Integrating Spaced Repetition in a Math Course Redesign

Rachid Ait Maalem Lahcen. Ph.D.
Professors often see that students do not remember key concepts from prerequisite courses. This could be due to students’ studying habits of cramming information right before exams. Cramming is the worst way of learning math. So, what can we do to help students with learning how study and to approach a great deal of concepts and skills that math courses contain? In this presentation, I’ll discuss implementation of spaced repetition or distributed practice strategy in a math course redesign. I’ll use Calculus I and/or college algebra to show great positive results.
“I Teach Students, Not Subjects.” Moje 1985

When you teach students, focus on how to help the students become a better version of themselves at the end of the course.
“Learning results from what the student does and thinks and only from what the student does and thinks. The teacher can advance learning only by influencing what the student does to learn.”

(Herbert Simon, 2001)

“It’s not teaching that causes learning. Attempts by the learner to perform cause learning, dependent upon the quality of feedback and opportunities to use it.”

(Grant Wiggins, 1993)
Redesign & Strategy

- Redesign process is to convert a course plan for successful student outcome.
- Spaced repetition or distributed practice.
- Ebbinghaus’s famous experiment of forgetting curve (1880-1885)
Memory Retention

First learned

Improved retention

Learning curve

Projected forgetting curve

Intervention

Time
Memory Retention

Learning curve

Time
Summary of main idea

Apply spaced repetition to objectives of interest (not all).

Use technology and written notes to supplement any objectives that don’t need much attention.

Improve design using course analytics after each run.
Method: Categorize Learning Objectives
Method: Sample from the Categories
Method: Apply Criterion

1. Direct: What’s important in the course? (survey) Or

2. Backward: Remediation, what’s important to be successful in subsequent course? (survey) Or

3. Data analysis: Diagnostic (pretest) – convenient for lower level: Intermediate/college algebra, Or

- Data analysis for new iterations: Course Analytics from previous or finished runs.
Calculus I Example
Calculus I: Learning Outcomes

Understand the concept of limit of a function.
Be able to calculate limits.
Understand and apply the concept of continuity of functions.
Understand the concept of limit at infinity and evaluate them.
Understand the concept of horizontal asymptote, and the relationship between the horizontal asymptote and limit at infinity.
Understand the definition of derivative and be able to apply it to compute the derivative of a function.
Understand the relation between average rate of change and instantaneous rate of change.
Differentiate functions using appropriate differentiation rules.
Calculus I: Learning Outcomes

Calculate derivatives using implicit differentiation.

Solve application problems involving related rates and optimization.

Understand and be able to apply the concepts of linear approximation and of differential.

Apply differentiation to problems about maximum and minimum values.

Understand the proofs of the Mean Value Theorem and Rolle’s Theorem and be able to apply them.

Recognize indeterminate forms of limits and know when and how to use l’Hospital’s rule to evaluate them.

Be able to sketch a curve using asymptotes, critical points, intervals of increasing and decreasing, inflection points and concavity.

Understand the concept of antiderivative.

Understand the relation between area under curves and integration using Riemann sums and the Fundamental Theorem of Calculus.

Solve basic integration using reverse differentiation rules and the Substitution Rule.

Overall, demonstrate stronger problem solving and critical thinking skills.

Be able to organize and communicate their work in a coherent way, using correct notation.
Criterion: What’s important in the course? (Survey)

- Limit concept
- Continuity
- Differentiation rules
  - Chain rule
- Related rates
- Curve sketching
- Optimization
- Definite Integral
- Substitution
- Fundamental Theorem of Calculus

Calculus I
Limits at infinity, indeterminate forms

Differentiation rules

Derivatives and antiderivatives of trig, log, and exponential functions

Chain rule

L’Hospital rule

Antiderivatives

Definite Integral

Substitution

Calculus I
How to integrate?

Planning Interventions on intervals

- Mid term exams use previous exams questions concerning spaced repetition

- Office hours/TAs Tutoring hours
- Knowledge check
- Proficiency Check
- Recorded lecture
- Class Activities
- Live Meeting
- Discussions
- Homework
- Recitations
How to integrate?

