Student Understanding of Domain & Range in Calculus 1

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The UNIVERSITY of OKLAHOMA

Introduction

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- More mathematical experience correlated with better performance on domain tasks (Dotson 2009)
- Low performance by twelfth-grade students on domain, particularly with non-polynomials (Alajmi 2019)
- Representational difficulties with notation and graphs (Cho & Moore-Russo 2014; Cho, Norris, Moore-Russo 2017)

Context & Data Gathered

- Two sections of Calculus I: 38 students and 28 students

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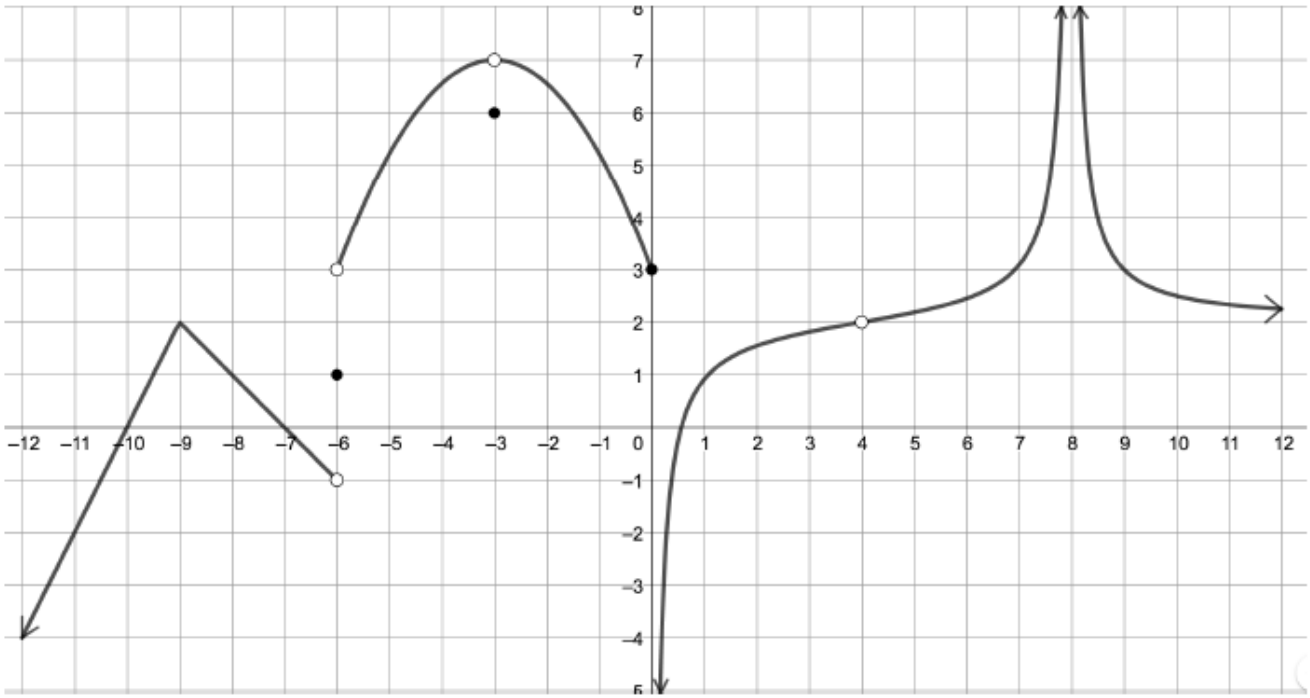
- Responses to week 4 midterm: E1, E2, E3
- Responses to week 6 quiz: Q1, Q2, Q3
- Final exam scores

