The Random Walks of a Mathematician:
Reflections on a Career

Lotfi Hermi
Florida International University

Math Education Seminar
December 4, 2020

Outline

- Mathematics for Business Decision
  - Description of Materials
  - Meet John Sanders… (Loan workout project)
- Learning Technologies and Mathematics Middle East Conference
- ~~Arizona Teacher Institute~~
- Voting Theory
  - John Sanders goes to the voting booth…

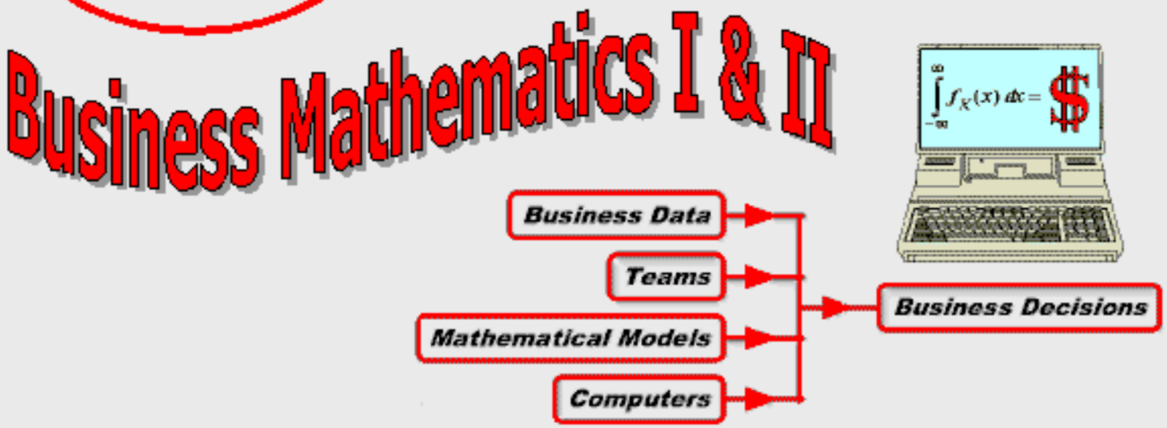
MBD

The electronic texts,
Mathematics for Business Decisions
are published by the
Mathematical Association of America
You can learn more about them.

Development and dissemination of the electronic texts has been partially supported by a grant from the National Science Foundation.

FOCUS on the projects

Department of Mathematics
University of Arizona



Tell me about *Business Mathematics I & II.*

Business Mathematics I: COURSE INFORMATION

Business Mathematics II: COURSE INFORMATION

View or download a two-page COLOR BROCHURE

Grantome

Home Search Services Blog Contact About

The Mathematics of Decision Making: A Model for Collaboration Between Mathematics and Other Fields

McCallum, William Thompson, Richard Lamoureux, Christopher Hughes-Hallett, Deborah

University of Arizona, Tucson, AZ, United States

For Information Contact:

The Mathematical Association of America
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Washington, DC 20036
or
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Abstract

Each year, hundreds of thousands of prospective students in business and public administration (BPA) degree programs take required mathematics courses because they need to learn to use mathematics effectively in making decisions. However, this need is often not addressed by the traditionally required courses. The main goal of this project is to produce materials that convince students that mathematics and modern computer technology are valuable tools in solving realistic problems and to teach them to utilize these tools effectively. We plan to develop, site-test, and nationally disseminate an archive of substantial, technology-based case studies (a realistic scenario which can be used to introduce the mathematical content) in the mathematics of decision-making. These will take between a few days and a few weeks to teach, and can be used instead of or in addition to a text. Each case study will be developed by teams consisting of faculty from mathematics and BPA, to be chosen from around the county. The co-PIs Thompson and Lamoureux, in a partnership between the University's Department of Mathematics and the College of Business and Public Administration, have produced case studies on whether to foreclose a commercial loan or work out an arrangement with the creditor, and on how to price a stock option. Hughes Hallett, in collaboration with E. Connally of Wellesley College and faculty at Harvard University's Kennedy School of Government, has produced case studies on the Austrian state pension system, population projection using US census data and data from the CIA World-book, and water flow in the Charles River using data from the US Geological Survey.

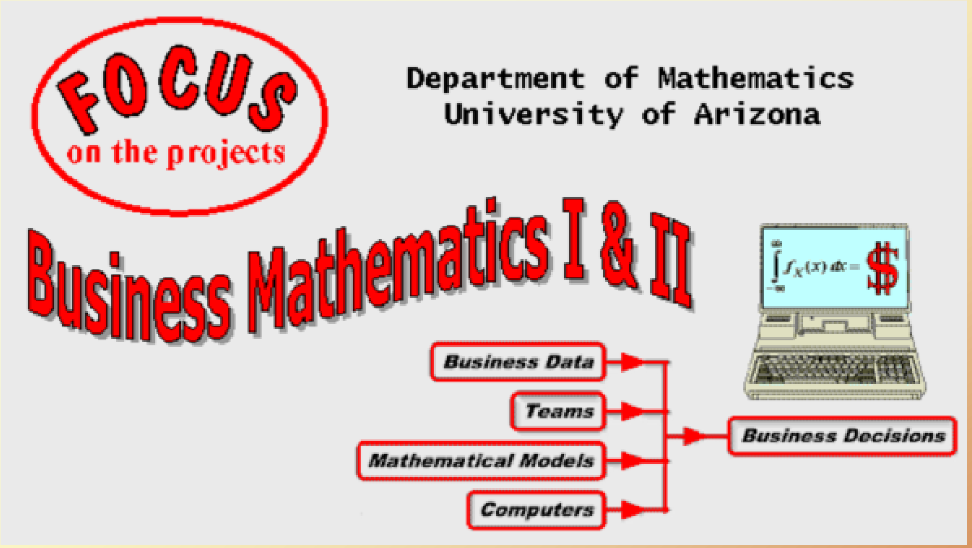
ICTM3

BUSINESS MATHEMATICS WORKSHOP

The University of Arizona Model for Integrating Technology into Learning

Saturday, July 1st, 2006
4:30 to 6:30 p.m.

Anadolu Room
Marmara Hotel
Istanbul, Turkey



***Lotfi Hermi*, Department of Mathematics, University of Arizona**

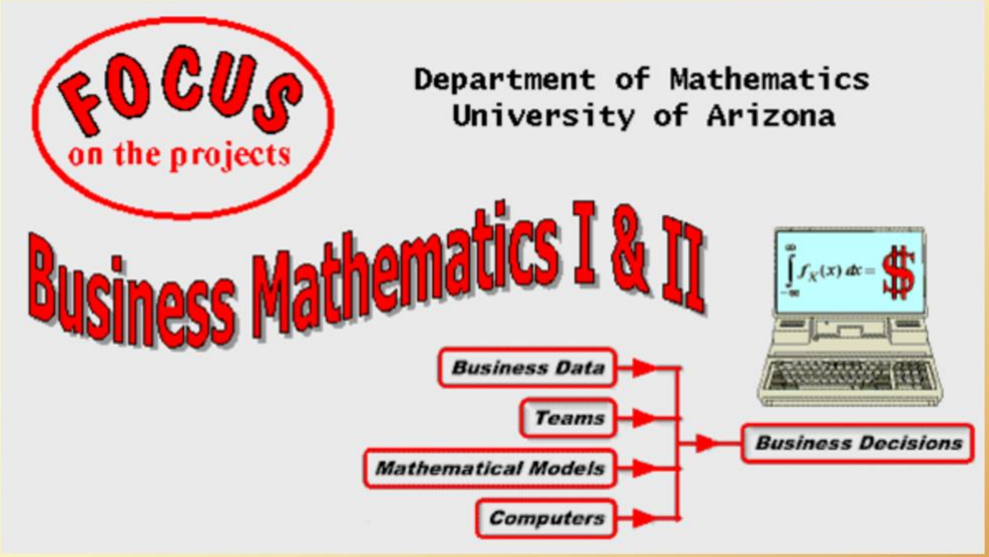
Learning Technologies and Mathematics Middle East Conference

BUSINESS MATHEMATICS WORKSHOP

The University of Arizona Model for Integrating Technology into Learning

Saturday, March 31, 2007
4:30 to 6:00 p.m.

Sultan Qaboos University
Muscat, Oman



Lotfi Hermi, **Department of Mathematics, University of Arizona**

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Department of Mathematics
University of Arizona

Business Mathematics I

Math 115a

Fall Semester 2005

The Course Homepage

Textbook Information

Math 115a Files

Interactive Tutorial

Course Policies

Syllabus: MWF Sections

TuTh Sections

Homework

To Accompany
Mathematics for Business Decisions Part 1
Alternative Edition, Release 1.5a, 2005

The Course Homepage

Most of your course information for ***Math 115a***, ***Business Mathematics I***, will come from the ***Course Homepage***. You can access this web site at

http://business.math.arizona.edu/~busmath

or by clicking on any ***Course Homepage*** box.

Course Homepage

What is on the ***Course Homepage***? Here is a partial list.

❖ General ***information*** about ***Business Mathematics I*** and ***II***.

❖ ***Readiness exercises*** to let you know if you have the necessary computer skills for the course.

❖ Current course ***Announcements***. Be in the know! Check this regularly.

(material continues)

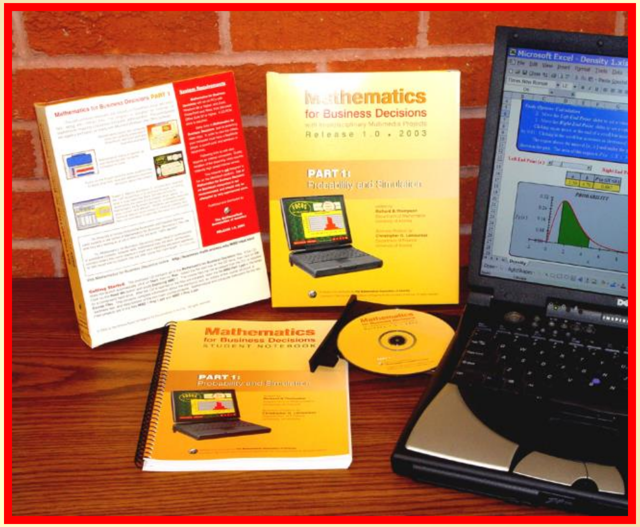
The Course Homepage: *page 2*

- ***Study guides*** and their solutions.
- ***Homework solutions***.
- ***Policies, team assignment, and project information***; provided by the instructor of your individual section of ***Math 115a***.

The last item is of particular importance. Each section of ***Business Mathematics I*** has its own page on the course web site. Your instructor will use this to keep you informed about activities in your own section.

Explore the ***Course Homepage*** now and check it regularly during the semester.

Textbook Information



MATHEMATICS FOR BUSINESS DECISIONS PART 1

Math 115a uses the Alternative Edition, Release 1.5a, of the electronic text ***Mathematics for Business Decisions Part 1***, by Richard Thompson, Christopher Lamoureux, and Pamela Slaten. This is copyrighted by the Arizona Board of Regents and is published by the Mathematical Association of America.

Copies of the e-text can be purchased from the University of Arizona bookstore. The material consists of a set of *PowerPoint*, *Excel* and *Word* files, along with many links to streaming video and internet sites. It is packaged as ***boxed software***, with an ***installation CD*** and a ***Student Notebook***, containing screen captures of all of the *PowerPoint* slides. The CD can be used to install all of the e-text files onto a computer hard drive or as a way to run the files directly from the CD drive on any computer.

SYSTEM REQUIREMENTS

Mathematics For Business Decisions Part 1 will run on **PC's** with *Windows 98* or higher; and *Excel*, *PowerPoint*, and *Word* from *Microsoft Office Suite 2000* or higher. It is ***not*** designed to run on the *Macintosh* platform. Some people have found that the course files run acceptably on *Macintosh* computers, others have had major problems with that operating system. Use the texts on *Macintosh* systems at your own risk.

USING THE CD-ROM

Running From The CD

1. Place the course CD into the CD drive and allow time for the computer to recognize the CD.

2. Left click on the ***My Computer*** desktop icon, or expand the ***My Computer*** folder in *Windows Explorer*.

3. Right click on the icon for the drive containing the course CD, and then left click on ***Open*** in the pull down menu.

4. Double left click on the subfolder ***MBD Part 1a***, and then double left click on its subfolder ***Course Files Alt***.

5. Select and open the *PowerPoint*, *Excel*, or *Word* file that you wish to view. If you are using *PowerPoint* from *Microsoft Office 97* and will be following links to *Excel* or *Word*, open these applications before starting a *PowerPoint* file.

Installing On A Hard Drive

1. Place the course CD into the CD drive and allow time for the computer to recognize the CD.

2. If the CD does not open automatically, click on ***Start***, then on ***Run***. Select the file ***start.exe*** on the CD drive, then click on ***OK***.

