22.08\% \]
Election 1960

Animal Farm, April 2, 1961 (Washington Post)

Electoral reapportionment in many states had failed to keep up with population shifts. Rural districts with a few inhabitants had greater representation than urban ones. “The integrity of representative government was in many cases endangered.”… In a 1964 decision, regarded by then Chief Justice Earl Warren as “the most important of his tenure”, the Supreme court issued a “one man, one vote” ruling designed to correct the imbalance in representation.

Source: Library of Congress; URL: http://www.loc.gov/
Why 435?

It will surprise some of us that within the framework of the present laws and with population figures not very different from those of the last census, in some freak political constellation, a minority of just a little over 22 per cent could elect the president.

APPLIED PROBLEMS

Few high school students will become mathematicians, but many more will become users of mathematics. Mathematics is used to solve applied problems, that is, practical or scientific problems which are not purely mathematical. Does the high school pay sufficient attention to the needs of prospective users of mathematics?

The problem of formulating a problem. When we treat an applied problem, our first task is to give it a mathematical formulation—express it in mathematical language, reduce it to mathematical concepts. This first task may easily be the most important, the heaviest with consequences, and the most delicate.

\[
W \geq \frac{269 - 2s}{874} + \frac{s}{437N}.
\]

Hence, the minimum value of \(W/T\) will be attained if equality is valid in (4) and the maximum value of \(s\) is attained. Therefore, we have to collect the greatest possible number \(s\) of states which have jointly precisely 269 electoral votes. Obviously, in collecting them we should start with the least populous states, each having just one representative, and then pass successively to the higher values 2, 3, 4, \ldots of \(r\). The necessary simple computations are displayed in Table 2, which should be self-explanatory (but study it before you present it to your class). We find that there are 38 states each with no more than twelve electoral votes apiece and that these 38 states have jointly 260 electoral votes. Remove from these 38 states one which has 4 electoral votes and add one which has 13 electoral votes; the set of 38 states so obtained commands precisely 269 electoral votes. Any set of 39 states, however, has at least 273 electoral votes, and so the desired maximum value of \(s\) is 38. Hence, the minimum value of the fraction of the popular vote that can elect a president, see (4), is

\[
\frac{269 - 76}{874} + \frac{38}{437N} = \frac{193}{874} + \frac{38}{437N} = 0.220824.
\]
Why 435?

How to Win the Presidency With Just 17.56% of the Popular Vote

Chuck Wessell

With the U.S. presidential election fast approaching we will often be reminded that the candidate who receives the most votes is not necessarily elected president. Instead, the winning candidate must receive a majority of the 538 electoral votes awarded by the 50 states and the District of Columbia. Someone with a curious mathematical mind might then wonder: What is the smallest fraction of the popular vote a candidate can receive and still be elected president?

In 1961 George Pólya, who certainly had a curious mathematical mind, considered exactly this question in a paper he published in *The Mathematics Teacher*. Pólya’s formulation of the question is an excellent example of how to simplify a real-world problem so it can be analyzed mathematically. Pólya’s simplification involved making three assumptions:

- The number of votes cast in a state is exactly proportional to the number of that state’s representatives in the U.S. Congress;
- There are two presidential candidates; and
- Each state gives all its electoral votes to the candidate with the largest number of popular votes in that state.

The mathematical argument Pólya made was in the following:

Using (2) to replace the first numerator, we obtain

$$\frac{w}{v} \geq \frac{270 - 2n}{872} + \frac{n}{436k}. \quad (4)$$

Plugging $n = 40$ into (4) and using the 2008 vote total leads to

$$\frac{270 - 2(40)}{872} + \frac{40}{131,370,793} \approx 0.21789.$$
What is the ideal size of a parliament?

Cubic Root Law

Let $N$ denote the number of deputees in the national assembly, in a fixed population $P$. What is the best value of $N$ to maximize the number of communication channels (within the parliament, and with the people)

$$C_{within} = \binom{N}{2} \sim \frac{N^2}{2}$$

$$C_{Populace} = \frac{P}{N}$$

One then needs to minimize the cost

$$C(N) = C_{Populace} + C_{within}$$

$$= \frac{P}{N} + \frac{N^2}{2}$$

This minimum is achieved when

$$N = P^{1/3}.$$ 

This was discovered heuristically studying the size of the British parliament over two hundred years.
Cubic Root Law for Election 2020