Exam Questions Studied

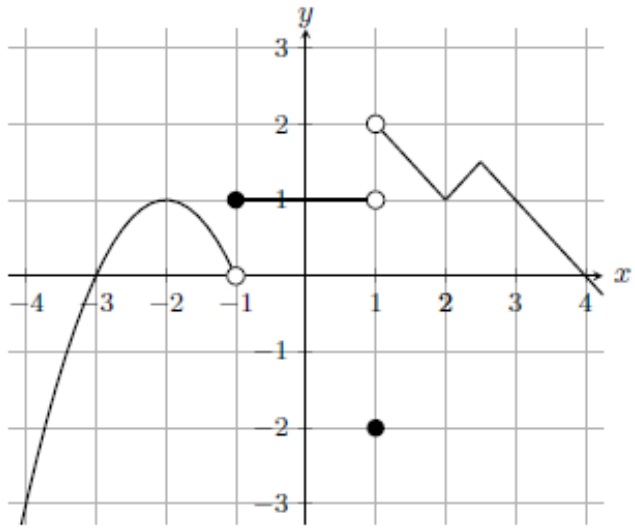
E1 Find the range of the function $y = 2 + \cos x$

E2 Find the domain of the function $g(x) = \dfrac{x-3}{5\sqrt{12-2x}}$

E3 Find the domain of f (below). State your answer using interval notation.



Quiz Questions Studied



Q1 Write the range of $g(x)$ using interval notation

Q2 Write the domain of $g(x)$ using interval notation

Q3 Determine the domain of $f(x) = \dfrac{x}{1-2x} + \sqrt{3x}$

Research Question 1

How do students perform on nontrivial domain and range tasks after a cursory review of algebraic topics in a Calculus 1 class prior to instruction on derivatives?

Research Question 1 - Data 1

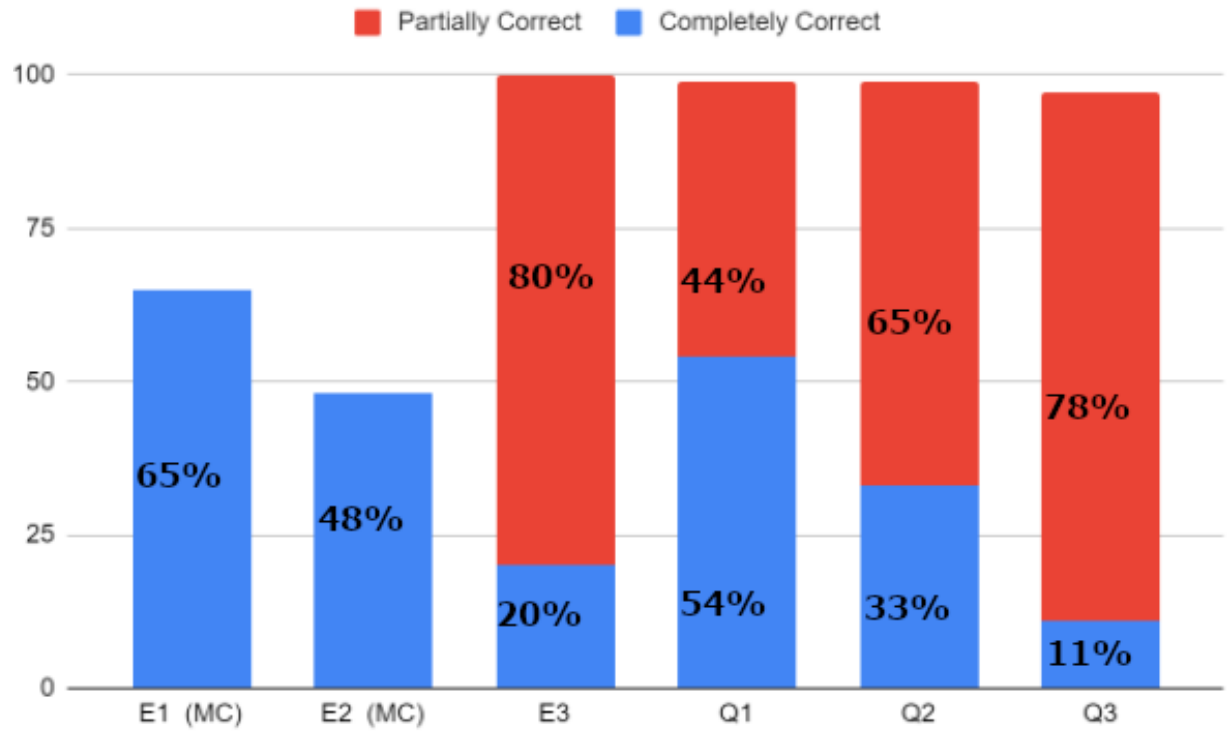


Figure: Stacked Bar Graph of Student Performance on Items

Research Question 1 - Data 2

Item	Type	% Completely Correct
E1	R	63%
Q1	R	54%
E2	D	50%
Q2	D	33%
E3	D	20%
Q3	D	11%