3. When the splash page of the CD appears, click on ***Install to Hard Drive***, and follow the on-screen instructions.

4. Read the ***License Agreement*** and, if you are willing to accept its terms, continue.

(material continues)

5. Accept the default installation folder ***C:\MBD Part 1a***, or click ***Change*** to install to a different folder.

6. Use the ***My Computer*** desktop icon or *Windows Explorer* to open the subfolder ***Course Files*** in the folder that you have just installed. All of the files for your e-text are in this folder.

Complete instructions for ***restoring*** or ***uninstalling*** the files are given the file ***Exploring MBD Part 1***. This is available in either *Word* or *PDF* form in the folder ***Read First***.

The main content of ***Mathematics For Business Decisions, Part 1a*** is in three *PowerPoint* files ***MBD Part 1a.ppt***, ***MBD 1 Proj 1a.ppt***, and ***MBD 1 Proj 2a.ppt***. These, and all other instructional files, are located in the folder ***Course Files Alt***. ***MBD Part 1a.ppt*** is the starting point for study of the material. It contains information about the course and its project structure. There is also an extensive section on how to use the electronic text effectively, and a self-grading test over the necessary prerequisite computer skills. All parts of the file may be accessed from links in its ***Table of Contents***, or located with its electronic ***Index***.

COPYRIGHT & REGISTRATION

Since the Mathematical Association of America is a not for profit organization, the e-texts are sold at lower prices than would have resulted from commercial publication. Any royalties paid to Professor Thompson on sales of the material at the University of Arizona will be transferred to the University for further development of the project to benefit business students.

For more information on the e-text visit its web site

http://business.math.arizona.edu/MBD/mbd.html.

Math 115a Files

In addition to the e-text, ***Mathematics for Business Decisions Part 1***, you will use a set of files that apply specifically to ***Math 115a*** at the University of Arizona. Your individual instructor may also create course files for you to download or copy. These will contain information that applies only to your section of the course.

To download the ***Math 115a Files***, go to the ***Course Homepage*** and click on ***Business Mathematics I*** in "Business Mathematics I: COURSE INFORMATION", then click on ***FILES*** in "Download FILES for *Math 115a* at the University of Arizona." The download will be a self-extracting Zipped file, ***M115a.exe***. Save this in a folder or on the Desktop, then double click on the file to extract the full set of ***Math 115a*** files.

The ***Math 115a Files*** and other course material can also be ***viewed*** or ***copied*** onto a *ZIP Disk* or CD from a computer in the ***Information Commons*** in the Integrated Learning Center (***ILC***). For directions, click on

ILC Bus Math Files.ppt.

Math 115a Files: *page 2*

The following files are needed for ***Math 115a*** at the University of Arizona.

PowerPoint

UA Bus Math I.ppt
ILC Bus Math Files.ppt

Word

Study Guide 1.doc
Study Guide 1 Solutions.doc
Study Guide 2.doc
Study Guide 2 Solutions.doc
Team Contract.doc

Interactive Tutorial

An interactive, basic level tutorial program is available on computers in the Integrated Learning Center, ***ILC***. This is a *PowerPoint* presentation, using animations, links to *Excel*, and voice narration to help you get started using your computer text. The tutorial shows how to get the most benefit from your course material, by the effective use of both the ***running*** and ***non-running modes*** of *PowerPoint*.

Topics are described with both audio and visual presentations, and are then illustrated with animations. The presentation pauses while you practice the new operations. A small amount of time spent with the tutorial may help you avoid some of the problems that can occur with the use of unfamiliar software. All of the material is presented at a ***very basic level***, assuming no prior experience with *PowerPoint* or *Excel*.

For directions on running the Interactive Tutorial, click on

ILC Bus Math Files.ppt.

Course Homepage

(material ends)

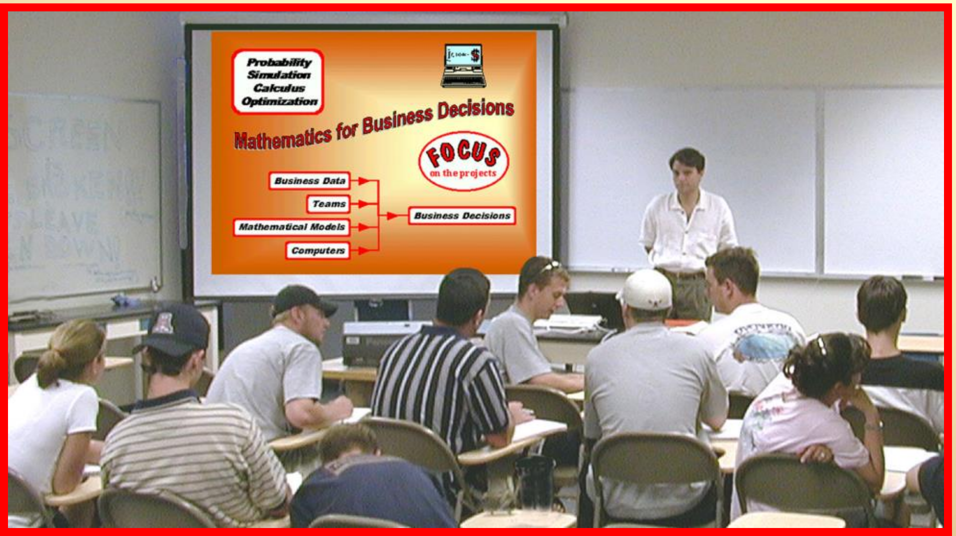
Menu

Course Policies

FALL SEMESTER, 2005

The following items apply to ***all*** sections of ***Math 115a*** at the University of Arizona. Office hours, assignments, final examination locations, and other matters that apply to individual sections of the course are listed under instructors' pages in the ***Course Homepage***.

Kick-Off. The ***Eller College of Management*** is holding a Kick-off for this course on ***Wednesday, September 7 at 5:00 p.m. in ILC 120***, with a make-up session on Saturday, September 10 at 9:00 am in MCLND 133. ***Participation in this event will give credit for Homework 1.***



Course Homepage

(material continues)

Menu

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Presentation Competition. The ***Eller College of Management*** is also organizing a presentation competition. All teams will give their oral reports on ***Project 1*** as shown in the *Syllabus*. For competition purposes, the presentations will be judged by members of the ***Eller Undergraduate Programs Team*** and students from the ***Eller Business Support Lab***. The winners of the first round will advance to the final round, which will be held on ***Saturday, September 22, 2005***. The final round will be judged by members of the local business community, and the first and second place teams will receive prizes. Your instructor will receive the evaluations of your presentation from the judges. However, for course grade purposes, the score for your presentation will be determined by your instructor.

Prerequisites. Successful completion of ***Introduction to Computing*** (***MIS 111***) and ***Business Problem Solving and Analysis*** (***B AD 112L***), and either successful completion of ***College Algebra with Data Analysis*** (***Math 109***), ***College Algebra*** (***Math 110***), or ***College Algebra Accelerated*** (***Math 112***) or an acceptable score on the Mathematics Readiness Test.

Text. ***Mathematics for Business Decisions, Part 1***, Release 1.5a, 2005, by Thompson and Lamoureux, Mathematical Association of America. Your instructor may require you to bring your CD to class on selected dates.

Course Materials Needed. (i) ***Textbook***. (ii) ***Math 115a Files***. (iii) Blank CD-R's, CD-RW's, Zip Disks, or 3-1/2″ Floppy Disks; as needed for reports and homework.

Turnitin*.** You will be required to have or create a ***Turnitin user profile and to join the ***Turnitin*** class created for your section of ***Business Mathematics I***. Your instructor will provide you with the class ID and enrollment password.

Examinations. ***Two midterm examinations*** will be given as shown in the *Syllabus*. ***Final examinations in all sections of Math 115a will be given at a common time, Tuesday, December 13, 2005, from 5-7 p.m.*** Your instructor will notify you about the location of the examination. ***See the Course Homepage, for each instructor's policy on missed examinations.***

Course Policies: *page 4*

Daily Homework. The *Syllabus* lists the topics that are to be studied during each class meeting. The text contains exercises, ***all*** of which should be completed while studying the material.

Graded Homework. Exercise sets will be provided by your instructor and collected on a regular basis. The due dates for these assignments will be announced in class. ***Your instructor will establish a policy on whether graded homework may be submitted by individuals or by teams.***

Reports. Teams will give preliminary and both oral and written final reports on the projects. All written reports must be printout of either word processing or *Excel* workbooks. Any member of a team may be called upon to explain material during the oral report. Your instructor will inform you of the report grading policy in your section.

Additional Work. Your instructor may give credit for announced or unannounced ***quizzes***, and for various types of additional work, including *Excel* assignments.

(material continues)

Course Policies: *page 5*

Course Grades. Each midterm test will be worth 100 points, the homework will be worth 100 points, the two team projects will be worth 150 points each, and the final examination will be worth 200 points. Other work, if any, may earn additional points. Your course grade will be based upon the fraction of the total possible points that you earn. If you earn at least 55% of the total points for individual work ***and*** at least 55% of the total points for team work, the following grading scale will be used.

A: 90% of total points
B: 80% of total points
C: 70% of total points
D: 60% of total points

Otherwise, you will receive a failing grade for the course. For course grade purposes, individual work is defined to include the midterm examinations, the final examination, and the quizzes, if any. Team work is defined to include the preliminary and final reports on the projects and the homework.

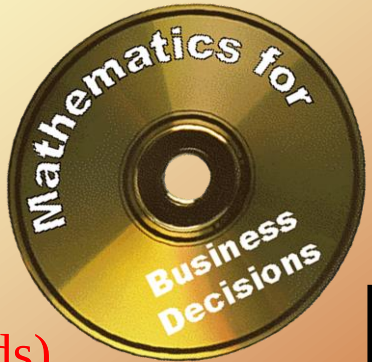
Course Homepage

(material continues)

🡐 ***Menu*** 🡒

Academic Integrity. You are assumed to be familiar with, and to abide by, all University of Arizona policies and procedures, particularly the ***Code of Academic Integrity*** and the ***Student Code of Conduct***. A discussion of these issues in the context of ***Business Mathematics*** is given in the ***Business Mathematics Academic Integrity Policy***. Your instructor may provide additional information about the expectations for your section. Students found to be in violation of any of these policies will be appropriately sanctioned.

Objectives and Structure. ***Business Mathematics I*** is specifically designed to prepare students for subsequent work in their business majors and for their future careers in the business community. To read a full statement of the course objectives and mandatory team and report structure, click on **Objectives and Structure**.



Syllabus, MWF

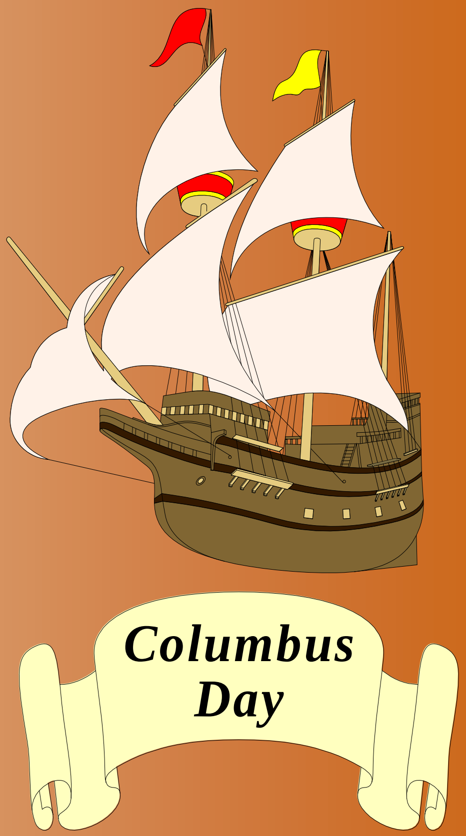
August 22 (M):	Introduction to Course
August 24 (W):	Using the Computer Text (Teams Formed)
August 26 (F):	Introduction to *Project 1*
August 29 (M):	Basic Probability
August 31 (W):	Basic Probability
September 2 (F):	Basic Probability
September 5 (M):	Labor Day – no classes
September 7 (W):	Word Processing Mathematics ***Business Mathematics Kick-Off*** 5:00-6:45 p.m., ILC Room 120.
September 9 (F):	Summation Notation
September 12 (M):	Expected Value
September 14 (W):	Expected Value

(material continues)

Menu

Syllabus, MWF: *page 2*

September 16 (F):	Preliminary Reports on *Project 1*
September 19 (M):	Database Functions and Filtering
September 21 (W):	Database Functions and Filtering
September 23 (F):	Conditional Probability
September 26 (M):	Conditional Probability
September 28 (W):	Bayes' Theorem
September 30 (F):	Bayes' Theorem
October 3 (M):	Management Example
October 5 (W):	***Test 1***, material through September 30
October 7 (F):	Discussion of *Project 1*



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Menu

Syllabus, MWF: *page 3*

October 10 (M):	Reports on *Project 1*
October 12 (W):	Reports on *Project 1*
October 14 (F):	Reports on *Project 1* Last day to drop the course.
October 17 (M):	Introduction to *Project 2*
October 19 (W):	Histograms
October 21 (F):	Histograms
October 22 (Sa):	***Presentation Competition, Final Round***
October 24 (M):	Preliminary Reports on *Project 2*
October 26 (W):	Exponential Growth
October 28 (F):	Exponential Growth
October 31 (M):	Probability Distributions
November 2 (W):	Probability Distributions

(material continues)

← Menu →

Syllabus, MWF: *page 4*

November 4 (F):	Probability Distributions
November 7 (M):	Random Sampling
November 9 (W):	Random Sampling
November 11 (F):	Veteran's Day – no classes
November 14 (M):	Random Sampling
November 16 (W):	Monte Carlo Methods
November 18 (F):	Monte Carlo Methods
November 21 (M):	Monte Carlo Methods
November 23 (W):	Wholesale Ordering Example
November 25 (F):	Thanksgiving Recess – no classes
November 28 (M):	***Test 2***, material from October 17 through November 21
November 30 (W):	Discussion of *Project 2*

(material continues)

Menu

December 2 (F): Reports on *Project 2*

December 5 (M) : Reports on *Project 2*

December 7 (W): Reports on *Project 2*

December 13 (Tu): ***Final Examination***, All course material

Final examinations in all sections of Math 115a will be given at a common time, Tuesday, December 13, 2005, from 5-7 p.m. Your instructor will notify you about the location of the examination.

