Student Understanding of Domain & Range in Calculus 1

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Introduction

- Students confuse domain and range in college calculus courses (Özkana & Ünala 2009)
- More mathematical experience correlated with better performance on domain tasks (Dotson 2009)
- Low performance by twelfth-grade students on domain, particularly with non-polynomials (Alajmi 2019)
- Representational difficulties with notation and graphs (Cho & Moore-Russo 2014; Cho, Norris, Moore-Russo 2017)
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Context & Data Gathered

- Two sections of Calculus I: 38 students and 28 students
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- Two sections of Calculus I: 38 students and 28 students
- Fall 2019 semester
Context & Data Gathered

- Two sections of Calculus I: 38 students and 28 students
- Fall 2019 semester
- Responses to week 4 midterm: E1, E2, E3
- Responses to week 6 quiz: Q1, Q2, Q3
- Final exam scores
Exam Questions Studied

E1  Find the range of the function \( y = 2 + \cos x \)

E2  Find the domain of the function \( g(x) = \frac{x - 3}{5\sqrt{12 - 2x}} \)

E3  Find the domain of \( f \) (below). State your answer using interval notation.
Quiz Questions Studied

Q1 Write the range of $g(x)$ using interval notation
Q2 Write the domain of $g(x)$ using interval notation
Q3 Determine the domain of $f(x) = \frac{x}{1 - 2x} + \sqrt{3x}$
Research Question 1

How do students perform on nontrivial domain and range tasks after a cursory review of algebraic topics in a Calculus 1 class prior to instruction on derivatives?
Research Question 1 - Data 1

Figure: Stacked Bar Graph of Student Performance on Items
Research Question 1 - Data 2

Table: Relative Frequency of Student Performance by Item

<table>
<thead>
<tr>
<th>Item</th>
<th>Type</th>
<th>% Completely Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>R</td>
<td>63%</td>
</tr>
<tr>
<td>Q1</td>
<td>R</td>
<td>54%</td>
</tr>
<tr>
<td>E2</td>
<td>D</td>
<td>50%</td>
</tr>
<tr>
<td>Q2</td>
<td>D</td>
<td>33%</td>
</tr>
<tr>
<td>E3</td>
<td>D</td>
<td>20%</td>
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<tr>
<td>Q3</td>
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<td>11%</td>
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R = Range, D = Domain

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<table>
<thead>
<tr>
<th>Correct Responses</th>
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<tbody>
<tr>
<td>0</td>
<td>22%</td>
</tr>
<tr>
<td>1</td>
<td>6%</td>
</tr>
<tr>
<td>2</td>
<td>28%</td>
</tr>
<tr>
<td>3</td>
<td>20%</td>
</tr>
<tr>
<td>4</td>
<td>11%</td>
</tr>
<tr>
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### Research Question 1 - Data 2

#### Item Type

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*Table: Relative Frequency of Student Performance by Items Completely Correct*
Research Question 1 - Initial Conclusions

- Poorer performance on free-response items than multiple-choice items
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- No deep understanding, but some exists
Research Question 1 - Initial Conclusions

- Poorer performance on free-response items than multiple-choice items
- No deep understanding, but some exists
- Domain more difficult than range
Research Question 2

Is there a difference in students’ task performance when tasks are expressed symbolically or graphically or when tasks involve different types of functions?
Research Question 2 - Data 1

Figure: Stacked Bar Graph of Performance by Item Category
Research Question 2 - Data 2

**Figure:** Stacked Bar Graph of Performance by Item Type
Research Question 2 - Conclusions

- No significant differences in performance by item category
- No patterns in performance when looking at item type
Research Question 2 - Conclusions

- No significant differences in performance by item category
- No patterns in performance when looking at item type
- Conjecture: students are able to switch between representations of domain and range, but have persistent issues with identifying relevant information for either.
Research Question 3

Does student understanding of domain and range early in a Calculus 1 course seem to play a role in performance at the end of the course (as measured by the final exam)?
Research Question 3 - Data