R = Range, D = Domain

Table: Relative Frequency of Student Performance by Item

Correct Responses	% Students
0	22%
1	6%
2	28%
3	20%
4	11%
5	13%
6	0%

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Research Question 2

Is there a difference in students' task performance when tasks are expressed symbolically or graphically or when tasks involve different types of functions?

Research Question 2 - Data 1

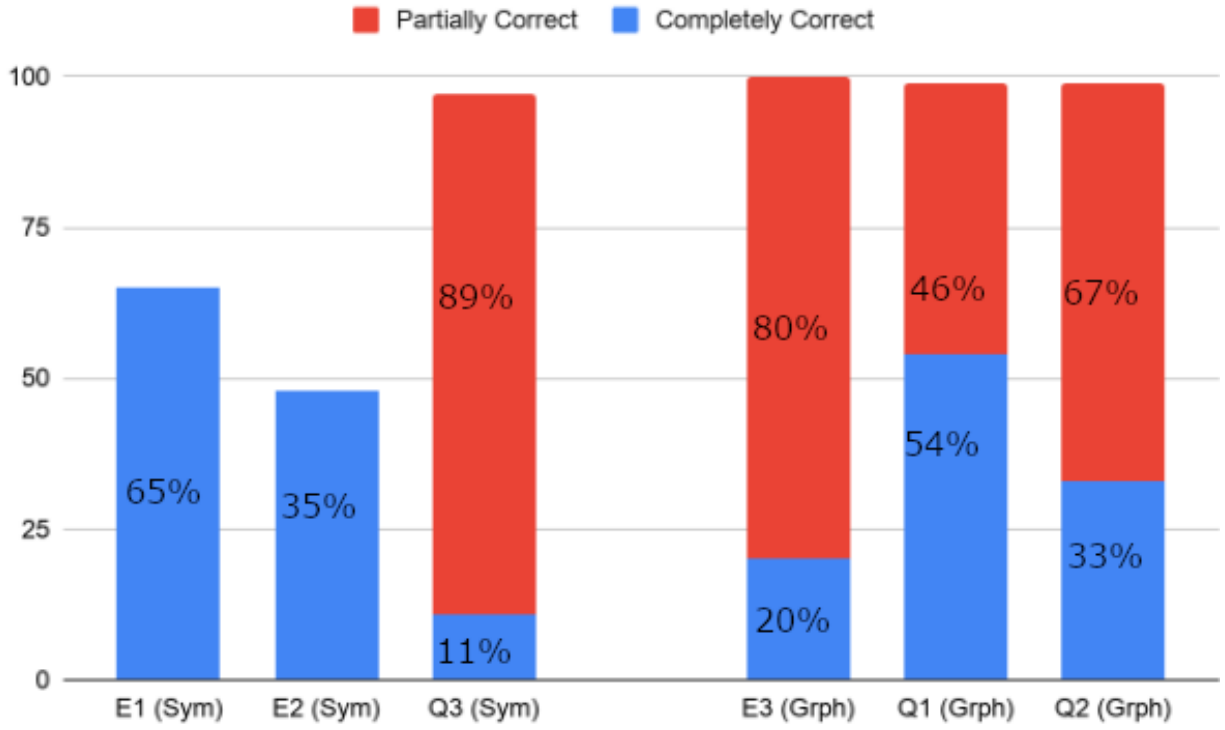

Figure: Stacked Bar Graph of Performance by Item Category

Research Question 2 - Data 2

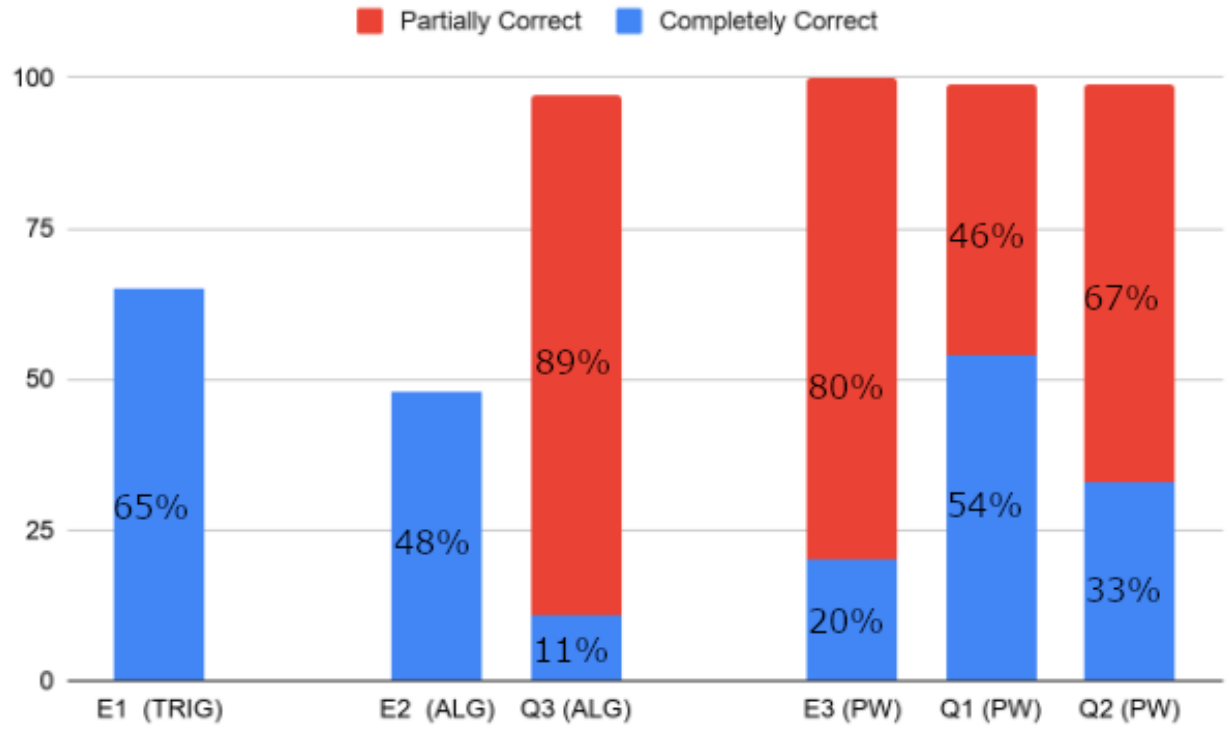

Figure: Stacked Bar Graph of Performance by Item Type

Research Question 2 - Conclusions

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- No patterns in performance when looking at item type
- Conjecture: students are able to switch between representations of domain and range, but have persistent issues with identifying relevant information for either.

Research Question 3

Does student understanding of domain and range early in a Calculus 1 course seem to play a role in performance at the end of the course (as measured by the final exam)?

Research Question 3 - Data

Initial Items Correct	Students	Final Exam %		
		Min	Median	Max
0	12*	36%	75%	87%
1	3	63%	80%	94%
2	15	46%	76%	93%
3	11**	66%	85%	97%
4	6	66%	80%	94%
5	7	58%	76%	97%
6	n/a	n/a	n/a	n/a

* 2 students did not take the final from this group

** 1 student did not take the final from this group

Table: Student Performance on Final Exam, Accounting for Performance on Items

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- No pattern in the relationship between initial understanding of domain/range and the final exam.
- Minimum & maximum scores have little variation
- Consistent median across all groups