(material ends)

Syllabus, TuTh

August 23 (Tu):	Introduction to Course Using the Computer Text
August 25 (Th):	Introduction to *Project 1* (Teams Formed)
August 30 (Tu):	Basic Probability
September 1 (Th):	Basic Probability
September 6 (Tu):	Word Processing Mathematics
September 7 (W):	***Business Mathematics Kick-Off*** 5:30-7:30 p.m., ILC Room 120.
September 8 (Th):	Summation Notation Expected Value
September 13 (Tu):	Expected Value
September 15 (Th):	Preliminary Reports on *Project 1* Database Functions and Filtering

(material continues)

Menu

Syllabus, TuTh: *page 2*

September 20 (Tu):	Database Functions and Filtering
September 22 (Th):	Conditional Probability
September 27 (Tu):	Conditional Probability Bayes' Theorem
September 29 (Th):	Bayes' Theorem
October 4 (Tu):	Management Example
October 6 (Th):	***<u>Test 1</u>***, material through September 29 Discussion of *Project 1*
October 11 (Tu):	Reports on *Project 1*
October 13 (Th):	Reports on *Project 1*
October 14 (F):	Last day to drop the course.
October 18 (Tu):	Introduction to *Project 2* Histograms

(material continues)

Syllabus, TuTh: *page 3*

October 20 (Th):	Histograms
October 22 (Sa):	***Presentation Competition, Final Round***
October 25 (Tu):	Preliminary Reports on *Project 2* Exponential Growth
October 27 (Th):	Exponential Growth
November 1 (Tu):	Probability Distributions
November 3 (Th):	Probability Distributions
November 8 (Tu):	Random Sampling
November 10 (Th):	Random Sampling
November 15 (Tu):	Monte Carlo Methods
November 17 (Th):	Monte Carlo Methods
November 22 (Tu):	Wholesale Ordering Example
November 24 (Th):	Thanksgiving Recess – no classes

(material continues)

← *Menu* →

November 29 (Tu): ***Test 2***, material from October 18 through November 17
Discussion of *Project 2*

December 1 (Th): Reports on *Project 2*

December 6 (Tu): Reports on *Project 2*

December 13 (Tu): ***Final Examination***, All course material

Final examinations in all sections of Math 115a will be given at a commontime, Tuesday, December 13, 2005, from 5-7 p.m. Your instructor will notify you about the location of the examination.

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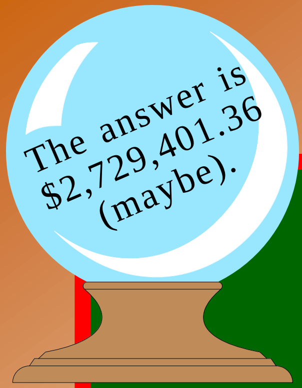
Graded Homework



Graded homework assignments are to be handed in at the start of class on the announced due dates.

These exercises will be listed in *Word* files, which will be posted on your instructor's web page. To complete an assignment, solve the problems, open the appropriate file in *Word*, and enter the required information on the cover page. Word process your solutions, then save and print the file. On some assignments, you will also need to include material copied from *Excel* files that you have created.

Your instructor will indicate whether these assignments are to be completed individually, or by more than one member of a ***currently operating project team***. Credit for the assignment will be given to those students whose signatures appear on the cover page. Work done by members of two or more project teams is not acceptable and can be viewed as plagiarism.

(material ends)

Menu

Meet John Sanders…

FOCUS on the projects

Department of Mathematics
University of Arizona

Business Mathematics I

Math 115a

Spring Semester 2007

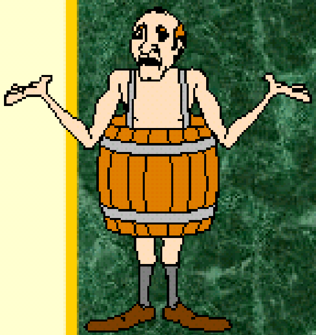
Loan Workout Class Project

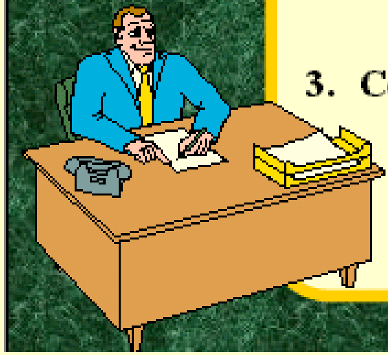
Project 1. *Loan Work Outs*

Video introduction by

Prof. Christopher Lamoureux
Department of Finance
University of Arizona.

1. Project Description
   - A. Business Background
   - B. Class Project
2. Mathematical Tools
   - A. Basic Probability
   - B. Summation Notation
   - C. Expected Value
   - D. Conditional Probability
   - E. Bayes' Theorem
   - F. Management Example
3. Computer Tools
   - A. Word Processing Mathematics
   - B. Database Functions and Filtering





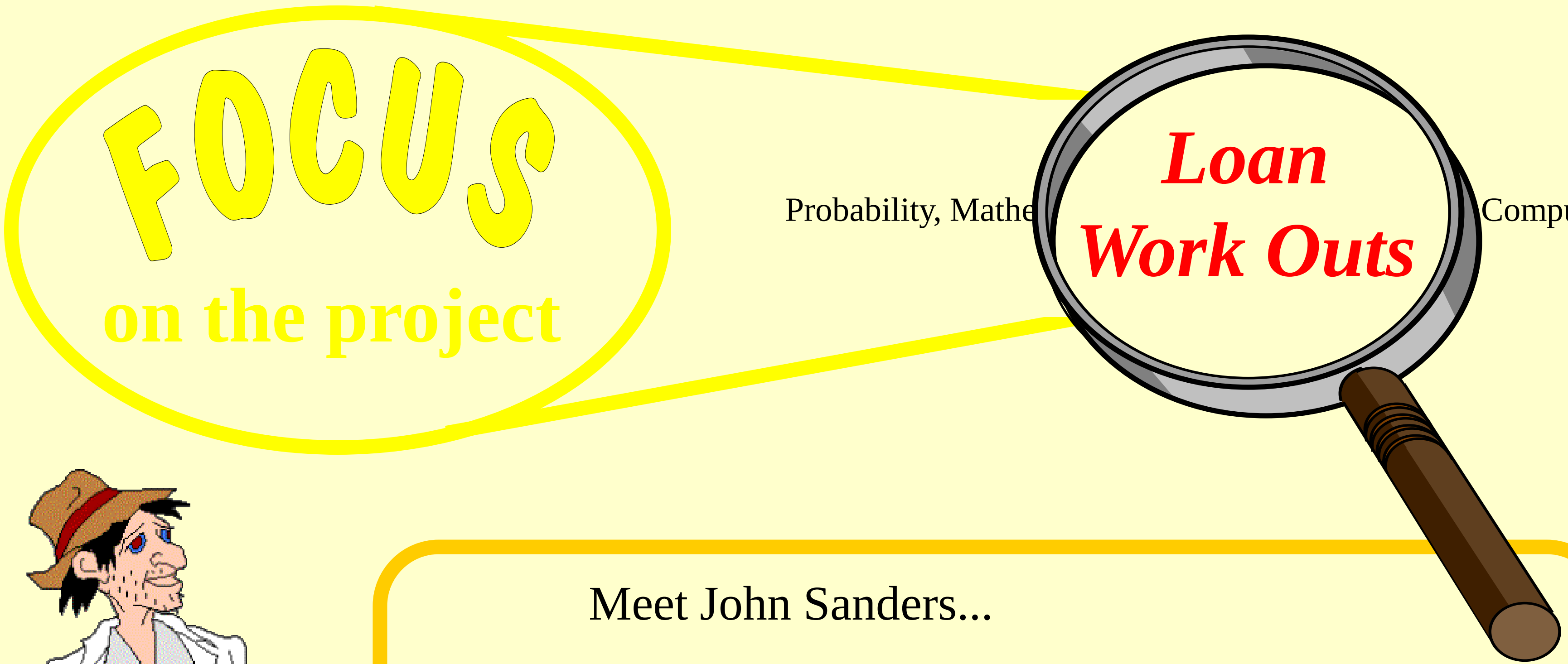
T C I

Loan Records

ACADIA BANK *LOAN RECORDS*

Acadia Bank has records of 8,226 business loans in which work out agreements were made. These come from three banks; BR, Cajun, and DuPont; whose merger formed Acadia. The three former banks each kept records on different aspects of the loans.

Bank Information		Borrower			Result
Customer Number	Former Bank	Years In Business	Education Level	State Of Economy	Loan Paid Back?
1	Cajun		Bachelor's Degree		no
2	BR	11			no
3	BR	13			yes
4	Cajun		Graduate Degree		no
5	DuPont			Normal	no
6	BR	20			no
7	BR	1			yes
8	DuPont			Boom	no
9	BR	7			no
10	BR	12			yes
11	DuPont			Normal	no



Probability, Mathe Computers

Meet John Sanders...

Years in Business	Education	Economy
7 years	Bachelor's	Normal

Loan Focus.xls

FOCUS

on the project

Probability, Mathe

Computers

Loan Details

Full Value	Foreclosure Value	Default Value
$ 4,000,000	$2,100,000	$ 250,000

Loan Records.xls

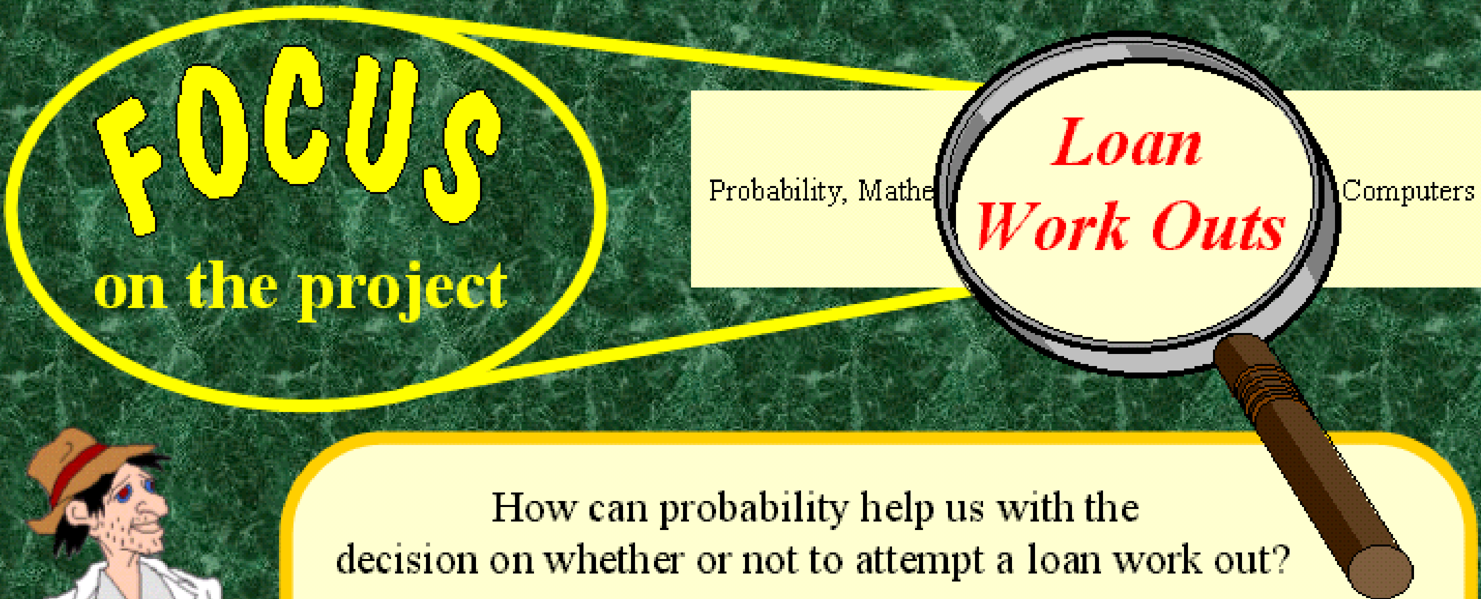
FOCUS
on the project

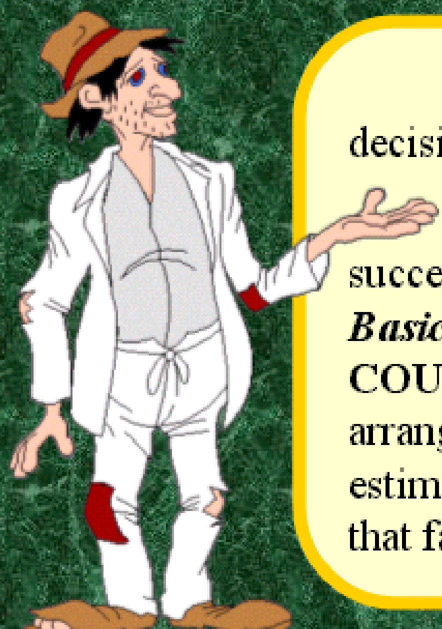
Probability, Mathe Computers

Problem:

Armed with this data, should Acadia Bank enter into a work out agreement with John, or should it foreclose on the loan?