The size of a US parliament (unicameral and bicameral) should be: $N = 677$

World Cubic Root Law, $N = 0.336 \, P^{0.3283}$
(Source: [http://world.byvmap.org/Population.html](http://world.byvmap.org/Population.html), downloaded 9 Jan 2013)

$$N = P_a^{0.3238} \text{ with } P_a = 0.036 \, P$$
Suppose we use the exact formula (it does not change much, but for fun):

\[
C(N) = \frac{P}{N} + \binom{N}{2} = \frac{P}{N} + \frac{N(N-1)}{2}
\]

The ideal size of parliament solves the equation

\[
N^3 = P + \frac{N}{2}
\]

This root is given exactly by

\[
N = \sqrt[3]{P} \sum_{k=0}^{\infty} \frac{1}{(2k+1)2^k} \left( \frac{2k+1}{3} \right) P^{-k/3}.
\]

The first few terms read:

\[
N = \sqrt[3]{P} \left( 1 + \frac{1}{6} P^{-1/3} + \frac{1}{36} P^{-2/3} + \frac{1}{324} P^{-1} 
- \frac{1}{11664} P^{-4/3} - \frac{7}{419904} P^{-5/3} + \ldots \right)
\]
1990 Priority List

<table>
<thead>
<tr>
<th>Seq.</th>
<th>State</th>
<th>Seat</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>CA</td>
<td>2</td>
<td>21,099,535.65</td>
</tr>
<tr>
<td>52</td>
<td>NY</td>
<td>2</td>
<td>12,759,391.63</td>
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<tr>
<td>53</td>
<td>CA</td>
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<td>12,181,821.46</td>
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<td>54</td>
<td>TX</td>
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<td>12,063,103.59</td>
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<tr>
<td>55</td>
<td>FL</td>
<td>2</td>
<td>9,194,765.29</td>
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<td>56</td>
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<td>PA</td>
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<td>IL</td>
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<tr>
<td>60</td>
<td>NY</td>
<td>3</td>
<td>7,366,637.51</td>
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<tr>
<td>61</td>
<td>TX</td>
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<td>62</td>
<td>CA</td>
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<td>63</td>
<td>MG</td>
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<tr>
<td>68</td>
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<td>4,924,741.41</td>
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Mike Peters’ Alice. Source: Slate.

Where does this list originate?

Source: Congressional Research Service
### Electoral College

#### 1990 Census

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Article 1, Section 2 of the U.S. Constitution:

Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other persons. …

The Number of Representatives shall not exceed one for every thirty thousand, but each State shall have at least one Representative;
The Huntington Method

Adopted by Congress in 1941, signed by Roosevelt

Recommended by National Academy of Sciences

Morse, Von Neumann & Eisenhart Report to Academy of Sciences

Method Devised in the 1920s based on an earlier scheme by Joseph Hill

Belongs to the Family of Methods of Jefferson, Webster, Adams, and Dean

Mathematical Name: Method of Equal Proportions

Belongs to a larger class of schemes called “Divisor Methods”
**Divisor Methods**

**Equal Proportions**

\[ \infty, \frac{P_1}{\sqrt{2}}, \frac{P_1}{\sqrt{6}}, \frac{P_1}{\sqrt{12}}, \frac{P_1}{\sqrt{20}}, \ldots \]

\[ \frac{P_2}{\sqrt{2}}, \frac{P_2}{\sqrt{6}}, \frac{P_2}{\sqrt{12}}, \frac{P_2}{\sqrt{20}}, \ldots \]

...  

**Adams**

\[ \infty, \frac{P_1}{1}, \frac{P_1}{2}, \frac{P_1}{3}, \frac{P_1}{4}, \ldots \]

\[ \frac{P_2}{1}, \frac{P_2}{2}, \frac{P_2}{3}, \frac{P_2}{4}, \ldots \]

...  

**d’Hondt**

\[ \frac{P_1}{1}, \frac{P_1}{2}, \frac{P_1}{3}, \frac{P_1}{4}, \ldots \]

\[ \frac{P_2}{1}, \frac{P_2}{2}, \frac{P_2}{3}, \frac{P_2}{4}, \ldots \]

...  

**Sainte-Laguë**

\[ \frac{P_1}{1}, \frac{P_1}{3}, \frac{P_1}{5}, \frac{P_1}{7}, \ldots \]

\[ \frac{P_2}{1}, \frac{P_2}{3}, \frac{P_2}{5}, \frac{P_2}{7}, \ldots \]

...
Too Many Names…

\[ n_i = \frac{P_i}{P} \]

- \( P_i \): population of state \( i \) (or votes for party \( i \))
- \( n_i \): Number of Representatives for state \( i \)
- \( P \): Total population of the US (total vote)
- \( N \): total number of representatives (MPs)

Danziger. *Chicago Trib.*
http://www.danzigercartoons.com
LR-Methods

Hare (1859)/Hamilton (1792)

\[ n_i = \left[ \frac{P_i}{P} N \right] \]

Droop (1868)

\[ n_i = \left[ \frac{P_i}{P} (N + 1) \right] \]

LR-Imperiali (1948, Italy)

\[ n_i = \left[ \frac{P_i}{P} (N + 2) \right] \]
The Alabama Paradox

One of three paradoxes discovered under the Hamilton Method (Hare) as the House grew in size.

