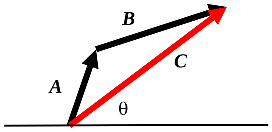
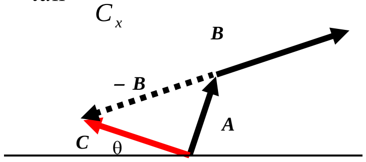
# Equation sheet Exam 1 PHY2048

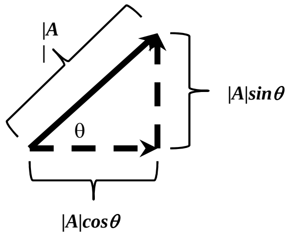
**Chapter 1: Measurements, Estimation, Vectors**

$$\vec{A} - \vec{B} = \vec{C}$$
$$A_x - B_x = C_x$$
$$A_y - B_y = C_y$$
$$|\vec{C}| = \sqrt{C_x^2 + C_y^2}$$
$$\theta = \tan^{-1}\frac{C_y}{C_x}$$

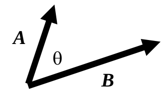


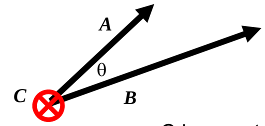

$$\vec{A} + \vec{B} = \vec{C}$$
$$A_x + B_x = C_x$$
$$A_y + B_y = C_y$$
$$|\vec{C}| = \sqrt{C_x^2 + C_y^2}$$
$$\theta = \tan^{-1}\frac{C_y}{C_x}$$




$$\vec{A} \cdot \vec{B} = C = |\vec{A}||\vec{B}|\cos\theta = A_xB_x + A_yB_y + A_zB_z$$




C is a scalar quantity that corresponds to the projection of A on B




C is a vector that is perpendicular to both vectors A and B

θ

$$\vec{A} \times \vec{B} = \vec{D}$$
$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta$$
$$\vec{D} = (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

**Chapter 2&3: Motion in a Straight Line and in a Plane**

**Equations of motion that apply generally**

$$\int_{x_0}^{x} d\vec{x} = \int_{t_0}^{t} \vec{v}(t)dt \rightarrow \vec{x} = \vec{x}_0 + \int_{t_0}^{t} \vec{v}(t)dt \qquad \int_{v_0}^{v} d\vec{v} = \int_{t_0}^{t} \vec{a}(t)dt \rightarrow \vec{v} = \vec{v}_0 + \int_{t_0}^{t} \vec{a}(t)dt$$

Displacement: A "vector" $\Delta\vec{r}$ that points between two locations. The vector begin at the initial location and ends at the final.

$$\text{displacement} = \Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

Distance: A "scalar" quantity that describes the length of the path taken between

$$\text{distance} = |\Delta\vec{r}| = \sqrt{(x_f - x_i)^2 + (y_f - y_i)^2}$$

Definition of velocity and acceleration. note: velocity and acceleration are vector quantities

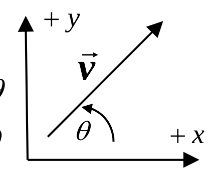
$$\vec{v}_{average} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t}, \quad \vec{v}_{inst} = \lim_{\Delta t \to 0}\frac{\vec{x}_f - \vec{x}_i}{\Delta t} = \frac{d\vec{x}}{dt}$$
$$\vec{a}_{average} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}, \quad \vec{a}_{inst} = \lim_{\Delta t \to 0}\frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{d\vec{v}}{dt}$$

The equations of motion in 2 or 1 dimension when the acceleration is constant are given below. The time t is the quantity that is common to both dimensions for 2-D problems

$$v_{2x} = v_{1x} + a_x t$$
$$x_2 = x_1 + v_{1x}t + \frac{1}{2}a_x t^2$$
$$v_{2x}^2 = v_{1x}^2 + 2a_x(x_2 - x_1)$$
$$v_{2y} = v_{1y} + a_y t$$
$$y_2 = y_1 + v_{1y}t + \frac{1}{2}a_y t^2$$
$$v_{2y}^2 = v_{1y}^2 + 2a_y(y_2 - y_1)$$