Adjust Grading with reduction of high-stake assignments 2021

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework (due 11:00 pm)</td>
<td>13%</td>
</tr>
<tr>
<td>Calculus Proficiency Check</td>
<td>5%</td>
</tr>
<tr>
<td>Calculus Knowledge Check</td>
<td>5%</td>
</tr>
<tr>
<td>Lecture Activities (due 6:00 pm)</td>
<td>7%</td>
</tr>
<tr>
<td>Four Exams</td>
<td>50%</td>
</tr>
<tr>
<td>Recitation and orientation activities, Discussions</td>
<td>15%</td>
</tr>
</tbody>
</table>

Cramming is the worst way to learn math.
# Adjust Grading with reduction of high-stake assignments 2022

How to integrate?

<table>
<thead>
<tr>
<th>Assignments</th>
<th>Percentage</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptive Homework</td>
<td>20%</td>
<td>Two lowest scores are dropped</td>
</tr>
<tr>
<td>Calculus Proficiency Initial Check</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td>Calculus Knowledge Check</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td>Lecture Activities (See full schedule for due dates)</td>
<td>15%</td>
<td>Two lowest scores are dropped</td>
</tr>
<tr>
<td>Recitation Activities (TBA), Orientation Activity &amp; Discussions (See full schedule for due dates)</td>
<td>5%</td>
<td>In recitation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Orientation and Discussions are online/Webcourses</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One lowest score is dropped</td>
</tr>
<tr>
<td>Exam 1 due 2/4/22 - Recitation period</td>
<td>36%</td>
<td>In recitation classroom per schedule</td>
</tr>
<tr>
<td>Exam 2 due 3/4/22-Recitation period</td>
<td></td>
<td>See full schedule for included topics. Exams will include topics from</td>
</tr>
<tr>
<td>Exam 3 due 4/8/22 - Recitation period</td>
<td></td>
<td>previous exams for repetition.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Best two scores are used, or lowest score is dropped</td>
</tr>
<tr>
<td>Final Exam</td>
<td>18%</td>
<td>In classroom</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cannot be dropped and does not replace another exam</td>
</tr>
<tr>
<td></td>
<td></td>
<td>See full schedule for included topics. Also, selected topics from</td>
</tr>
<tr>
<td></td>
<td></td>
<td>exams 1, 2, and 3 may also be included</td>
</tr>
</tbody>
</table>
How to integrate?

Challenge:
• Some topics appear in second have or late in the course
• Introduce early in some form that allow to build and repeat

• Use prior knowledge
• Optimization using vertex can be introduced earlier
• Limits at infinity can be introduced early using graphs and transformations
• Integration can be introduced on the first day as a distance problem
• Chain rule with power rule
• Our students come knowing L’Hospital rule, they aren’t sure about indeterminate form.
Data analysis: Course Analytics from previous run Fall 2021
Top 10, % of Students Struggled (end up mastering)

<table>
<thead>
<tr>
<th>Topic</th>
<th>Struggled</th>
<th>Mastered</th>
<th>Didn’t Master</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the differentiability of piecewise functions</td>
<td>59%</td>
<td>86%</td>
<td>14%</td>
</tr>
<tr>
<td>Combine the product and quotient rules</td>
<td>55%</td>
<td>86%</td>
<td>14%</td>
</tr>
<tr>
<td>Graph the derivative of a function</td>
<td>56%</td>
<td>89%</td>
<td>11%</td>
</tr>
<tr>
<td>Use related rates to solve problems involving volume</td>
<td>53%</td>
<td>96%</td>
<td>4%</td>
</tr>
<tr>
<td>Determine the continuity of a piecewise function</td>
<td>39%</td>
<td>96%</td>
<td>4%</td>
</tr>
<tr>
<td>Use the chain rule with the power rule</td>
<td>28%</td>
<td>89%</td>
<td>11%</td>
</tr>
<tr>
<td>Explain the three conditions for continuity at a point</td>
<td>28%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>Find a higher-order derivative of a sine or cosine function</td>
<td>23%</td>
<td>95%</td>
<td>5%</td>
</tr>
<tr>
<td>Use the chain rule with the product or quotient rule</td>
<td>15%</td>
<td>89%</td>
<td>11%</td>
</tr>
<tr>
<td>Distinguish between three kinds of discontinuity</td>
<td>13%</td>
<td>98%</td>
<td>2%</td>
</tr>
</tbody>
</table>
### Data analysis: Course Analytics from previous run Fall 2021
### Top 10, Easiest (Mastered easily)