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- 313 coded instances
- Identified four overall categories for confusion: *Continuity*, *Intervals*, *Notation*, *Switch*
  - Continuity: 225 instances
  - Intervals: 43 instances
  - Notation: 32 instances
  - Switch: 13 instances

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- C-4: Confusion with removable discontinuities (point removed) (22)
- C-5: Confusion with asymptotes (14)
- C-6: Confusion with domain of radical (13)
- C-7: Confusion with removable discontinuities (overlapping intervals) (2)

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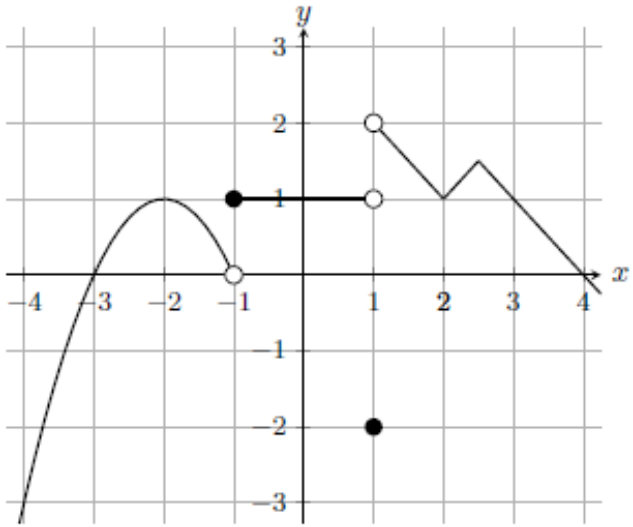

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Figure: C-1 & I-1 confusion on Q1

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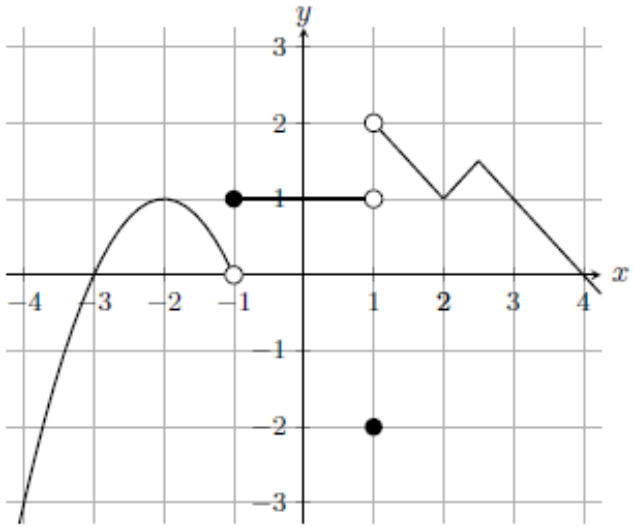

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Figure: C-1, C-2, I-1 confusion on Q2

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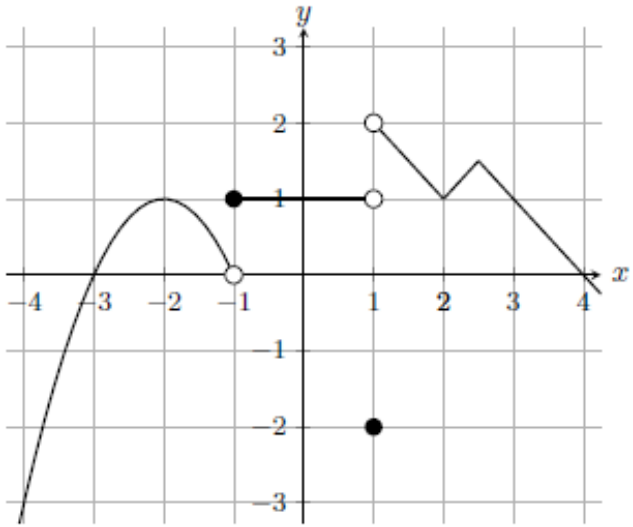