For projectile motion in the plane

$$a_y = -g$$
$$a_x = 0$$
$$v_{(2/1)x} = v_{2/1}\cos\theta$$
$$v_{(2/1)y} = v_{2/1}\sin\theta$$




For motion in a circle the acceleration vector points towards the center of the circle with a magnitude given by: $a_{rad} = \frac{v^2}{R}$

**Chapter 4&5: Newton's Law and Applications**

Newton's Laws of Motion

- 1st : A body at rest will remain at rest a body in motion will remain in motion unless acted upon by an external force

$$\sum \vec{F} = 0$$

- 2nd :The net sum of forces accelerates an object by an amount proportional to its mass and in the direction of the net forces.

$$\sum \vec{F} = m\vec{a}$$

- 3rd :For every action there is an opposite and equal reaction. Action reaction pairs never act on the same object

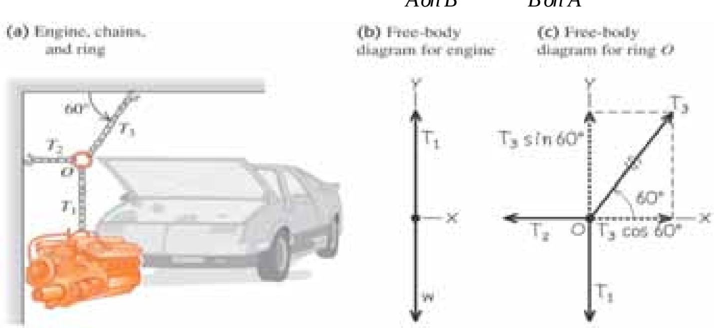
$$\vec{F}_{A\,on\,B} = -\vec{F}_{B\,on\,A}$$

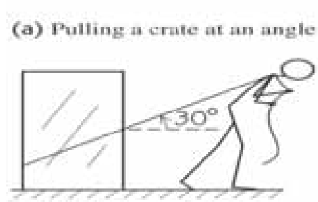
Force of friction comes in two flavors. Static frictional forces apply when the object is at rest with respect to the surface. Kinetic frictional forces apply when the object is moving with respect to the surface. Both frictional forces always act parallel to the surface and are proportional to the normal force.

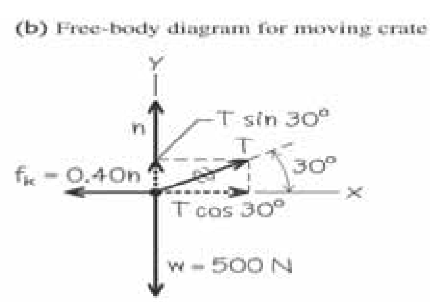
$$\vec{F}_{friction} = \mu_k \vec{N}$$
$$\vec{F}_{friction} \le \mu_s \vec{N}$$

Force due to spring is given by: $\vec{F}_{Spring} = -k\vec{x}$

Use FBD for spring problems, the sign of the force should be clear from the diagram



# Equation sheet Exam II

## Chapter 6: Work, Energy and Power

Work definition

$W = \vec{F}\cdot\vec{s} = Fs\cos\theta$

Work by varing force or curved path

$W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s} = \int_{s_1}^{s_2} F\cos\theta ds$

Work-energy relation, definition of *KE*

$W = \Delta K = KE_2 - KE_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Definition of Power

$P_{av} = \frac{\Delta W}{\Delta t}, \quad P_{av} = F_{\parallel}v_{av}$

$P = \frac{dW}{dt}, \quad P = F_{\parallel}v$

## Chapter 7: Potential Energy and Conservation of Energy

Potential Energy ($U$) (conservative forces)

$W_{gravity} = U_{grav,1} - U_{grav,2} = -\Delta U_{grav} = mg(y_1 - y_2)$

$W_{elastic} = U_{elas,1} - U_{elas,2} = -\Delta U_{elas} = \frac{1}{2}k(x_1^2 - x_2^2)$