<table>
<thead>
<tr>
<th>Topic</th>
<th>% Struggled</th>
<th>% Mastered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find the limit of a polynomial or rational function using limit laws and direct substitution</td>
<td>0</td>
<td>100%</td>
</tr>
<tr>
<td>Use the constant, constant multiple, and power rules</td>
<td>0</td>
<td>96%</td>
</tr>
<tr>
<td>Evaluate limits graphically</td>
<td>2</td>
<td>100%</td>
</tr>
<tr>
<td>Understand the limit of a function and evaluate a limit from a table</td>
<td>2</td>
<td>100%</td>
</tr>
<tr>
<td>Find absolute extrema</td>
<td>3</td>
<td>76%</td>
</tr>
<tr>
<td>Find the derivative of other trigonometric functions</td>
<td>5</td>
<td>95%</td>
</tr>
<tr>
<td>Define a vertical asymptote</td>
<td>7</td>
<td>98%</td>
</tr>
<tr>
<td>Understand differentiability and when a function does not have a derivative</td>
<td>11</td>
<td>100%</td>
</tr>
<tr>
<td>Find the derivative of a natural logarithmic function using properties of logarithms</td>
<td>11</td>
<td>96%</td>
</tr>
<tr>
<td>Use related rates to solve problems involving area</td>
<td>15</td>
<td>93%</td>
</tr>
</tbody>
</table>
Results Spring 2022

How are things going so far with this course? 71% great

Do you think you will review a past review even if it was not required that week for an assignment or an exam? 50% yes

Lecture activities helped me with time management and quick practice. 89% yes

Discussions are not common in math courses, but they intend to help you write or research and learn from writing and classmates. 50% agree
Spaced repetition or distributed practice is used in this course by bringing previous questions from previous exams/assignments. It helped me with remembering them.

95% agree, 2% disagree. 4% no comment

I'm confident I can talk about Calculus in public.

84% Agree
College Algebra

Example
Combining Keller’s Plan with Spaced Repetition

- Spaced repetition based
  - on Data analysis:
    - Diagnostic (pretest)
    - Weekly analytic reports

- PSI or Keller’s Plan:
  - Modularized content,
  - Prepare written material,
  - Pacing,
  - Mastery,
  - Human proctor.

- Monitor time on task.
- Adaptive system.
Overall Target Course Success (ABC) by Prerequisite Course Attempt Source

MAC 1105 College Algebra
students from UCF
overall success in subsequent or target course
MAC 1140 Precalculus Algebra
is 91.9%, higher when Compared to students who transferred in college algebra or used another way, 80.6%.
Overall Target Course Success (ABC) by Prerequisite Course Attempt Source

MAC 1105 College Algebra students from UCF overall success in subsequent or target course MAC 1114 College Trigonometry is 92.8%, higher when Compared to students who transferred in college algebra or used another way, 86%.
Final Thoughts
• Use first day of class to justify reasons behind design. Ask students for input.
• Work on human connection from the first day.
• Include relevance.
• Assign roles if groups are used.
• Post materials early.
• Using LMS and Conference technology (Zoom/Teams).
• Embed undergraduate or graduate assistant in meetings.
• Have activities for students to connect ideas or explore to create new ideas.
• Give prompt feedback.
• Facilitate collaboration, engagement, peer evaluation and/or peer reflection
• Facilitate environment in which students should feel secure to communicate
• Adopt flexibility and empathy.
• Model the behavior that you want to see: Curiosity, use of textbook, solving more problems, etc.
References


• No Title. (n.d.).


• Iaroslav, B., Post, V., Student, G., Petersburg, S., & Федерация, Б. Я. В. (1885). IMPROVING LISTENING SKILLS IN LANGUAGE LEARNING WITH SPACED Baranov I. V. ( Russian Federation ) Email : Baranov340@scientifictext.ru ПОВТОРЕНИЙ. 1–5.


• Spaced Repetition Learning as a Tool for Orthopedic Surgical Education A Prospective Cohort Study on a Training Examination Elsevier Enhanced Reader.pdf. (n.d.).


• Lafleur, L. (2020). The Indirect Spaced Repetition Concept. 9(2), 9–16.

Thank you!