Preliminary Decision





How can probability help us with the decision on whether or not to attempt a loan work out?

Let S be the event that an attempted work out is successful and let F be the event that it fails. In the sheet ***Basic Prob*** of the *Excel* file ***Loan Focus.xls*** we use the **COUNTIF** function to find the fraction of past work out arrangements which were successful. This fraction is our estimate for $P(S)$. Likewise, we find the fraction of attempts that failed and use this as our estimate for $P(F)$.

Loan Focus.xls ***Class Project*** (material continues) ← ***T*** ***C*** ***I*** →

Overall Records

Total Records	8226				
Customer Number	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
Paid Loan	3818		P(S)	46%	
Customer Number	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
					Yes
Did not pay	4408		P(F)	54%	
Customer Number	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
					No

Preliminary Decision

Conditional Probability. *Independent Events: page 8*

Exercise 10 As in the exercises in *Database Functions*; use Y, T, and C as the events that a randomly selected borrower from **Acadia Bank** has the same number of years experience, the same educational level as your team's specific borrower, and that the economic times are the same as in your team's loan, respectively.

Let Z_Y and Z_T be the random variables giving the amounts of money, in dollars, that **Acadia Bank** receives from a future loan work out attempt to a borrower with the same number of years experience or educational level, respectively, as your team's borrower. Let Z_C be the random variable giving the amounts of money, in dollars, that **Acadia Bank** receives from a future loan work out attempt to a borrower in the same economic times as your team's loan.

(i) Use the bank records to estimate $P(S|Y)$, $P(F|Y)$, $P(S|T)$, $P(F|T)$, $P(S|C)$, and $P(F|C)$. (ii) Compute $E(Z_Y)$, $E(Z_T)$, and $E(Z_C)$. See the ***Focus On The Project*** section for the corresponding work with the class project.

Years in the Business

Total Records	239				
Customer Number	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
		7			
Paid Loan	105		P(S|Y)	44%	
Customer Number	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
		7			Yes
Did not pay	134		P(F|Y)	56%	
Customer Number	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
		7			No

Records / Overall SF / **Years** / Training / Conditions

Educational Training

Total Records	1154				

Customer Number	Former Bank	Years In Business	Education Level	State Of Economy	Loan Paid Back?
			Bachelor		

Paid Loan	510		P(S|T)	44%	

Customer Number	Former Bank	Years In Business	Education Level	State Of Economy	Loan Paid Back?
			Bachelor		Yes

Did not pay	644		P(F|T)	56%	

Customer Number	Former Bank	Years In Business	Education Level	State Of Economy	Loan Paid Back?
			Bachelor		No

Records / Overall SF / Years / Training / Conditions

Economic Conditions

Total Records	1547

Customer Number	Former Bank	Years In Business	Education Level	State Of Economy	Loan Paid Back?
				Normal	

Paid Loan	807	P(S|C)	52%

Customer Number	Former Bank	Years In Business	Education Level	State Of Economy	Loan Paid Back?
				Normal	Yes

Did not pay	740	P(F|C)	48%

Customer Number	Former Bank	Years In Business	Education Level	State Of Economy	Loan Paid Back?
				Normal	No

Records / Overall SF / Years / Training / **Conditions**

Preliminary Decision

	Overall Records	Years	Training	Conditions
Success	46%	44%	44%	52%
Failure	54%	56%	56%	48%

What is next?

❖ **How to combine the information?**

$$P(S \mid Y \cap T \cap C) \qquad\qquad P(F \mid Y \cap T \cap C)$$

❖**How to use the loan values (Full Value, Foreclosure Value, Default Value)**

$$E(Z) = P(S \mid Y \cap T \cap C) * FV + P(F \mid Y \cap T \cap C) * DV$$

❖**Business Decision:**

$$Foreclosure \geq E(Z)$$ **Foreclose**

$$Foreclosure < E(Z)$$ **Workout**

Problem

- **These numbers are not accessible**

$$P(S \mid Y \cap T \cap C) \qquad P(F \mid Y \cap T \cap C)$$

- **Bayes' Theorem**

$$P(S \mid Y \cap T \cap C) = \frac{P(Y \cap T \cap C \mid S) * P(S)}{P(Y \cap T \cap C \mid S) * P(S) + P(Y \cap T \cap C \mid F) * P(F)}$$

$$P(F \mid Y \cap T \cap C) = 1 - P(S \mid Y \cap T \cap C)$$

Assumptions

❖ **Events are independent**

$$P(Y \cap T \cap C \mid S) = P(Y \mid S) * P(T \mid S) * P(Y \mid S)$$

$$P(Y \cap T \cap C \mid F) = P(Y \mid F) * P(T \mid F) * P(Y \mid F)$$

❖**Database merger problem: partial info represents what happens in Acadia Bank**

$$P(Y \mid S) \approx P(Y_{BR} \mid S_{BR}) = \frac{P(Y \cap S_{BR})}{P(S_{BR})}$$

What would lead "bank analysts" to reverse their decision?

❖ **Sensitivity analysis: "Range of Years": within 1 and 2 years**

$$P(S \mid Y' \cap T \cap C) \qquad\qquad P(F \mid Y' \cap T \cap C)$$

❖ **What is the least default value that would lead to a reversal of decision?**

$$Foreclose \geq P(S \mid Y \cap T \cap C) * FV + P(F \mid Y \cap T \cap C) * DV$$

$$DV \geq \frac{Foreclose - P(S \mid Y \cap T \cap C) * FV}{P(F \mid Y \cap T \cap C)}$$

LEARNING TECHNOLOGIES AND MATHEMATICS MIDDLE EAST CONFERENCE

SULTAN QABOOS UNIVERSITY
OMAN
MARCH 31-APRIL 2, 2007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Learning Technologies and Mathematics Middle East Conference: Workshops

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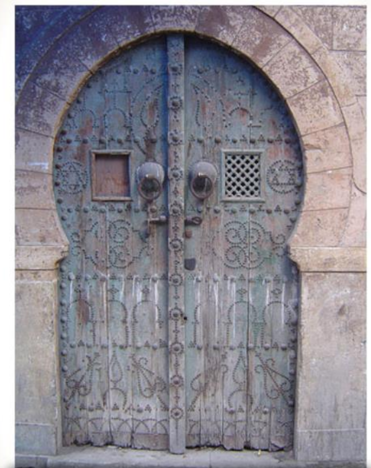
https://www.math.arizona.edu/~atp-mena/conference/presentations/invited.html

De Carthage vers le monde
Le problème isopérimétrique de la reine Didon
et ses ramifications mathématiques

From Carthage to the World
The Isoperimetric Problem of Queen Dido
and its Mathematical Ramifications

Carthage, Tunisia
May 24-29, 2010



CALL FOR APPLICATIONS

Summer 2021 African Diaspora Joint Mathematics Workshop (ADJOINT) in Berkeley, California

Photo: ADJOINT 2020 Participants

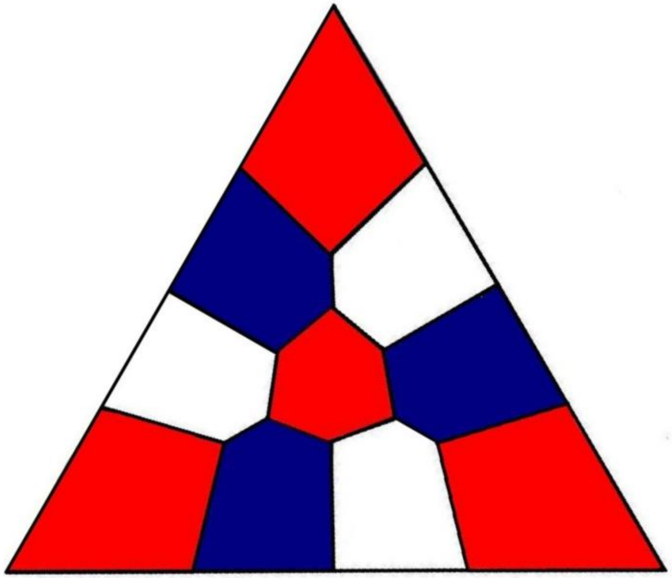
The Mathematical Sciences Research Institute invites applications for its 2021 ADJOINT workshop taking place June 21 - July 2 in Berkeley, California.

John Sanders Votes:
Democracy and its Discontents …

Why Stop at Business Math?

一人一票

A Mathematical Exploration of Apportionment Procedures Around the World



Lotfi Hermi; Deborah Hughes Hallett; William G. McCallum

- *Cover*
- *Preface*
- *Table of Contents*
- CHAPTER I
- CHAPTER II
- CHAPTER III
- CHAPTER IV
- CHAPTER V
- EXCEL SUPPLEMENT
- GLOSSARY
- LINKS

一人一票

A Mathematical Exploration of Apportionment Procedures Around the World

Download Chapter I

CHAPTER I
DEMOCRACY AND ITS MATHEMATICAL DISCONTENTS

EXCEL FILES

- World Voting Systems
- Filtering Data (Use of the "Filter" and "Countif" commands in Excel)
- Women in World Parliaments, April 2004
- South Africa Elections (April 2004)
- injustice.xls

LINKS

- The International IDEA Handbook of Electoral System Design (PDF)
    - The Electoral Systems of Independent States (IDEA, 2002)
    - Basic Kit from IDEA
    - Electoral System Families Flowchart (IDEA)
- The Global Distribution of Electoral Systems (ACE Project)
- IFES **democracy@large** (International Foundation for Election Systems)
- electionguide.org
- **Independent Electoral Commission of South Africa**
- Women in National Parliaments (April 2004)
- **"Ladies First":** Read the reasons why Rwanda has the highest women in parliament rate
- Psephos Election Archive
- News Coverage: Lexis-Nexis Service