Conservation of Energy

$E_{Total,2} = E_{Total,1}$

$E_{Total,1} = KE + U_{elas} + U_{grav} + W_{other}$

$KE_2 + U_{grav,2} + U_{elas,2} = KE_1 + U_{grav,1} + U_{elas,1} + W_{other}$

Force from Energy is

$F_x(x) = -\frac{dU(x)}{dt}$

In 3-D

$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial x}\hat{k}\right)$

## Chapter 8: Momentum, Impulse, Conservation of Momentum

$\vec{p} = m\vec{v} : p_x = mv_x, p_y = mv_y$

Momentum is conserved if there are no external forces

$\sum \vec{F}_{ext} = \frac{\Delta\vec{p}}{\Delta t} = 0 \rightarrow \vec{p}_{T,initial} = \vec{p}_{T,final}$

Impulse is defined as:

$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} dt$

Impulse-momentum theorem:

$\vec{J} = \vec{p}_2 - \vec{p}_1$

Center of Mass and Momentum

$x_{cm} = \frac{x_1m_1 + x_2m_2 + x_3m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$

$y_{cm} = \frac{y_1m_1 + y_2m_2 + y_3m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$

**Collisions (momentum is cnrv'd)**

**Elastic:** KE is conserved implies the following velocity relation

$\left(\vec{\mathbf{v}}_{B,f} - \vec{\mathbf{v}}_{A,f}\right) = -\left(\vec{\mathbf{v}}_{B,i} - \vec{\mathbf{v}}_{A,i}\right)$

**Inelastic:** KE is not conserved

**Completely inelastic:** objects stick together after they collide and

$\vec{\mathbf{v}}_{B,f} = \vec{\mathbf{v}}_{A,f}$

## Chapter 9: Rotational Motion

Angular velocity and acceleration

$\omega_{inst} = \frac{d\theta}{dt}; \quad \alpha_{inst} = \frac{d\omega}{dt}$

Angular kinematic relations

$\omega_2 = \omega_1 + \alpha t$

$\theta_2 = \theta_1 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega_2^2 = \omega_1^2 + 2\alpha(\theta_2 - \theta_1)$

Angular to linear relations

$s_{arc-length} = r\theta$

$v_{tangential} = \omega r$

$a_{tangential} = \alpha r$

$a_{radial} = \frac{v_{tangential}^2}{r} = \omega^2 r$

Potential Energy

$U = Mgy_{CM}$

Moment of Inertia, general form, see table 9.2

$I = \sum_i m_i r_i^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots$

## Chapter 10: Rotational Dynamics

The torque $\tau$ is given by $\vec{\tau} = \vec{\mathbf{r}}\times\vec{\mathbf{F}}$, the magnitude of $|\vec{\tau}| = |\vec{\mathbf{r}}||\vec{\mathbf{F}}|\sin\theta$, where $\vec{\mathbf{r}}$ is a vector pointing from the pivot point to the where the $\vec{\mathbf{F}}$ acts. The angle $\theta$ is the smallest angle between the vectors when located tail to tail.

$\sum\vec{\tau} = I\vec{\alpha}$, for rigid bodies the relation $a = r\alpha$ is useful

Kinetic Energy of object with rotational and linear motion

$KE_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

Rolling without slipping

$v_{bottom} = 0, v_{CM} = R\omega, v_{top} = 2R\omega$

Angular Momentum

$L = I\omega$, where $I$ is the moment of inertia and $\omega$ is the angular velocity

$L = mvl$, for a single particle where $l$ is perpendicular distance from axis and $mv$ is the linear momentum

Conservation of Angular Momenta follows from Newton's 2nd Law

$\frac{dL}{dt} = \sum\tau_{ext} = I\alpha = I\frac{d\omega}{dt}$

Work Energy theorem for angular motion

$W = \frac{1}{2}I_{CM}(\omega_2^2 - \omega_1^2)$

thus $L_{Total,1} = L_{Total,2}$ if and onf if $\sum\tau_{ext} = 0$, *ie.*, the are no external torques