<table>
<thead>
<tr>
<th>Initial Items Correct</th>
<th>Students</th>
<th>Final Exam %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
</tr>
<tr>
<td>0</td>
<td>12*</td>
<td>36%</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>63%</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>46%</td>
</tr>
<tr>
<td>3</td>
<td>11**</td>
<td>66%</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>66%</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>58%</td>
</tr>
<tr>
<td>6</td>
<td>n/a</td>
<td>n/a</td>
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</tbody>
</table>

* 2 students did not take the final from this group
** 1 student did not take the final from this group

Table: Student Performance on Final Exam, Accounting for Performance on Items
Research Question 3 - Conclusions

- No pattern in the relationship between initial understanding of domain/range and the final exam.
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- No pattern in the relationship between initial understanding of domain/range and the final exam.
- Minimum & maximum scores have little variation
- Consistent median across all groups
Confusion Categories

- Student responses were analyzed and coded for specific elements of confusion
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- 313 coded instances
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- Identified four overall categories for confusion: *Continuity, Intervals, Notation, Switch*
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- 313 coded instances
- Identified four overall categories for confusion: *Continuity, Intervals, Notation, Switch*
  - Continuity: 225 instances
  - Intervals: 43 instances
  - Notation: 32 instances
  - Switch: 13 instances
Confusion Categories - Continuity

- Refers to students’ difficulty with the idea of continuity of a function
Confusion Categories - Continuity

- Refers to students’ difficulty with the idea of continuity of a function
- C-1: Confusion with overlapping points (88)
- C-2: Confusion between open/closed endpoints (43)
- C-3: Confusion at isolated points (43)
- C-4: Confusion with removable discontinuities (point removed) (22)
- C-5: Confusion with asymptotes (14)
- C-6: Confusion with domain of radical (13)
- C-7: Confusion with removable discontinuities (overlapping intervals) (2)
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Confusion Categories - Examples

\[ (-\infty, 0) \cup (0, 1) \cup (1, 2) \cup (2, -\infty) \]

**Figure:** C-1 & I-1 confusion on Q1
Confusion Categories - Examples

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

**Figure:** C-1, C-2, I-1 confusion on Q2
Confusion Categories - Intervals

- Refers to students’ difficulty parsing intervals and how they interact

  I-1: Confusion between domain/range involving interval endpoints (28)
  I-2: Confusion with horizontal segments (8)
  I-3: Confusion on how to treat sharp/cusp points (4)
  I-4: Looks at intervals independently (3)
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Figure: C-1 & I-1 confusion on Q1

\[(-\infty, 0) \cup (0, 1) \cup (1, 2) \cup (2, -\infty)\]
Confusion Categories - Notation & Switch

- Notation refers to how students reported their answers

- N-1: Included ±∞ as an endpoint (16)
- N-2: Strange notation but correct answer (7)
- N-3: Wrote range top to bottom or left to right (5)
- N-4: Strange notation and incorrect answer (4)

- S-1: Switched between domain/range midway through task (potentially multiple times) (12)
- S-2: Switched domain/range at onset of task (1)
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Confusion Categories - Examples

\[(x \in \mathbb{R} \quad -\infty, \infty, \text{ iff } x \neq 4 \quad x \neq 8)\]

**Figure**: N-2 confusion on E3
Confusion Categories - Examples

\[ \left(-\infty, -6\right) \cup (-6, -3) \cup (-3, 0] \cup (-\infty, 4) \cup (4, \infty) \cup (-\infty, \infty) \]

**Figure:** C-1, I-1, S-1 confusion on E3
Conclusions & Future Work

- Results agree with previous studies: lack of deep understanding of domain/range
- Domain more difficult than range
- Content of tasks has greater impact than presentation or type of function

Investigate struggles specifically with rational/radical functions, as well as more complex combinations
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Thank You for Coming!

We hope the rest of your semester goes well!