一人一票

A Mathematical Exploration of Apportionment Procedures Around the World

COMPLETE LISTING OF EXCEL SUPPLEMENTS

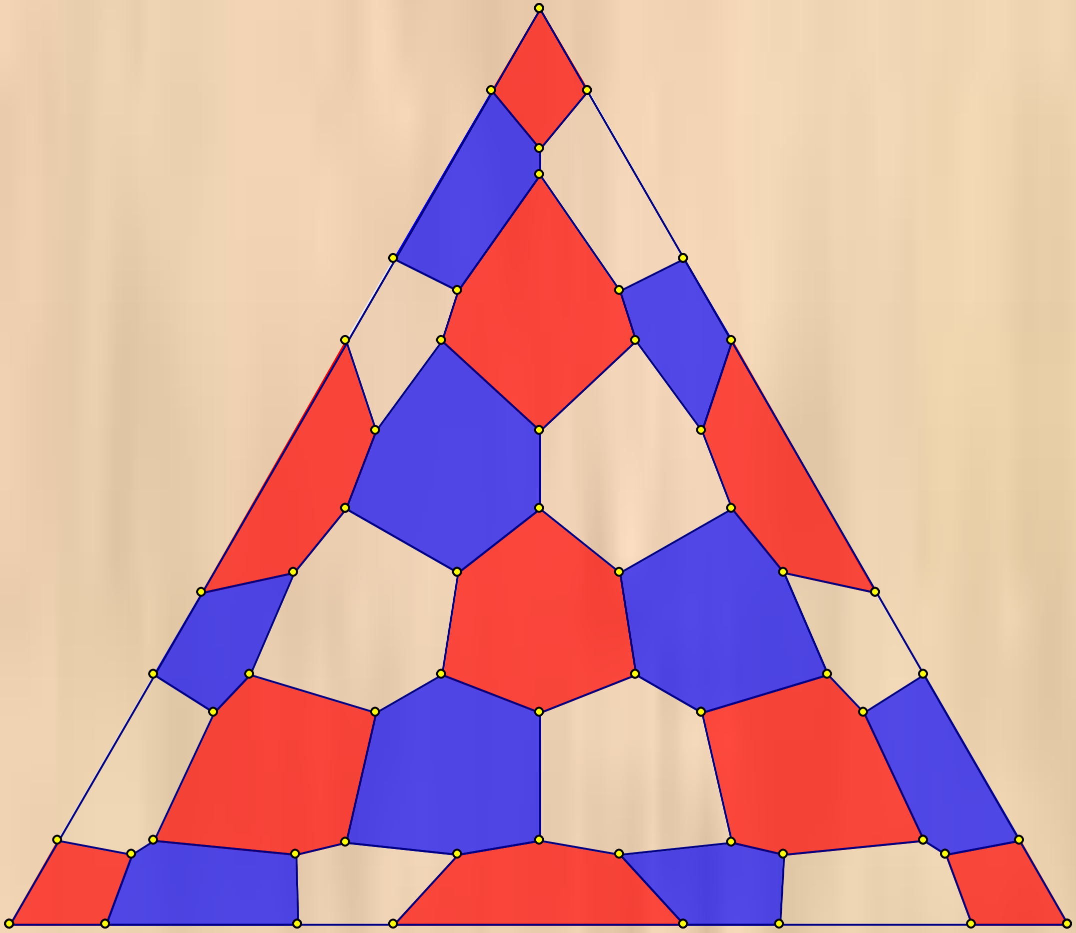
Methods

- largest-remainder.xls (handles up to 300 parties, unlimited number of seats, 703 KB)
- divisor-methods5p700s.xls
  (Eight most commonly used divisor methods for an apportionment of up to 5 parties, 700 seats, 5.6 MB)
- divisor-methods5p1000s.xls
  (Eight most commonly used divisor methods for an apportionment of up to 5 parties, 1000 seats, 7.6 MB)
- dhondt50parties700seats.xls
  (d'Hondt method for an apportionment of up to 50 parties, 700 seats, 6 MB)
- dhondt50parties1000seats.xls
  (d'Hondt method for an apportionment of up to 50 parties, 1000 seats, 8.4 MB)
- sainte-lague50parties700seats.xls
  (Sainte Lague method for an apportionment of up to 50 parties, 700 seats, 6 MB)
- sainte-lague50parties1000seats.xls
  (Sainte Lague method for an apportionment of up to 50 parties, 700 seats, 8.4 MB)
- huntington50parties500seats.xls
  (Huntington method for an apportionment of up to 50 parties, 500 seats, 5.6 MB)
- huntington50parties700seats.xls
  (Huntington method for an apportionment of up to 50 parties, 700 seats, 8.1 MB)
- huntington50parties1000seats.xls
  (Huntington method for an apportionment of up to 50 parties, 1000 seats, 11.6 MB)
- enpp-disproportionality.xls (effective number of political parties, Index of Disproportionality)

Extras

- army-divisions.xls
- imf.xls
- india2004parties.xls
- india2004states.xls
- mozambique1999.xls
- netherlands2003.xls
- south-africa.xls

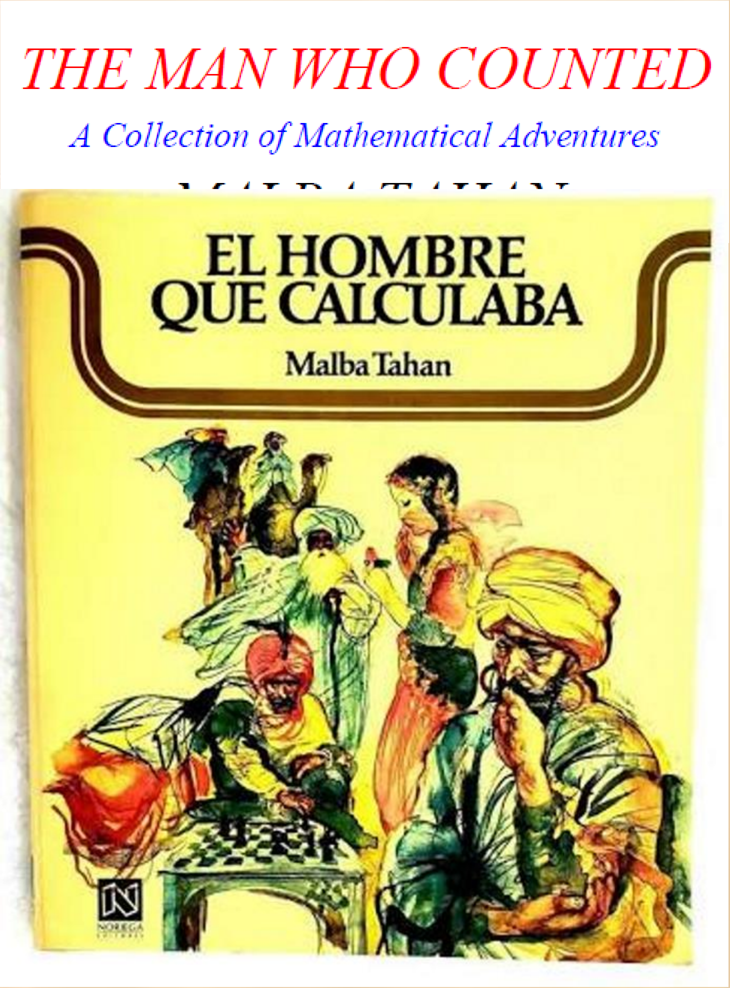
Democracy and its Mathematical Discontents
Biden v Trump



OUTLINE

- Winning Elections with the Least Popular Vote
- The Electoral College Priority List
- The Huntington Method of Apportionment
- Proportional Methods in the World
  - Divisor Methods
  - Largest Remainders’ Methods
- The Alabama Paradox
- How to Measure Injustice?

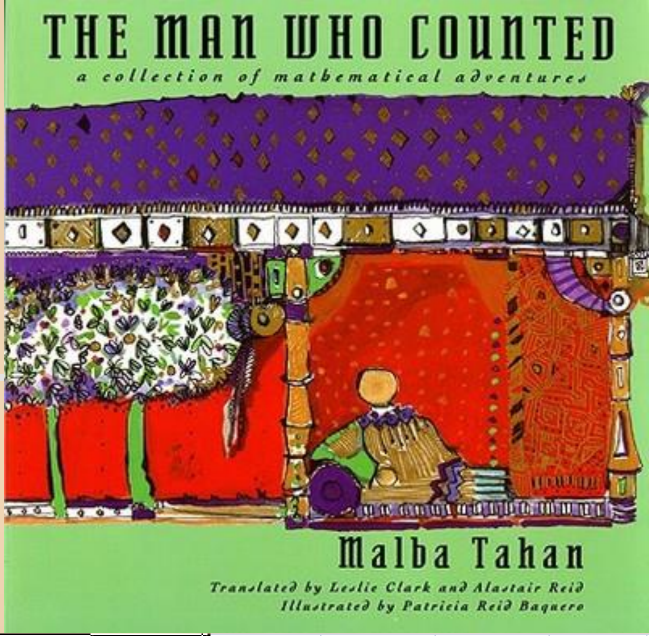
The 17 Camel Conundrum



	Total	17	17	18
Party	Votes	Hare	Droop	Imperiali
1	0.500	9	9	10
2	0.333	6	6	6
3	0.111	2	2	2

What if the father left 24 camels?
Solution using Voting Theory and Excel

Exercise: What if the father left 35 camels?
Solution in "*The Man who counted*"
By the Brazilian author
Malba Tahan, 1895 - 1974



Party	P1	P2	P3	P4	P5
Votes	0.667	0.333	0.125		

OUTPUT

Imperiali

Party	P1	P2	P3	P4	P5	TOTAL
Seats	15	7	2	0	0	24

d'Hondt

Party	P1	P2	P3	P4	P5	TOTAL
Seats	15	7	2	0	0	24

Modified Sainte-Lague

Party	P1	P2	P3	P4	P5	TOTAL
Seats	14	7	3	0	0	24

Sainte-Lague

Party	P1	P2	P3	P4	P5	TOTAL
Seats	14	7	3	0	0	24

Equal Proportions Method

Party	P1	P2	P3	P4	P5	TOTAL
Seats	14	7	3	0	0	24

Danish

Party	P1	P2	P3	P4	P5	TOTAL
Seats	14	7	3	0	0	24

Major fractions

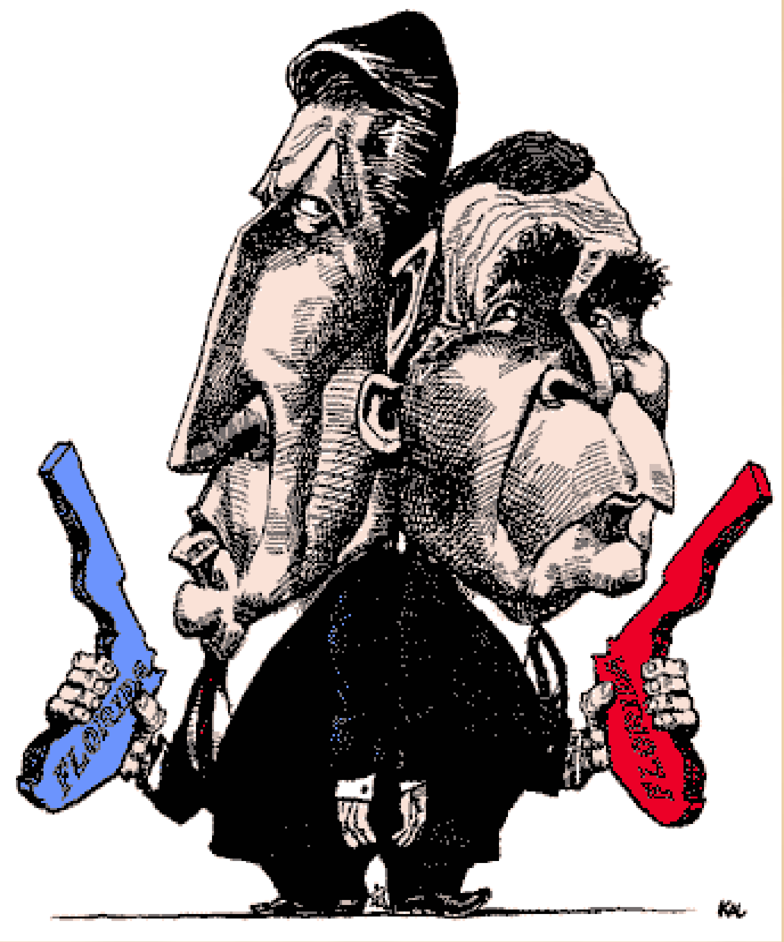
Party	P1	P2	P3	P4	P5	TOTAL
Seats	14	7	3	0	0	24

Adams method

Party	P1	P2	P3	P4	P5	TOTAL
Seats	14	7	3	0	0	24

http://math.arizona.edu/~voting-theory

Election 2000



Florida Shootout. *Source: Slate*.

2000

BUSH vs. GORE

PRESIDENT	EV	STATES	%	VOTES
BUSH	271	30	47.87	50,460,110
GORE	266	21	48.38	51,003,835

Source: http://www.uselectionatlas.org/USPRESIDENT

What is the least fraction of the popular vote that will elect either candidate to office?

Winning Fraction

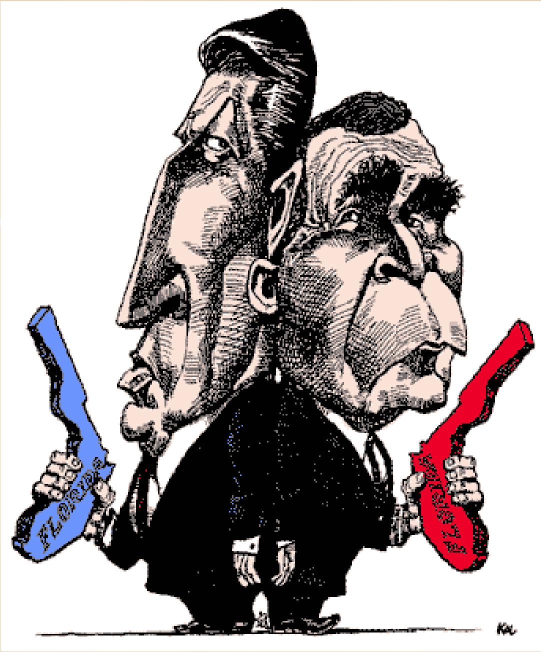
Electoral Votes: 538
Half of Electoral Votes: 269
Needed: 270 EVs to win.
T: Total number of votes.
W: Votes carried by the winning candidate.
r_i: Number of Representatives of state *i*.
r_i +2: Number of electoral votes for state *i*.

