Classical Physics
Ph.D. Qualifying Exam Fall 2017
Florida International University Department of Physics

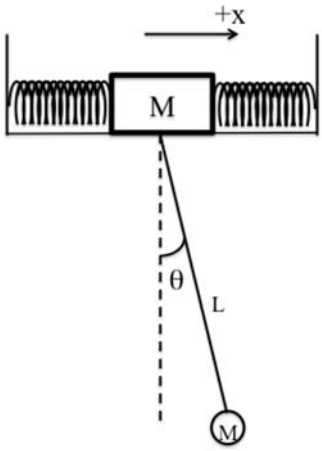
Instructions: There are nine problems on this exam. Four on Mechanics (Section A), three on Electricity and Magnetism (Section B), and two on Statistical Physics and Thermodynamics (Section C). You must solve a total of six problems with at least two from Section A, two from Section B, and one from Section C. You must also do the problems marked **Required**.

Do each problem on its own sheet (or sheets) of paper. Turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Write your student ID number on each page but not your name.

You may use a calculator and the math handbook as needed.

Section A: Mechanics

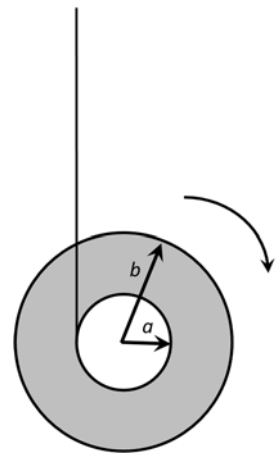
1) **[Required]** The following system has two objects with equal mass (M), shown below. There are also two massless springs both with $k = \mathrm{Mg/L}$ arranged so that at the equilibrium position, the springs are not compressed or stretched. Assuming small oscillations, determine the position $x(t)$ and the angle $\theta(t)$, if the pendulum was released from an initial angle of θ_0, and the block was initially at rest at the equilibrium position.



2) A bead of mass m slides without friction along a wire which has the shape of a parabola: $y = Ax^2$ with the y-axis vertical in the Earth's gravitational field g.
   a. Find the Lagrangian, taking as generalized coordinate the horizontal displacement x.
   b. Write down Lagrange's equations of motion. [Note: you do not need to solve the equations of motion!]

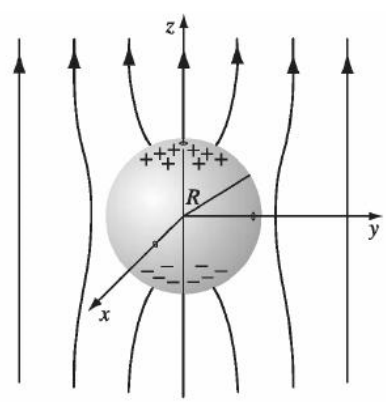
3) A spherical raindrop falling through fog or mist *accumulates* mass due to condensation at a rate proportional to its cross-sectional area multiplied by the velocity.
   a. Calculate the acceleration of the raindrop in terms of its radius and velocity. The raindrop starts from rest and has almost zero size.
   b. If a terminal velocity of 10 m/s is reached at a raindrop radius of 1 mm, find the proportionality constant for the increase in the raindrop's mass with velocity.
   c. find the relationship between the raindrop radius, *r*, and the height dropped, y. [Hint: $v = {dy}/{dt}$]
   d. How far does the raindrop fall as it grows to a size of 1 mm?

4) A yo-yo is allowed to unwind vertically as shown. Treat the yo-yo as a disk $(I = \frac{1}{2}mR^2)$ of mass *m* and radius *b* with an inner spindle (another solid disk) with diameter of radius *a*. You can ignore the mass of the inner spindle.
   a. Write the Lagrangian and the equation of constraint. Do not yet incorporate the constraint thereby having two coordinates in the Lagrangian.
   b. Find the equations of motion using Lagrange multipliers and solve for the vertical acceleration and the angular acceleration of the yo-yo.
   c. Identify the generalized forces and calculate them.



