Classical Physics
Ph.D. Qualifying Exam Fall 2016
Florida International University Department of Physics

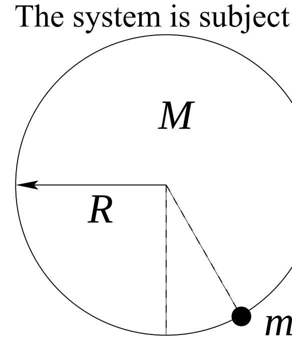
Instructions: There are nine problems on this exam. Three on mechanics (Section A), four on electricity and magnetism (Section B), and two on thermodynamics and statistical physics (Section C). You must solve a total of six problems with at least two from Section A, two from Section B, and one from Section C. You must also do the problems marked **Required**.

Do each problem on its own sheet (or sheets) of paper. Turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Write your student ID number on each page but not your name.

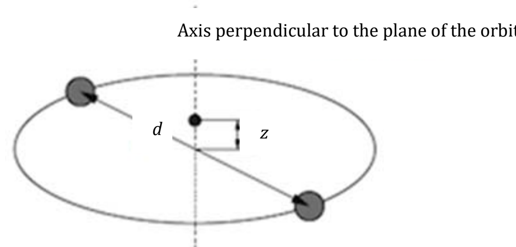
You may use a calculator and the math handbook as needed.

Section A

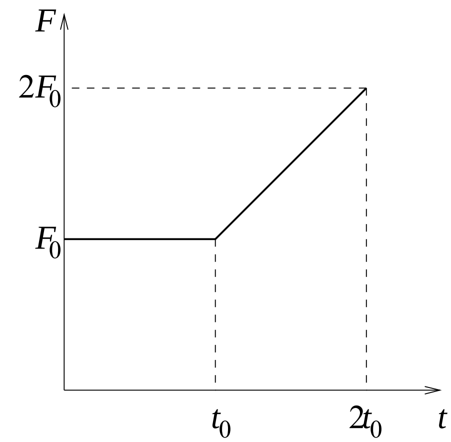
1. (**Required**) A disk of radius R and mass M is free to rotate about a light axle running through its center. It also has a mass m, imbedded on its rim. The system is subject to gravity.
   a. Write down the Lagrangian for the system.
   b. Find the equation of motion from the Lagrangian.
   c. Find the *stable* equilibrium position for the system. It may be obvious but some proof is needed.
   d. Find the frequency, ω, for small oscillations about the equilibrium position.



2. Two stars, each of mass M and separated by a distance d, orbit about their center of mass. A planetoid of mass m ($m<<M$) moves along the axis of this system perpendicular to the orbital plane of the starts with very small displacement z as shown in the figure. Determine the following in terms of the given symbols and fundamental constants:
   a. Determine T_S, the period of the stars around the common CM.
   b. Determine the ratio T_p/T_S of the period of planetoid oscillation to the period of star's orbit. (T_p represents the period of oscillation of the planetoid along the perpendicular axis).



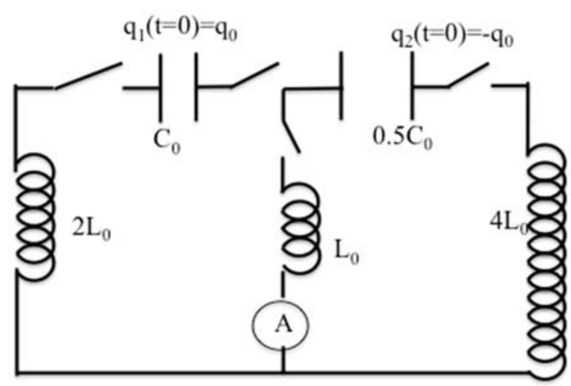
3. An object of mass m is initially at rest. A constant force of F_0 acts on the object for a time of t_0. The force then increases linearly with time such that at time $2t_0$ the force is $2F_0$. This force as a function of time is shown in the figure. Find the total distance traveled by the mass in the interval from $t = 0$ to $t = 2t_0$.