Winner needs

$$(\boldsymbol{r}_1 + 2) + (\boldsymbol{r}_2 + 2) + \ldots + (\boldsymbol{r}_s + 2) \geq 270$$

Number of votes cast is proportional to number of representatives

$$v_i = Kr_i$$



$$K = \frac{T}{435} = \frac{101,463,220}{435} \approx 233,250$$

To win *(r+2)* ev's of a given state, a candidate needs at least *(r K/2) + 1* votes

$$W \geq \left(\frac{1}{2}Kr_1 + 1\right) + \left(\frac{1}{2}Kr_2 + 1\right) + \ldots + \left(\frac{1}{2}Kr_s + 1\right)$$

$$\frac{W}{T} \geq \frac{270 - 2s}{870} + \frac{s}{435K}$$

r_i	Number of Reps for state i
v_i	Number of votes in state i
s	Number of states to win 270 EV's
T	Total votes
W	Winner's votes

Largest States

2000 Elections

State	EV
California	54
New York	33
Texas	32
Florida	25
Pennsylvania	23
Illinois	22
Ohio	21
Michigan	18
New Jersey	15
North Carolina	14
Georgia	13
Total	**270**

$$\frac{W}{T} \geq \frac{248}{870} \approx 28.81\%$$

2020 Elections

	State	Population	EV
1	California	37,253,956	53
2	Texas	25,145,561	36
3	New York	19,378,102	27
4	Florida	18,801,310	27
5	Illinois	12,830,632	18
6	Pennsylvania	12,702,379	18
7	Ohio	11,536,504	16
8	Michigan	9,883,640	14
9	Georgia	9,687,653	14
10	North Carolina	9,535,483	13
11	New Jersey	8,791,894	12
12	Virginia	8,001,024	11
13	Washington	6,724,540	10
14	Massachusetts	6,547,629	9
		Total	278

$$\frac{W}{T} \geq 28.05\,\%$$

Smallest States

EV	*States*	EV*States	sum	sum of ev's
3	8	24	8	24
4	6	24	14	48
5	4	20	18	68
6	2	12	20	80
7	3	21	23	101
8	6	48	29	149
9	2	18	31	167
10	2	20	33	187
11	4	44	37	231
12	2	24	39	255
13	2	26	41	281

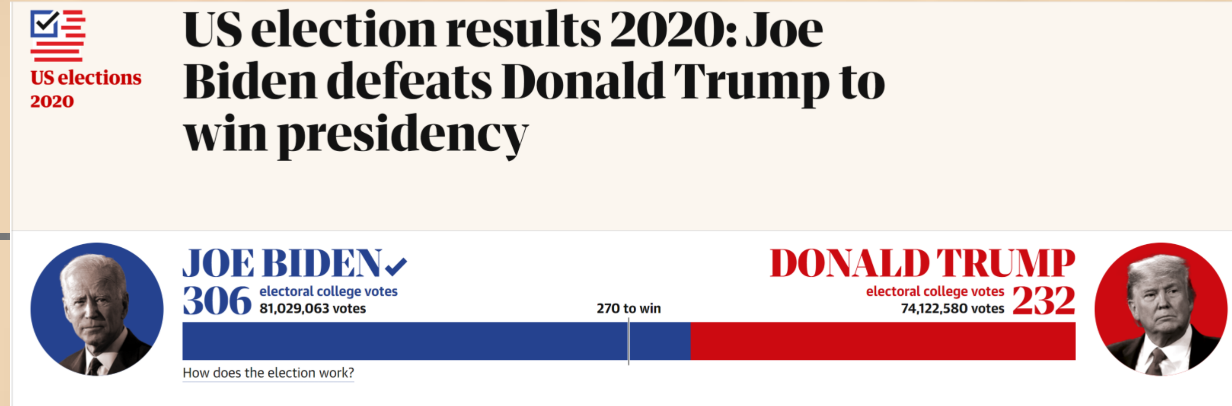
$$\frac{W}{T} \geq \frac{190}{870} \approx 21.69\%$$

Elections 2020

EV	States	EV*States	sum of states	sum of ev
1	8	8	8	8
2	5	10	13	18
3	3	9	16	27
4	6	24	22	51
5	3	15	25	66
6	2	12	27	78
7	3	21	30	99
8	4	32	34	131
9	4	36	38	167
10	1	10	39	177
11	1	11	40	188
12	1	12	41	200
13	1	13	42	213
14	2	28	44	241
16	1	16	45	257
18	2	36	47	293

$$\frac{W}{T} \geq 22.76\,\%$$

Number of votes cast is proportional to number of representatives



$$K = \frac{T}{435} = \frac{155,151,643}{435} \approx 356,670$$

To win *(r+2)* ev's of a given state, a candidate needs at least *(r K/2) + 1* votes

$$W \geq \left(\frac{1}{2}Kr_1 + 1\right) + \left(\frac{1}{2}Kr_2 + 1\right) + \ldots + \left(\frac{1}{2}Kr_s + 1\right)$$

$$\frac{W}{T} \geq \frac{270 - 2s}{870} + \frac{s}{435K}$$

r_i	Number of Reps for state i
v_i	Number of votes in state i
s	Number of states to win 270 EV's
T	Total votes
W	Winner's votes

Electoral Votes: 537

PRESIDENT	EV	STATES	%	VOTES
KENNEDY	303	23	49.72	50,456,169
NIXON	219	26	49.55	50,996,116

Largest States:

$$\frac{W}{T} \geq 28.03\%$$

Smallest States:

$$\frac{W}{T} \geq 22.08\%$$

Election 1960

Animal Farm, April 2, 1961 (Washington Post)

Electoral reapportionment in many states had failed to keep up with population shifts. Rural districts with a few inhabitants had greater representation than urban ones. "The integrity of representative government was in many cases endangered."… In a 1964 decision, regarded by then Chief Justice Earl Warren as "the most important of his tenure", the Supreme court issued a "one man, one vote" ruling designed to correct the imbalance in representation.

Source: Library of Congress; URL: http://www.loc.gov/

Why 435?

It will surprise some of us that within the framework of the present laws and with population figures not very different from those of the last census, in some freak political constellation, a minority of just a little over 22 per cent could elect the president.

Applied problems

Few high school students will become mathematicians, but many more will become users of mathematics. Mathematics is used to solve *applied problems*, that is, practical or scientific problems which are not purely mathematical. Does the high school pay sufficient attention to the needs of prospective users of mathematics?

The problem of formulating a problem. When we treat an applied problem, our first task is to give it a mathematical formulation—express it in mathematical language, reduce it to mathematical concepts. This first task may easily be the most important, the heaviest with consequences, and the most delicate.

JSTOR

*The minimum fraction of the popular vote that can elect the President of the United States**

G. PÓLYA, *Stanford University, Stanford, California.*

"When we treat an applied problem, our first task is to give it a mathematical formulation."

$$\frac{W}{T} \geq \frac{269-2s}{874} + \frac{s}{437N}. \qquad (4)$$

Hence, the minimum value of W/T will be attained if equality is valid in (4) *and* the maximum value of s is attained. Therefore, we have to collect the greatest possible number s of states which have jointly precisely 269 electoral votes. Obviously, in collecting them we should start with the least populous states, each having just one representative, and then pass successively to the higher values 2, 3, 4, $\cdots$ of r. The necessary simple computations are displayed in Table 2, which should be self-explanatory (but study it before you present it to your class). We find that there are 38 states each with no more than twelve electoral votes apiece and that these 38 states have jointly 260 electoral votes. Remove from these 38 states one which has 4 electoral votes and add one which has 13 electoral votes; the set of 38 states so obtained commands precisely 269 electoral votes. Any set of 39 states, however, has at least 273 electoral votes, and so the desired maximum value of s is 38. Hence, the minimum value of the fraction of the popular vote that can elect a president, see (4), is

$$\frac{269-76}{874} + \frac{38}{437N} = \frac{193}{874} + \frac{38}{437N} = 0.220824.$$

Why 435?

270

How to Win the Presidency With Just 17.56% of the Popular Vote

Chuck Wessell

With the U.S. presidential election fast approaching we will often be reminded that the candidate who receives the most votes is not necessarily elected president. Instead, the winning candidate must receive a majority of the 538 electoral votes awarded by the 50 states and the District of Columbia. Someone with a curious mathematical mind might then wonder: What is the smallest fraction of the popular vote a candidate can receive and still be elected president?

In 1961 George Pólya, who certainly had a curious mathematical mind, considered exactly this question in a paper in he published in *The Mathematics Teacher*. Pólya's formulation of the question is an excellent example of how to simplify a real-world problem so it can be analyzed mathematically. Pólya's simplification involved making three assumptions:

- The number of votes cast in a state is *exactly* proportional to the number of that state's representatives in the U.S. Congress;
- There are two presidential candidates; and
- Each state gives all its electoral votes to the candidate with the largest number of popular votes in that state.

The mathematical argument Pólya made was in the

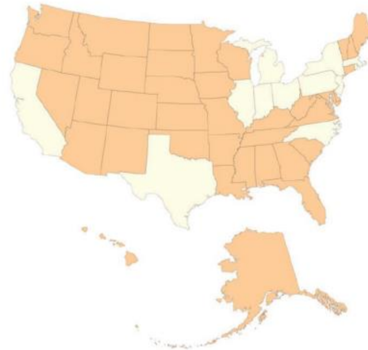


Figure 1. Pólya's 1960 solution estimated that by winning the 38 orange-shaded states by the smallest possible margin while receiving no votes in the unshaded states a candidate could be elected president with 22.1 percent of the popular vote. Using the actual vote totals from that election one can show that the 1960 winner could have received as little as 19.12 percent of the popular vote.

To apply Pólya's methodology to the 2012 election requires a slight reworking of his solution to take into account the 23rd Amendment, which gave electoral votes

Using (2) to replace the first numerator, we obtain

$$\frac{w}{v} \geq \frac{270-2n}{872} + \frac{n}{436k}. \qquad (4)$$

Plugging $n = 40$ into (4) and using the 2008 vote total leads to

$$\frac{270-2(40)}{872} + \frac{40}{131{,}370{,}793} \approx 0.21789.$$

What is the ideal size of a parliament?

Cubic Root Law

Let N denote the number of deputees in the national assembly, in a fixed population P. What is the best value of N to maximize the number of communication channels (within the parliament, and with the people)

$$C_{within} = \binom{N}{2} \sim \frac{N^2}{2}$$

$$C_{Populace} = \frac{P}{N}$$

One then needs to minimize the cost

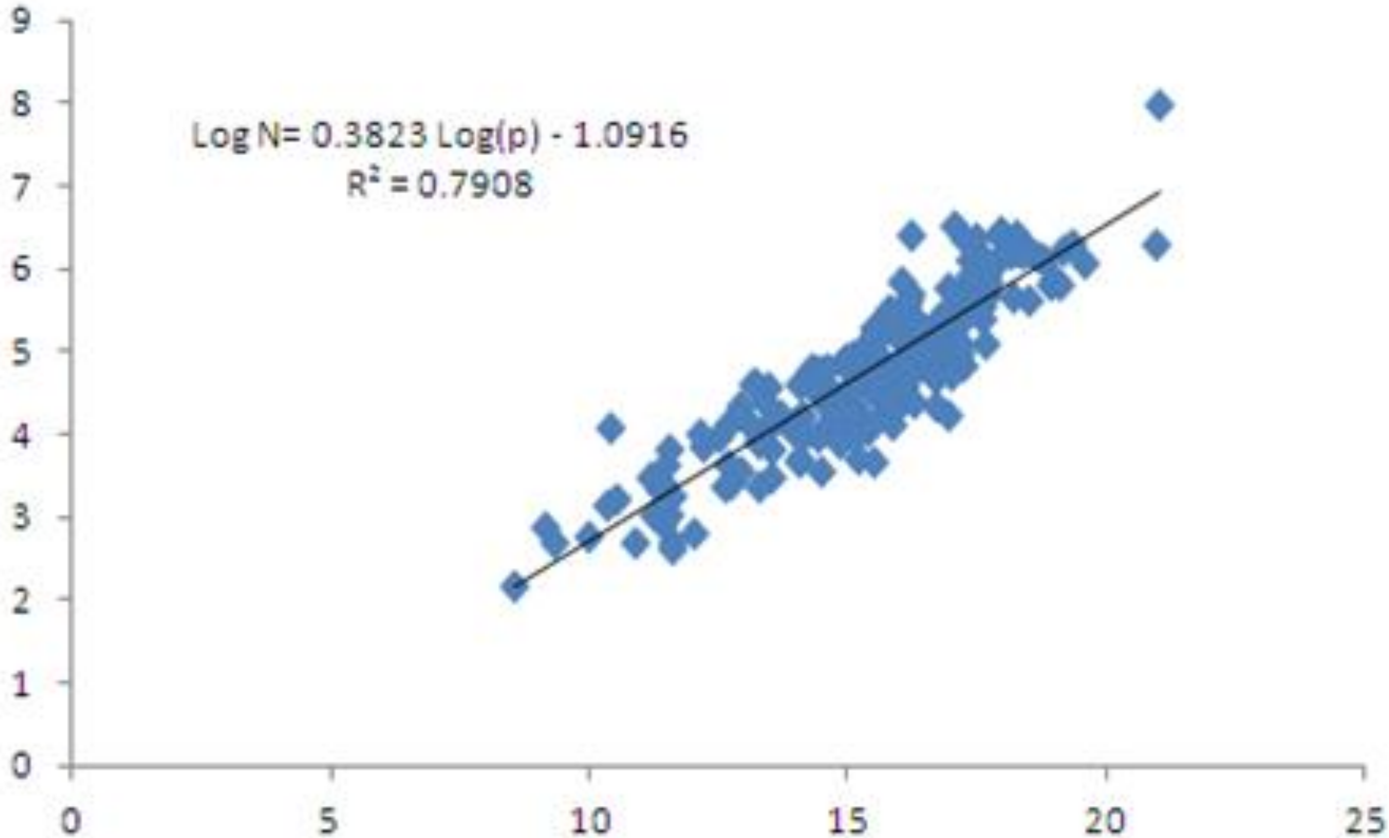
$$\begin{aligned} C(N) &= C_{Populace} + C_{within} \\ &= \frac{P}{N} + \frac{N^2}{2} \end{aligned}$$

This minimum is achieved when

$$N = P^{1/3}.$$

This was discovered heuristically studying the size of the British parliament over two hundred years.