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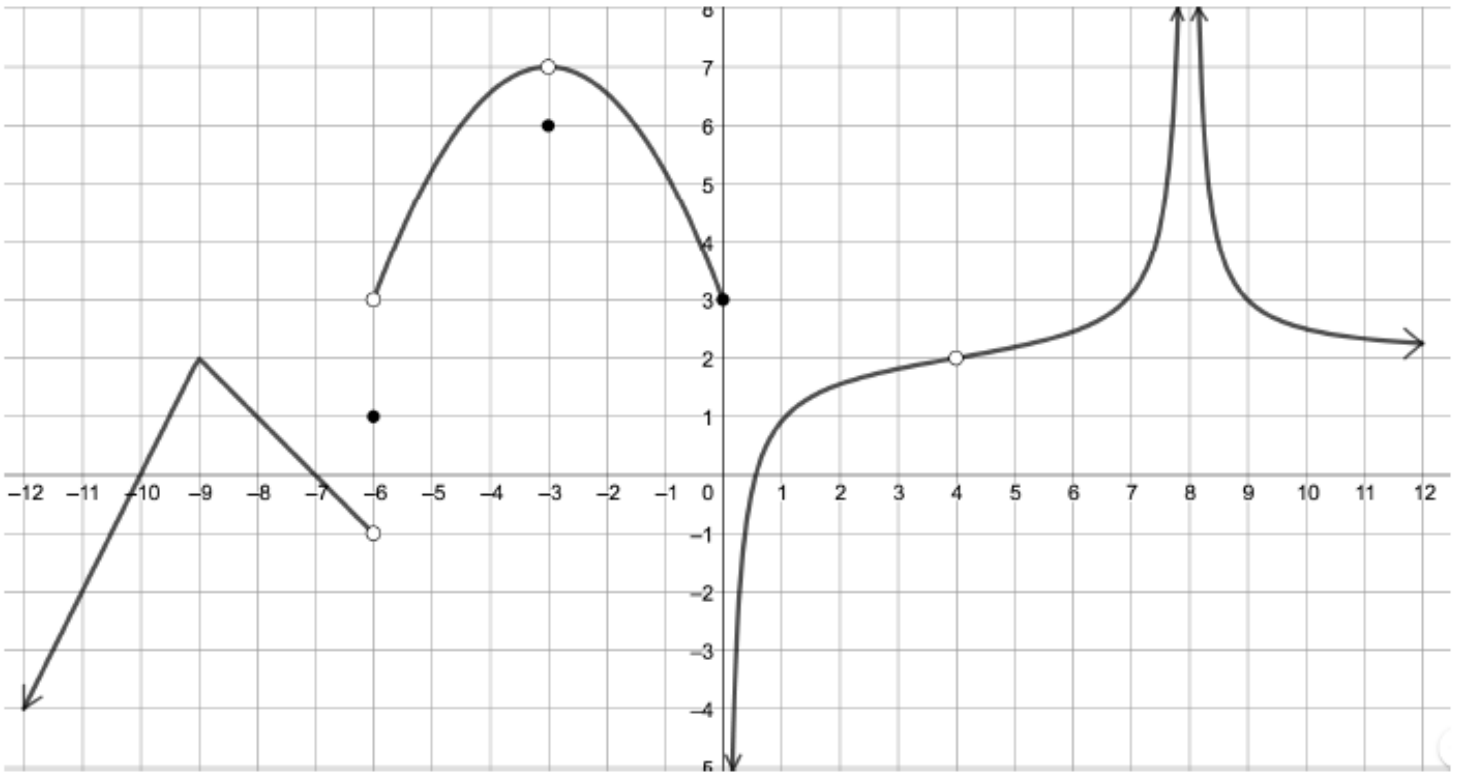