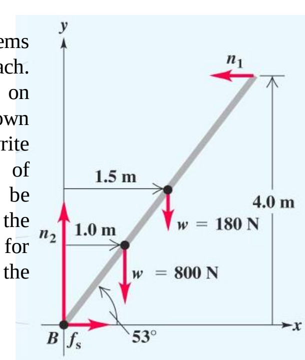
## Chapter 11: Equilibrium

Equillibrium of rigid body

$\sum\vec{F} = 0 : \sum F_x = 0, \sum F_y = 0$ and $\sum\vec{\tau} = 0$

FBD for man w/ladder on friction less wall, The torque is computed about axis at B but you can use anywhere else as axis of rotation

To solve equilibrium problems begin by using FBD approach. Each force is now applied on extended object. Next write down $\Sigma F = 0$, in x and y, and then write down $\Sigma\ \tau = 0$, take the axis of rotation for the torques to be anywhere on the object that is the most convenient. Finally solve for the unkwnown(s) from the resulting three equation.



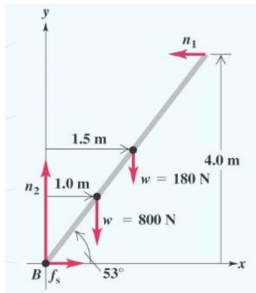
# Equation sheet Exam III, PHY2048

## Chapter 11: Equlibrium

Equillibrium of rigid body: $\sum \vec{F} = 0$ : and $\sum \vec{\tau} = 0$

FBD for man w/ladder on friction less wall
Torque computed about axis at B but anywhere else is OK.

To solve equilibrium problems begin by using FBD approach. Each force is now applied on extended object. Next write down $\Sigma F = 0$, in x and y, and then write down $\Sigma \tau = 0$, take the axis of rotation for the torques to be anywhere on the object at equilibrium that's convenient. Finally Inspect three equation and solve for unknown(s).




## Chapter 12: Fluid Mechanics

Density and pressure

$$\rho = \frac{m}{V};$$

$$p = \frac{dF_\perp}{dA}$$

Archimede's principle

$F_B = \rho_f g V$

$\sum \mathbf{F} = 0$

$F_B - mg = 0$

The Buoyant is the upward force excerted by fluid on a body immersed equal to amount of fluid displaced.

Pressure in a fluid at rest (Pascal's Law)

$p_2 = p_2 + \rho g h$, Where the $p_2$ and $p_1$ are the pressures at pts 2 and 1, $\rho$ is the density of the fluid, g is the gravitational acceleration and $h$ is the distance in the vertical btw pts 2&1

Absolute pressure: The total pressure including atmospheric. Gauge pressure: The excess pressure above atmospheric pressure $p_{gauge} = p_{ab} - p_{atm}$

Bernoulli's equation

$$p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

## Chapter 13: Gravitation

Newton's Law of Universal Gravity. ***F*** is the force exerted by objects of mass $m_1$ and $m_2$ on one another at a distance $r$ apart. The direction of force is along the line joining the two objects. These force obey Newton's third law and can be considered an action reaction pair.

$$\vec{F} = G_N \frac{m_1 m_2}{r^2}\hat{r}$$

Equations for satellite motion in a circular orbit. Here $r$ is the distance between the satellite and the object being orbited. $G_N$ is Newton's constant and $T$ is the period or time it takes the satellite to go one time around.

$$F_G = m a_{rad} \Rightarrow F_G = m_{Sat}\frac{v^2}{r}$$

$$G_N \frac{m_{Sat} m_{Earth}}{r^2} = m_{Sat}\frac{v^2}{r} \Rightarrow G_N \frac{m_{Earth}}{r} = v^2$$

$$G_N \frac{m_{Earth}}{r} = \left(\frac{2\pi r}{T}\right)^2 \Rightarrow T = \frac{2\pi}{\sqrt{G_N m_{Earth}}} r^{3/2}$$

$$G_N = 6.67\times 10^{-11}\, N\cdot m^2/kg^2$$

Kepler's Laws
1. Each planet moves in an elliptical orbit with the sun at one of the foci of the ellipse
2. A line from a planet to sun sweeps out equal area in equal times
3. The periods of the planets are proportional to the 3/2 power of the major axis length of their orbit