Cubic Root Law for Election 2020
The size of a US parliament (unicameral and bicameral) should be: $N = 677$



World Cubic Root Law, $N = 0.336\ P^{0.3283}$
(Source: http://world.bymap.org/Population.html, downloaded 9 Jan 2013)

$N = P_a^{0.3238}$ with $P_a = 0.036\ P$

Suppose we use the exact formula
(it does not change much, but for fun):

$$C(N) = \frac{P}{N} + \binom{N}{2} = \frac{P}{N} + \frac{N(N-1)}{2}$$

The ideal size of parliament solves the equation

$$N^3 = P + \frac{N}{2}$$

This root is given exactly by

$$N = \sqrt[3]{P} \sum_{k=0}^{\infty} \frac{1}{(2k+1)2^k} \binom{\frac{2k+1}{3}}{k} P^{-k/3}.$$

The first few terms read:

$$\begin{aligned} N = \quad & \sqrt[3]{P}\Big(1 + \frac{1}{6}P^{-1/3} + \frac{1}{36}P^{-2/3} + \frac{1}{324}P^{-1} \\ & - \frac{1}{11664}P^{-4/3} - \frac{7}{419904}P^{-5/3} + \ldots\Big) \end{aligned}$$

Electoral College

1990 Priority List

Seq.	State	Seat	Priority
51	CA	2	21,099,535.65
52	NY	2	12,759,391.63
53	CA	3	12,181,821.46
54	TX	2	12,063,103.59
55	FL	2	9,194,765.29
56	CA	4	8,613,849.35
57	PA	2	8,432,043.16
58	IL	2	8,108,168.46
59	OH	2	7,698,501.20
60	NY	3	7,366,637.51
61	TX	3	6,964,635.46
62	CA	5	6,672,258.17
63	MG	2	6,596,446.31
64	NJ	2	5,479,111.55
65	CA	6	5,447,875.79
66	FL	3	5,308,599.72
67	NY	4	5,208,999.81
68	TX	4	4,924,741.41

Source: Congressional Research Service

Mike Peters' *Alice*. Source: Slate.

Where does this list originate?

Electoral College

1990 Census

State	Population	Sequence		
California	$29,760,021$	$\frac{29,760,021}{\sqrt{2}}$	$\frac{29,760,021}{\sqrt{6}}$	$\frac{29,760,021}{\sqrt{12}}$
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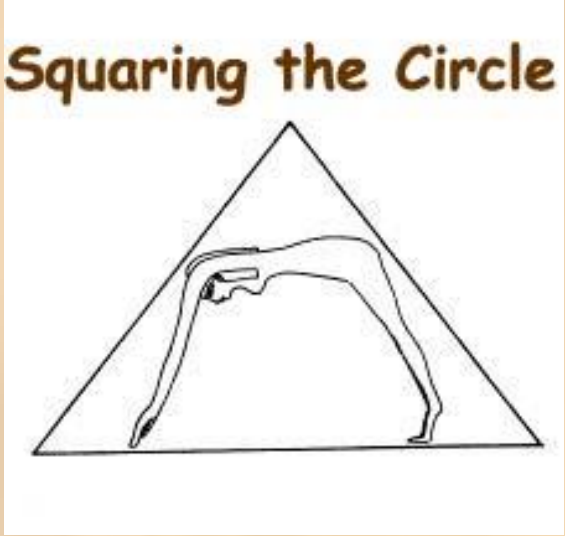
Texas	$16,986,510$	$\frac{16,986,510}{\sqrt{2}}$	$\frac{16,986,510}{\sqrt{6}}$	$\frac{16,986,510}{\sqrt{12}}$
Florida	$12,937,926$	$\frac{12,937,926}{\sqrt{2}}$	$\frac{12,937,926}{\sqrt{6}}$	$\frac{12,937,926}{\sqrt{12}}$
Pennsylvania	$11,881,643$	$\frac{11,881,643}{\sqrt{2}}$	$\frac{11,881,643}{\sqrt{6}}$	$\frac{11,881,643}{\sqrt{12}}$



Sequence of divisors: $0, \sqrt{2}, \sqrt{6}, \sqrt{12}, \sqrt{20}, \sqrt{30}, \sqrt{42}, \cdots, \sqrt{\boldsymbol{n}(\boldsymbol{n}-1)}, \cdots$

Electoral College

1990 Census

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∞ and the American polity

Article 1, Section 2 of the U.S. Constitution:

Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other persons. …

The Number of Representatives shall not exceed one for every thirty thousand, but each State shall have at least one Representative;

The Huntington Method

Adopted by Congress in 1941, signed by Roosevelt

Recommended by National Academy of Sciences

Morse, Von Neumann & Eisenhart Report to Academy of Sciences

Method Devised in the 1920s based on an earlier scheme by Joseph Hill

Belongs to the Family of Methods of Jefferson, Webster, Adams, and Dean

Mathematical Name: Method of Equal Proportions

Belongs to a larger class of schemes called "Divisor Methods"

Divisor Methods

Equal Proportions

$$\infty, \frac{P_1}{\sqrt{2}}, \frac{P_1}{\sqrt{6}}, \frac{P_1}{\sqrt{12}}, \frac{P_1}{\sqrt{20}}, \ldots$$

$$\infty, \frac{P_2}{\sqrt{2}}, \frac{P_2}{\sqrt{6}}, \frac{P_2}{\sqrt{12}}, \frac{P_2}{\sqrt{20}}, \ldots$$

$\ldots$

Adams

$$\infty, \frac{P_1}{1}, \frac{P_1}{2}, \frac{P_1}{3}, \frac{P_1}{4}, \ldots$$

$$\infty, \frac{P_2}{1}, \frac{P_2}{2}, \frac{P_2}{3}, \frac{P_2}{4}, \ldots$$

$\ldots$

d'Hondt

$$\frac{P_1}{1}, \frac{P_1}{2}, \frac{P_1}{3}, \frac{P_1}{4}, \ldots$$

$$\frac{P_2}{1}, \frac{P_2}{2}, \frac{P_2}{3}, \frac{P_2}{4}, \ldots$$

$\ldots$

Sainte-Laguë

$$\frac{P_1}{1}, \frac{P_1}{3}, \frac{P_1}{5}, \frac{P_1}{7}, \ldots$$

$$\frac{P_2}{1}, \frac{P_2}{3}, \frac{P_2}{5}, \frac{P_2}{7}, \ldots$$

$\ldots$

Too Many Names…

P_i : population of state *i* (or votes for party *i*)
n_i : Number of Representatives for state *i*
P : Total population of the US (total vote)
N : total number of representatives (MPs)

Danziger. *Chicago Trib.*
http://www.danzigercartoons.com

$$n_i = \frac{P_i}{P} N = \frac{P_i}{P / N}$$

LR-Methods

Hare (1859)/Hamilton (1792)

$$n_i = \left[\frac{P_i}{P} N \right]$$

Droop (1868)

$$n_i = \left[\frac{P_i}{P} (N+1) \right]$$

LR-Imperiali (1948, Italy)

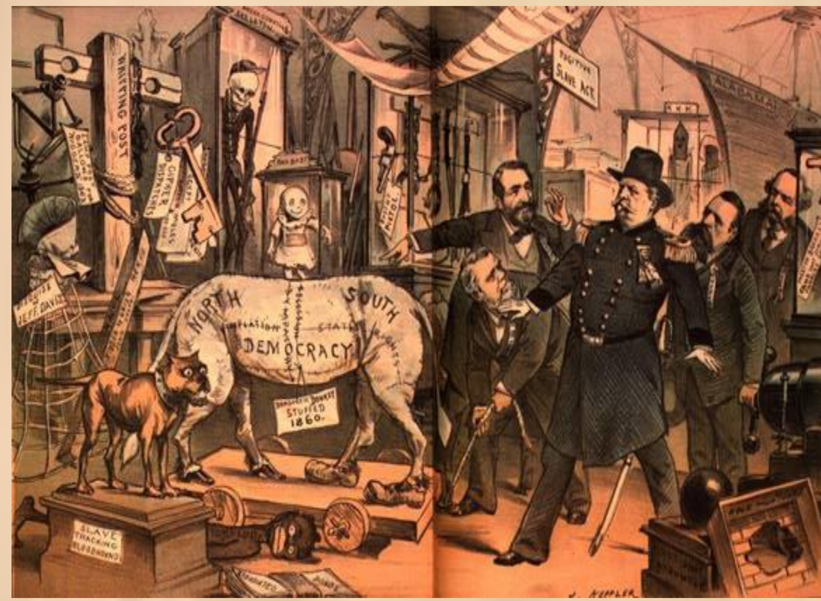
$$n_i = \left[\frac{P_i}{P} (N+2) \right]$$

The Alabama Paradox

One of three paradoxes discovered under the Hamilton Method (Hare) as the House grew in size

First observed in 1870: With a house of 270 members, RI got 2 reps; with a house of 280, it lost a seat

Observed again in 1880: With a house of 299, Alabama would have 8 reps; with a house of 300, it would lose a seat.



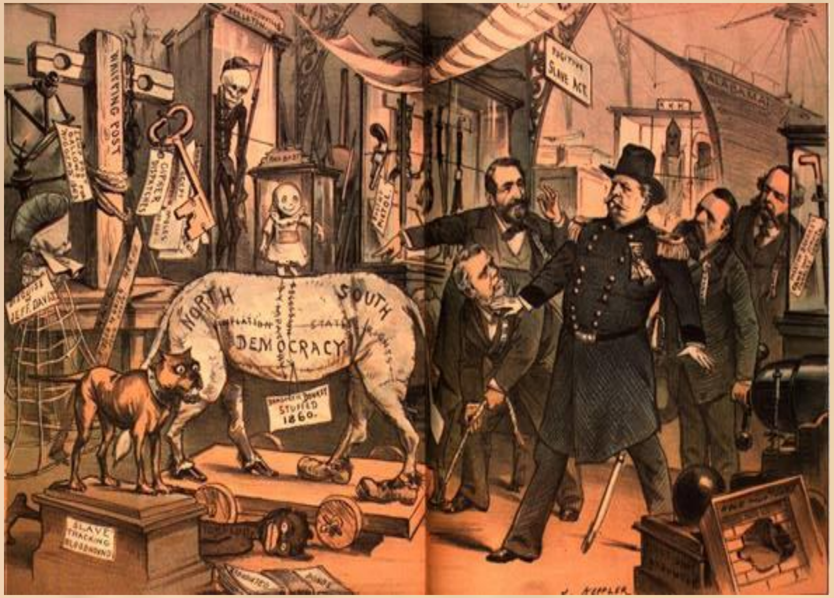
"Inspecting the Democratic Curiosity Shop"
September 1, 1880

Other Paradoxes

Population Paradox: *Observed in 1900: While Virginia was growing much faster than Maine, the first lost a seat, the second gained one.*

New States Paradox: *Observed in 1907 when Oklahoma became a state.*

From an initial size of 386, the House was increased to 391. When recalculating the apportionment, Maine gained a seat (from 3 to 4) while New York lost a seat (from 38 to 37).