First observed in 1870: With a house of 270 members, RI got 2 reps; with a house of 280, it lost a seat.

Observed again in 1880: With a house of 299, Alabama would have 8 reps; with a house of 300, it would lose a seat.

"Inspecting the Democratic Curiosity Shop"
September 1, 1880
**Population Paradox: **Observed in 1900: While Virginia was growing much faster than Maine, the first lost a seat, the second gained one.

**New States Paradox: **Observed in 1907 when Oklahoma became a state.

From an initial size of 386, the House was increased to 391. When recalculating the apportionment, Maine gained a seat (from 3 to 4) while New York lost a seat (from 38 to 37).
Polya Representation (d’Hondt)

A 6 seat, 3 party apportionment

600,
510, 501,
420, 411, 402,
330, 321, 312, 303
240, 231, 222, 213, 204
150, 141, 132, 123, 114, 105
060, 051, 042, 033, 024, 015, 006

d’Hondt tends to favor large parties
A 6 seat, 3 party apportionment

600,
510, 501,
420, 411, 402,
330, 321, 312, 303
240, 231, 222, 213, 204
150, 141, 132, 123, 114, 105
060, 051, 042, 033, 024, 015, 006

Webster is intermediate
Polya Representation (Huntington)

A 6 seat, 3 party apportionment

411, 321, 312, 231, 222, 213, 141, 132, 123, 114

Huntington excises away favoring large parties
Mathematical Problem

\[ P_1 \leq P_2 \leq P_3 \cdots \implies N_1 \leq N_2 \leq N_3 \cdots \]

\[ \sum P_i = P \quad \sum N_i = N \]

\[ \frac{P_i}{P} \approx \frac{N_i}{N} \]
Measures of Injustice

\[ \text{Min}_{i,j} \left| \frac{N_i}{P_i} - \frac{N_j}{P_j} \right| \quad \text{... leads to Sainte Laguë (Webster, Arithmetic Mean)} \]

\[ \text{Min}_{i,j} \left| \frac{P_i}{N_i} - \frac{P_j}{N_j} \right| \quad \text{... leads to Dean’s Method (Harmonic Mean)} \]

\[ \text{Min}_{i,j} \left| \frac{N_i/\sqrt{P_i}}{N_j/\sqrt{P_j}} - 1 \right| \quad \text{... leads to Huntington’s Method (Equal Proportions)} \]
Measures of Injustice

\[ \text{Min}_{i,j} \left| \frac{P_i}{N_i} - \frac{P_j}{N_j} \right| \]

... leads to Huntington’s Method (Equal Proportions)

\[ \text{Min}_{i,j} \left| N_i - N_j \frac{P_j}{P_j} \right| \]

... leads to Adam’s Method

\[ \text{Min}_{i,j} \left| P_i - P_j \frac{N_i}{N_j} \right| \]

... leads to Jefferson’s Method
Measures of Injustice

\[ \text{Max}_{i=1,\ldots,s} \text{Min} \frac{P_i}{N_i} \] leads to d’Hondt

\[ \text{Min}_{i=1,\ldots,s} \text{Max} \frac{P_i}{N_i} \] leads to Adam’s
Measures of Injustice

\[ \text{Min } \sum P_i \left( \frac{N_i}{P_i} - \frac{N}{P} \right)^2 \quad \ldots \text{leads to Sainte Laguë} \]

\[ \text{Min } \sum N_i \left( \frac{P_i}{N_i} - \frac{P}{N} \right)^2 \quad \ldots \text{leads to Huntington} \]

\[ \text{Min } \sum \left( \frac{N_i}{N} - \frac{P_i}{P} \right)^2 \quad \ldots \text{leads to Hare} \]
Case of Sainte Laguë (Webster)

Measure of Injustice

\[ Q = \sum P_i \left( \frac{N_i}{P_i} - \frac{N}{P} \right)^2 \]

Sainte Laguë’s scheme follows from the fact that

\[ 1 + 3 + \cdots + (2N - 1) = N^2 \]