Potential Energy ($U$) of a mass a distance $r$ from center of the Earth

$$U_E = -\frac{G_N m_E m}{r}$$

The weigth and gravitational constant g at surface of the earth is

$$w = F_g = \frac{G_N m_E m}{R_E^2}, \text{i} \qquad g = \frac{G_N m_E}{R_E^2}$$

## Chapter 14: Periodic Motion

Simple harmonic motion, restorative force proportional to $x$

$$F_x = -kx, \quad m a_x = -kx, \quad m\frac{d^2x}{dt^2} = -kx \rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Equations of motion for a mass attached to a spring

$$x(t) = A\cos(\omega t); \quad v_x = \frac{dx}{dt} = -\omega A\sin\omega t; \quad a_x = \frac{d^2x}{dt^2} = -\omega^2 A\cos\omega t; \quad \omega = \sqrt{\frac{k}{m}}$$

Total Energy of a mass attached to a spring on a frictionless surface

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2, \quad \rightarrow v_x = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

$A =$ Amplitude ($\pm$max displacement from equillibrium)$[m]$

$T =$ Period, time it takes to complete one cycle $[s]$

$f =$ Frequency, how many cycles per sec $[\text{Hz or } 1/s]$

$\omega =$ Angular frequency which is equal to $2\pi f\,[rad/s]$

$f = \frac{1}{T}$, frequency and period are inverse of each other

Angular simple harmonic motion is

$\omega = \sqrt{\frac{\kappa}{I}}$ , where $\kappa$ is the torsion constant and $I$ is the moment of inertia

The simple pendulum

$\omega = \sqrt{\frac{g}{L}}$ , where g is the gravitational acceleration, L is the length of the pendulum

The physical pendulum

$\omega = \sqrt{\frac{mgd}{I}}$, where $g$ is gravitational constant and $d$ is the distance btw center-of-mass and axis, $m$ is the mass and $I$ is the moment of inertia

## Chapter 15: Mechanical Waves

The speed of a mechanical wave is:

$v_{wave} = f\lambda$, where $f$ is frequency and $\lambda$ is the wavelength.

The magnitude of $v$ depends only on the physical properties of the media through which the wave is propagating. Thus for a string of length $l$ the wave speed

$v_{wave} = \sqrt{\frac{F_\perp}{\mu}}$, where $\mu$ is the linear mass density or mass per unit length of the string.

Power and Intensity and the Inverse square law

$$I = \frac{P}{4\pi r^2}; \qquad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Power in a sinousoidal wave,

$$P(x,t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t)$$

$$P_{max} = \sqrt{\mu F}\,\omega^2 A^2, \quad P_{ave} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$

Wave superposition

$$y_{total}(x,t) = y_1(x,t) + y_2(x,t)$$

Standing waves on a string

$$y(x,t) = A_{SW}\sin kx \sin\omega t$$

$$f_n = n\frac{v}{2L} = n f_1 \quad (n = 1,2,3...)$$

$$\lambda_n = \frac{2L}{n} = \frac{1}{n}\lambda_1 \quad (n = 1,2,3...)$$

Equation that describes a mechanical wave

$$y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right),$$

$$y(x,t) = A\cos(kx - \omega t),$$

$$\text{with } k \equiv \frac{2\pi}{\lambda} \text{ and } \omega \equiv \frac{2\pi}{T}$$

Velocity and acceleration of any particle on a transverse wave

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

$$a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t)$$

These eq. describe waves propagating to the right with a phase (), amplitude $A$, angular frequency $\omega$ and wave number $k$