Section B: Electricity and Magnetism

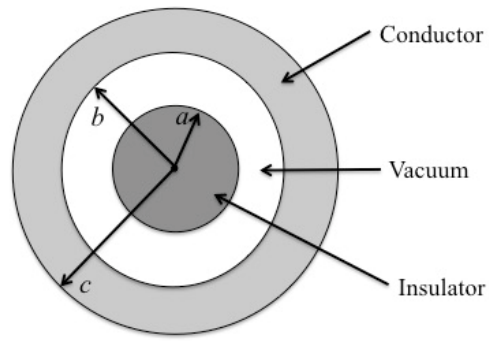
5) An uncharged metal sphere of radius *R* is placed in an otherwise uniform electric field $\vec{\boldsymbol{E}} = E_0\hat{\mathbf{z}}$. The field "pushes" positive charge to the "northern" surface of the sphere, and (symmetrically) negative charge to the "southern" surface. This induced charge, in turn, distorts the field in the neighborhood of the sphere (see figure to the right). Determine the **electric potential in the region outside the sphere** ($r > R$) and the **induced surface charge density**.



6) An insulating sphere of radius a carries a charge density of

$$\rho = \frac{k}{r}$$

in the region $r \leq a$, where k is a positive constant and r is the distance from the center of the sphere. The sphere is surrounded by a thick, concentric conducting metal shell with an inner radius b and an outer radius c. The conducting shell carries no net charge.

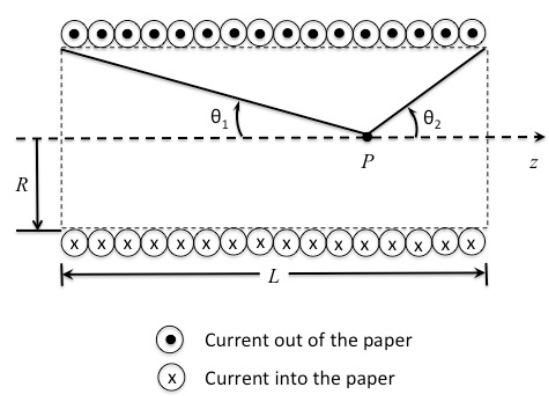


a. Determine the surface charge density at the inner ($r = b$) and outer surfaces ($r = c$) of the conductor.
b. Determine the electric field $\vec{\boldsymbol{E}}$ in all four regions (i) $r < a$, (ii) $a < r < b$, (iii) $b < r < c$, and (iv) $r > c$.
c. Determine the electric potential V at the center of the sphere using infinity ($r = \infty$) as a reference point.
d. If the outer shell is grounded, what would the potential at the center of the sphere be using the same reference point as in part (c).

7) Consider a cross-section of a finite solenoid shown in the figure below. The solenoid has length L, radius R, consists of n turns per unit length, and carries current I. Using the **Biot-Savart Law**, show that the magnetic field $\vec{\boldsymbol{B}}$ at any point P on the z-axis is given by:

$$\vec{\boldsymbol{B}} = \frac{\mu_0 n I}{2}[\cos\theta_1 + \cos\theta_2]\hat{\mathbf{z}}$$

where the angles θ_1 and θ_2 are defined in the figure below. **To receive full credit, you must fully develop the line integral for this finite solenoid.**



Section C: Statistical Physics & Thermodynamics

8) A heat engine operates between two heat reservoirs (T_H and T_L) by going through two isochoric and two isobaric processes (a rectangular cycle on P-V diagram). The working media is one mole of monoatomic ideal gas, with $P_H = 3P_L$ and $V_H = 2V_L$ (therefore $T_H = 6T_L$).
    a. Sketch the cycle and calculate the two intermediate temperatures at the end of the two isochoric processes, express them in terms of T_L.
    b. Calculate the work output during a single cycle.
    c. Calculate the efficiency of this heat engine and compare the efficiency with that of a Carnot engine.

9) A superconductor has a critical field curve that varies with temperature as $H_C(T) = H_{C-Max}[(T_C - T)/T_C]^{5/2}$ where H_{C-Max} is a constant. The difference $(C_S - C_n)$ between the heat capacities in the superconducting state and the normal state at the critical field H_C (which is a function of T) below T_C is given by

$$(C_S - C_n) = \frac{VT}{4\pi}\left(\frac{dH_C}{dT}\right)^2 + \frac{VT}{4\pi}H_C(T)\left(\frac{d^2H_C}{dT^2}\right)$$

Derive an expression that gives the temperature dependence of $(C_S - C_n)$.