Equation sheet Exam II

Chapter 6: Work, Energy and Power

Work definition

$$W = \vec{F}\cdot\vec{s} = Fs\cos\theta$$

Work by varing force or curved path

$$W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s} = \int_{s_1}^{s_2} F\cos\theta ds$$

Work-energy relation, definition of *KE*

$$W = \Delta K = KE_2 - KE_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Definition of Power

$$P_{av} = \frac{\Delta W}{\Delta t}, \quad P_{av} = F_{\parallel}v_{av}$$

$$P = \frac{dW}{dt}, \quad P = F_{\parallel}v$$

Chapter 7: Potential Energy and Conservation of Energy

Potential Energy (U) (conservative forces)

$$W_{gravity} = U_{grav,1} - U_{grav,2} = -\Delta U_{grav} = mg(y_1 - y_2)$$

$$W_{elastic} = U_{elas,1} - U_{elas,2} = -\Delta U_{elas} = \frac{1}{2}k(x_1^2 - x_2^2)$$

Conservation of Energy

$$E_{Total,2} = E_{Total,1}$$

$$E_{Total,1} = KE + U_{elas} + U_{grav} + W_{other}$$

$$KE_2 + U_{grav,2} + U_{elas,2} = KE_1 + U_{grav,1} + U_{elas,1} + W_{other}$$

Force from Energy is

$$F_x(x) = -\frac{dU(x)}{dt}$$

In 3-D

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial x}\hat{k}\right)$$

Chapter 8: Momentum, Impulse, Conservation of Momentum

$$\vec{p} = m\vec{v} : p_x = mv_x, p_y = mv_y$$

Momentum is conserved if there are no external forces

$$\sum \vec{F}_{ext} = \frac{\Delta\vec{p}}{\Delta t} = 0 \rightarrow \vec{p}_{T,initial} = \vec{p}_{T,final}$$

Impulse is defined as:

$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} dt$$

Impulse-momentum theorem:

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

Center of Mass and Momentum

$$x_{cm} = \frac{x_1m_1 + x_2m_2 + x_3m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$y_{cm} = \frac{y_1m_1 + y_2m_2 + y_3m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Collisions (momentum is cnrv'd)

Elastic: KE is conserved implies the following velocity relation

$$(\vec{\mathbf{v}}_{B,f} - \vec{\mathbf{v}}_{A,f}) = -(\vec{\mathbf{v}}_{B,i} - \vec{\mathbf{v}}_{A,i})$$

Inelastic: KE is not conserved

Completely inelastic: objects stick together after they collide and

$$\vec{\mathbf{v}}_{B,f} = \vec{\mathbf{v}}_{A,f}$$

Chapter 9: Rotational Motion

Angular velocity and acceleration

$$\omega_{inst} = \frac{d\theta}{dt}; \quad \alpha_{inst} = \frac{d\omega}{dt}$$

Angular kinematic relations

$$\omega_2 = \omega_1 + \alpha t$$

$$\theta_2 = \theta_1 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega_2^2 = \omega_1^2 + 2\alpha(\theta_2 - \theta_1)$$

Angular to linear relations

$$s_{arc-length} = r\theta$$

$$v_{tangential} = \omega r$$

$$a_{tangential} = \alpha r$$

$$a_{radial} = \frac{v_{tangential}^2}{r} = \omega^2 r$$

Potential Energy

$$U = Mgy_{CM}$$

Moment of Inertia, general form, see table 9.2

$$I = \sum_i m_i r_i^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots$$

Chapter 10: Rotational Dynamics

The torque τ is given by $\vec{\tau} = \vec{\mathbf{r}}\times\vec{\mathbf{F}}$, the magnitude of $|\vec{\tau}| = |\vec{\mathbf{r}}||\vec{\mathbf{F}}|\sin\theta$, where $\vec{\mathbf{r}}$ is a vector pointing from the pivot point to the where the $\vec{\mathbf{F}}$ acts. The angle θ is the smallest angle between the vectors when located tail to tail.

$\sum\vec{\tau} = I\vec{\alpha}$, for rigid bodies the relation $a = r\alpha$ is useful

Kinetic Energy of object with rotational and linear motion

$$KE_{total} = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}I_{cm}\omega^2$$

Rolling without slipping

$$v_{bottom} = 0, v_{CM} = R\omega, v_{top} = 2R\omega$$

Angular Momentum

$L = I\omega$, where I is the moment of inertia and ω is the angular velocity

$L = mvl$, for a single particle where l is perpendicular distance from axis and mv is the linear momentum

Conservation of Angular Momenta follows from Newton's 2nd Law

$$\frac{dL}{dt} = \sum\tau_{ext} = I\alpha = I\frac{d\omega}{dt}$$

Work Energy theorem for angular motion

$$W = \frac{1}{2}I_{CM}(\omega_2^2 - \omega_1^2)$$

thus $L_{Total,1} = L_{Total,2}$ if and onf if $\sum\tau_{ext} = 0$, *ie.*, the are no external torques

Chapter 11: Equilibrium

Equillibrium of rigid body

$$\sum\vec{F} = 0 : \sum F_x = 0, \sum F_y = 0 \text{ and } \sum\vec{\tau} = 0$$

FBD for man w/ladder on friction less wall, The torque is computed about axis at B but you can use anywhere else as axis of rotation

To solve equilibrium problems begin by using FBD approach. Each force is now applied on extended object. Next write down $\Sigma F = 0$, in x and y, and then write down $\Sigma\ \tau = 0$, take the axis of rotation for the torques to be anywhere on the object that is the most convenient. Finally solve for the unkwnown(s) from the resulting three equation.