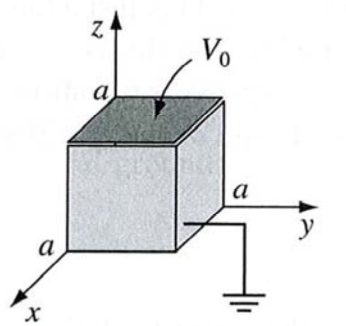
Section B

4. (**Required**) Write down Maxwell's equations (in differential form) and the charge-current continuity equation for a region that contains charge density ρ and current density $\boldsymbol{J}$. Explain the physical significance of each equation.
   Dr. Evil states that he has irrefutable evidence that magnetic monopoles exist. Write down Maxwell's equations (in differential form) and the charge-current continuity equations for a region that contains electric charge density ρ_e, magnetic charge density ρ_m, electric current density $\boldsymbol{J}_e$, and magnetic current density $\boldsymbol{J}_m$.

5. For the following LC circuit shown below, at time $t = 0$, the amount of charge on the left capacitor (capacitance C_0) was q_0, the amount of charge on the right capacitor (capacitance 0.5 C_0) is $-q_0$. The three inductors have inductance of $2L_0$, L_0, and $4L_0$ (from left to right) After all the switches are closed simultaneously, determine
   a. $q_1(t)$ and $q_2(t)$
   b. $I(t)$ as measured by the ammeter.

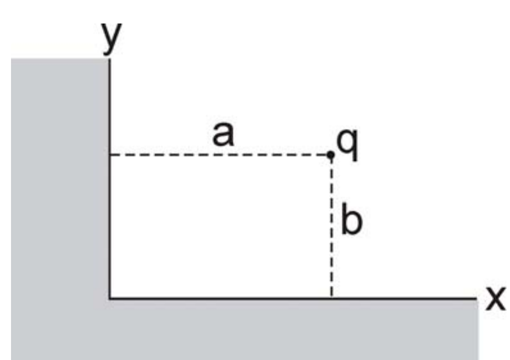


6. A cubical box (sides of length a) consists of five metal plates, which are welded together and grounded (see figure below). The top is made of a separate sheet of metal, insulated from the other sides, and held at constant potential V_0. Determine the electric potential inside the box.



7. A point charge q is located near two grounded, semi-infinite conductors shown in gray.

    a. Find the image charges that, together with q, will give the correct potential in the vacuum region.
    b. Find this potential, $\phi(x,y,z)$ in the vacuum region.
    c. Find the component $E_x(x,y,z)$ in the vacuum region and verify that it vanishes on the conducting plane for which it is the tangential component.
    d. Find the surface charge density on the surface for which E_x is the appropriate component to use.



Section C

8. One mole of an ideal *monatomic* gas, initially at pressure P_0 and occupying a volume V_0, is taken through the following thermodynamic cycle:
   - Step 1 – the gas is compressed *adiabatically* until its pressure has doubled.
   - Step 2 – the gas is compressed *isothermally* until its pressure is triple its initial value of P_0.
   - Step 3 – the gas is allowed to expand *adiabatically* until it has returned to its initial temperature.
   - Step 4 – the gas is allowed to expand *isothermally* until it has returned to its initial pressure and volume.
     a. Calculate T_0, the initial temperature of the gas, and the temperatures and volumes at the end of each of the four steps of the cycle.
     b. Calculate the work done *on* the gas during each of the four steps of the cycle.
     c. Calculate the heat transferred to the gas during each of the four steps of the cycle.
     d. Use the results of parts b and c to explicitly show that the first law of thermodynamics is satisfied for this cycle.

   Note that in any adiabatic process involving an ideal gas, the quantity PV^{γ} remains constant, where the adiabatic index γ is equal to the ratio of the molar specific heat at constant pressure to the molar specific heat at constant volume for that gas. Also, express all results in terms of P_0, V_0, and R.

9. Suppose that instead of the Maxwell-Boltzmann distribution, the distribution of molecular speeds in a classical gas was given by $n(v) = Ave^{-v/v_0}$ where A and v_0 are constants.
   a. Determine the constant A so that the total number of molecules in the gas is N.
   b. In terms of v_0, find the average speed, the *rms* speed, and the most likely speed (speed at the distribution peak) of the molecules in the gas.