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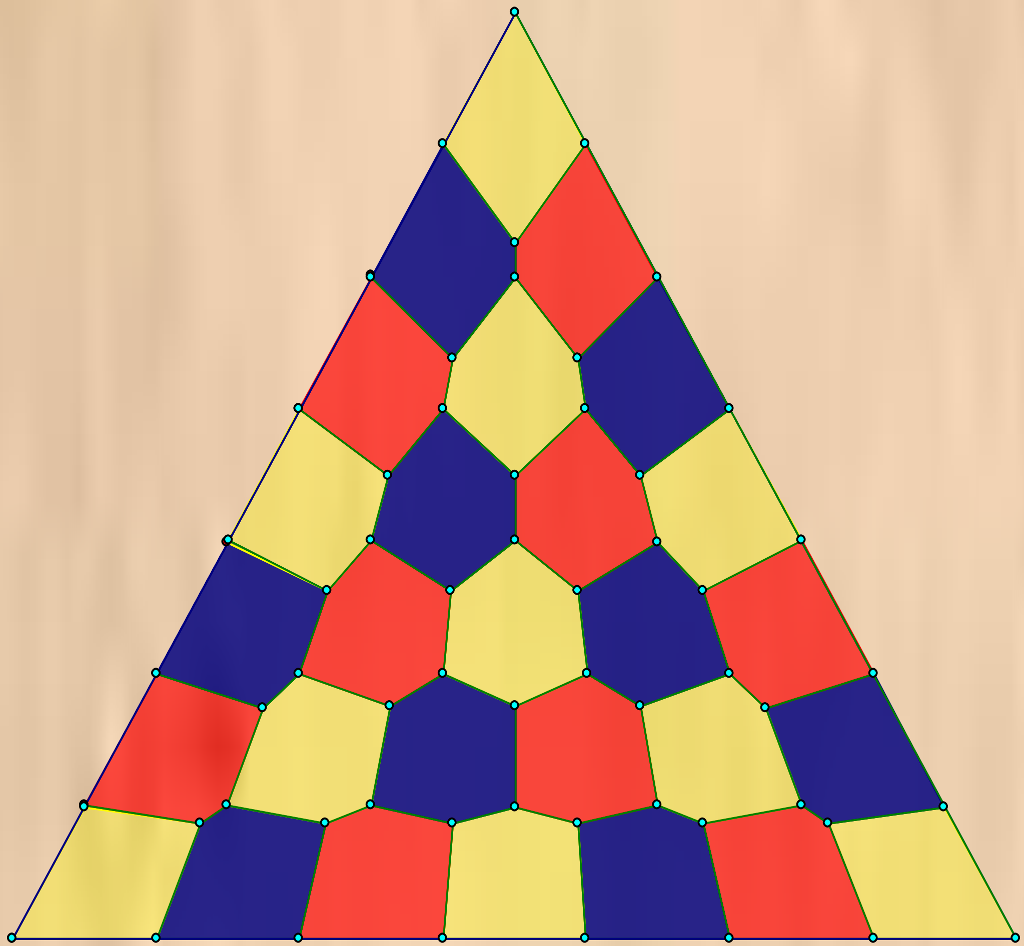
Polya Representation (d'Hondt)

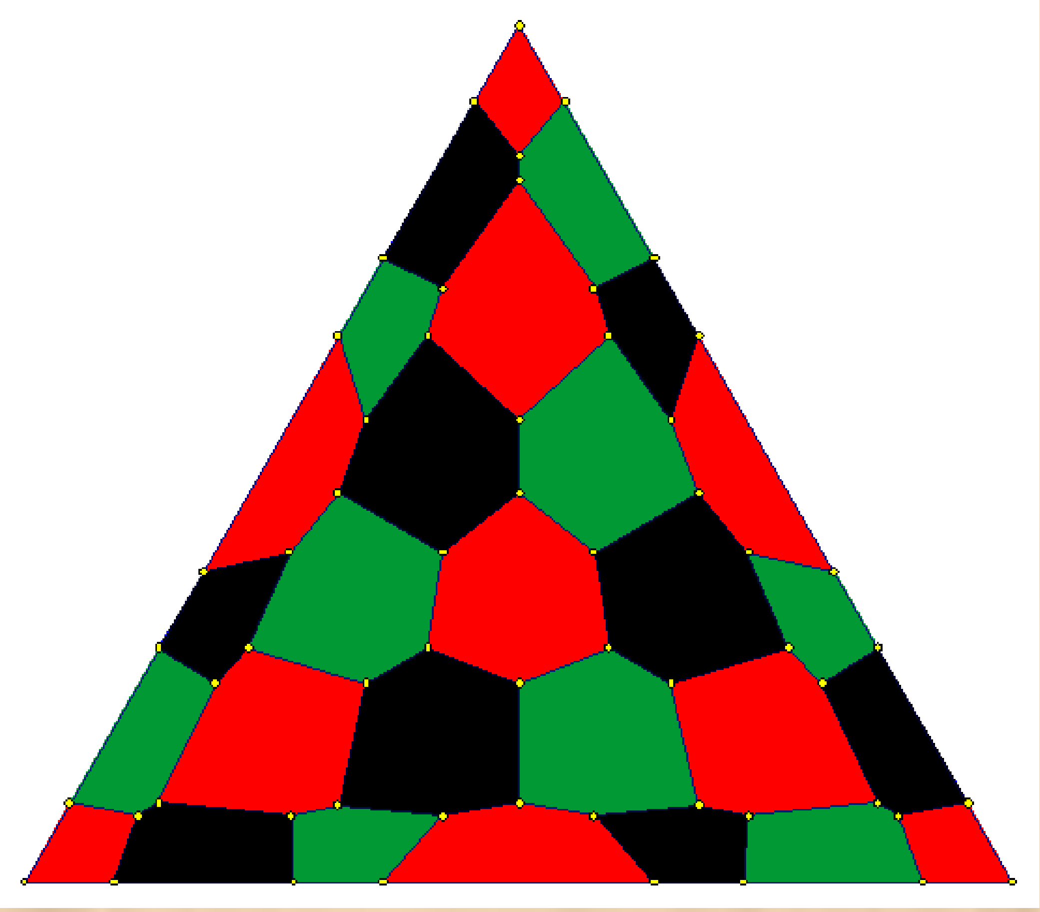


A 6 seat, 3 party apportionment

600,
510, 501,
420, 411, 402,
330, 321, 312, 303
240, 231, 222, 213, 204
150, 141, 132, 123, 114, 105
060, 051, 042, 033, 024, 015, 006

d'Hondt tends to favor large parties

Polya Representation (Webster)



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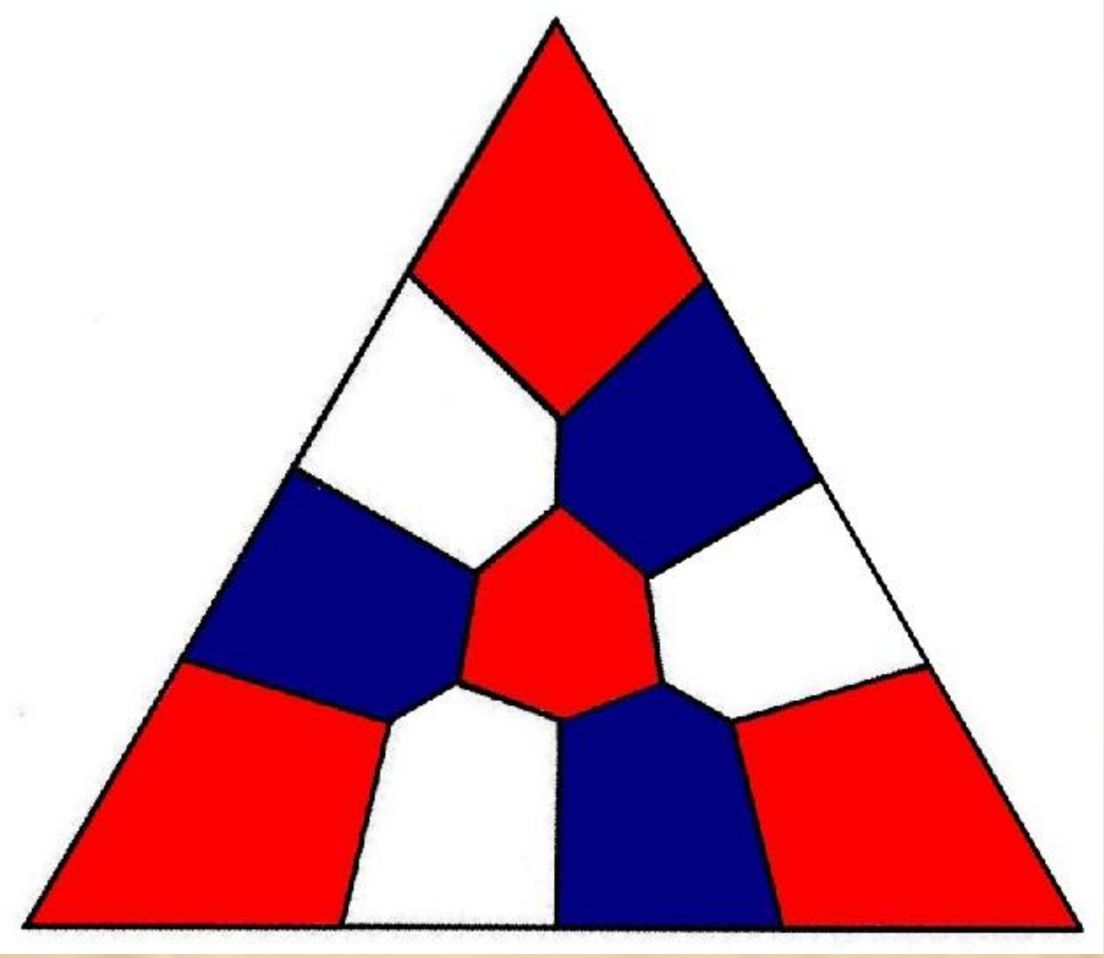
Webster is intermediate

Polya Representation (Huntington)



A 6 seat, 3 party apportionment

411,
321, 312
231, 222, 213
141, 132, 123, 114

Huntington excises away favoring large parties

Mathematical Problem

N	Number of Representatives
P	Total Population
N_i	Number of Reps for state i
P_i	Population of state i

$$P_1 \le P_2 \le P_3 \cdots \Rightarrow N_1 \le N_2 \le N_3 \cdots$$

$$\sum P_i = P \qquad \sum N_i = N$$

$$\frac{P_i}{P} \approx \frac{N_i}{N}$$

Measures of Injustice

$$\min_{i,j}\left|\frac{N_i}{P_i}-\frac{N_j}{P_j}\right|$$

… leads to Sainte Laguë (Webster, Arithmetic Mean)

$$\min_{i,j}\left|\frac{P_i}{N_i}-\frac{P_j}{N_j}\right|$$

… leads to Dean's Method (Harmonic Mean)

$$\min_{i,j}\left|\frac{N_i / P_i}{N_j / P_j}-1\right|$$

… leads to Huntington's Method (Equal Proportions)

Measures of Injustice

$$\min_{i,j} \frac{\left| \frac{P_i}{N_i} - \frac{P_j}{N_j} \right|}{\sqrt{\frac{P_i P_j}{N_i N_j}}}$$

… leads to Huntington's Method (Equal Proportions)

$$\min_{i,j} \left| N_i - N_j \frac{P_j}{P_j} \right|$$

… leads to Adam's Method

$$\min_{i,j} \left| P_i - P_j \frac{N_i}{N_j} \right|$$

… leads to Jefferson's Method

Measures of Injustice

$$\max_{i=1,\ldots,s} \min \frac{P_i}{N_i}$$

… leads to d'Hondt

$$\min_{i=1,\ldots,s} \max \frac{P_i}{N_i}$$

… leads to Adam's

Measures of Injustice

$$Min \sum P_i \left(\frac{N_i}{P_i} - \frac{N}{P} \right)^2$$

***... leads to* Sainte Laguë**

$$Min \sum N_i \left(\frac{P_i}{N_i} - \frac{P}{N} \right)^2$$

... leads to Huntington

$$Min \sum \left(\frac{N_i}{N} - \frac{P_i}{P} \right)^2$$

... leads to Hare

Case of Sainte Laguë (Webster)

Measure of Injustice

$$Q = \sum P_i \left(\frac{N_i}{P_i} - \frac{N}{P} \right)^2$$

$$\begin{aligned} Q &= \Sigma P_i \left(\frac{N^2{}_i}{P^2{}_i} - \frac{2NN_i}{PP_i} + \frac{N^2}{P^2} \right) \\ &= \Sigma \frac{N^2{}_i}{P_i} - 2\Sigma \frac{N}{P} N_i + \frac{N^2}{P^2} \Sigma P_i \\ &= \Sigma \frac{N^2{}_i}{P_i} - 2 \frac{N}{P} \cdot N + \frac{N^2}{P^2} \cdot P \\ &= \Sigma \frac{N^2{}_i}{P_i} - \frac{N^2}{P}. \end{aligned}$$

Sainte Laguë's scheme follows from the fact that

$$1 + 3 + \cdots + (2N - 1) = N^2$$

From a Zimbabwe Newspaper