$$(x \in \mathbb{R} \quad -\infty, \infty, \text{iff } x \neq 4 \quad x \neq 8$$

Figure: N-2 confusion on E3

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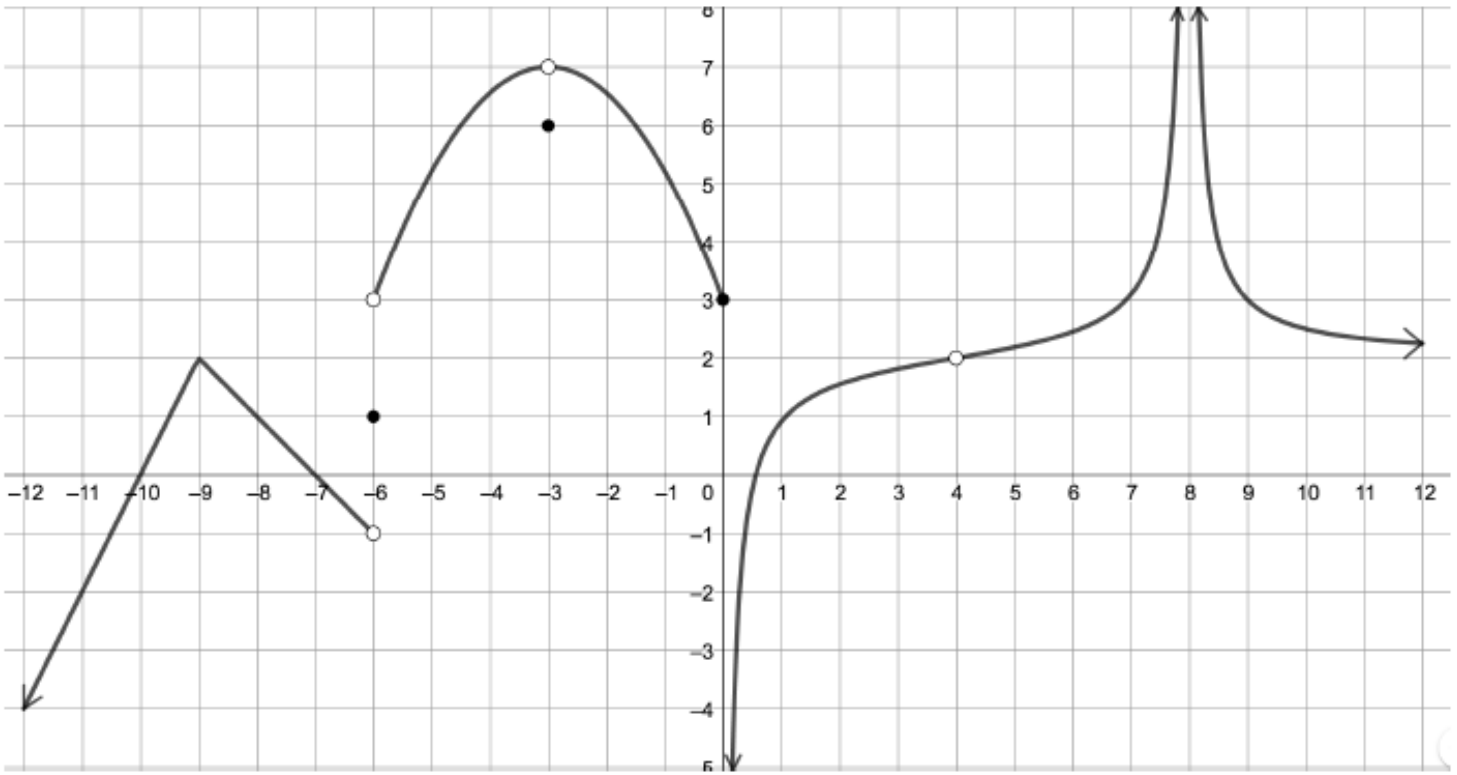

$$(-\infty, -6) \cup (-6, -3) \cup (-3, 0] \cup (-\infty, 4) \cup (4, \infty) \cup (-\infty, \infty)$$

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Conclusions & Future Work

- Results agree with previous studies: lack of deep understanding of domain/range
- Domain more difficult than range
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- Domain more difficult than range
- Content of tasks has greater impact than presentation or type of function
- Investigate struggles specifically with rational/radical functions, as well as more complex combinations

Thank You for Coming!

We hope the rest of your semester goes well!