Classical Physics
Ph.D. Qualifying Exam Fall 2018
Florida International University Department of Physics

Instructions: There are nine problems on this exam. Three on Mechanics (Section A), four on Electricity and Magnetism (Section B), and two on Statistical Physics and Thermodynamics (Section C). You must solve a total of six problems with at least two from Section A, two from Section B, and one from Section C.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Write your student ID on each page and the question number **but DO NOT WRITE your name**.

You may use a calculator and the math handbook as needed.

Section A: Mechanics

1) ``Ion traps'' are used to study the properties of individual atomic ions. The potential energy of a trapped ion can by written as:

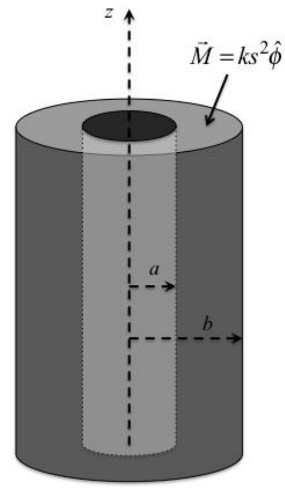
$$U(x, y) = \frac{1}{2}kr^2, \text{ where } r^2 = x^2 + y^2$$

Write:
   a. the Lagrangian, using coordinates x and y
   b. the two Lagrange equations of motion
   c. the equations of motion $x(t)$ and $y(t)$

2) A rocket with initial mass m_0 is far from an external gravitational field. It starts from rest and accelerates uniformly at a up to a final speed of v by expelling gas out the back end at a speed of u relative to the rocket. How much work is done by the rocket's engine?

3) Consider a thin equilateral triangle of mass m with sides of length a. The triangle is suspended from one apex such that the triangle is constrained to rotate in the plane of the triangle.
   a. Find the moment of inertia about the pivot.
   b. Find the frequency of small oscillations assuming the triangle is subject to gravity.
   c. Flip the triangle over and place the pivot at the center of one side so that the apex hangs down. Do parts (a) and (b) for this configuration.

Section B: Electricity and Magnetism



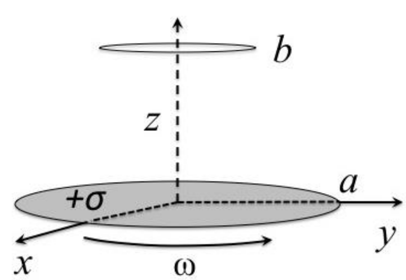
4) Consider the long cylindrical shell centered on the z-axis with inner radius a and outer radius b. The cylindrical shell carries a "frozen-in" magnetization, parallel to the xy-plane $\vec{\boldsymbol{M}} = ks^2\hat{\phi}$ where k is a constant and s is the distance from the axis.
   a. Determine all bound currents ($\vec{\boldsymbol{J}}_b$ and $\vec{\boldsymbol{K}}_b$).
   b. **Using the bound currents determined in part (a),** determine the magnetic field $\vec{\boldsymbol{B}}$ and the auxiliary field $\vec{\boldsymbol{H}}$ for the regions $s < a$, $a < s < b$, and $s > b$.

5) Two large parallel conducting plates are a distance d apart. The region between them is filled with two linear isotropic and homogeneous (l.i.h.) layers. The first of thickness x has conductivity σ_1 and permittivity ε_1. The second of thickness $d - x$ has conductivity σ_2 and permittivity ε_2. The plates are maintained at potentials V_1 and V_2, and there is a steady current flowing from one plate to the other.
   a. Find the potential at the interface between the two layer.
   b. Find the free surface charge density also at the interface between the two layers.

6) A conducting wire is wrapped around a vertical cylinder of radius a to form a helical coil with pitch angle α (i.e. any segment of the wire makes an angle α with the horizontal). The helix has N complete turns. If the wire carries a current I, show that the B field at the center of the helix is,

$$\frac{1}{2}\left(\frac{\mu_0 N I}{a}\right)(1 + \pi^2 N^2 \tan^2\alpha)^{-1/2}$$

7) An infinitesimally thin disk of radius a rotates about its symmetry axis with angular velocity ω. The disk carries a constant surface charge density +σ.
   a. Determine the magnetic dipole moment $\boldsymbol{m}$ of this spinning disk.
   b. Determine the (dipole approximation) magnetic field $\boldsymbol{B}$ anywhere on the z-axis due to this spinning disk.
   c. A small circular loop of wire (radius b) is held a distance z above the center of the spinning disk. The planes of the loop and the disk are parallel, and perpendicular to the z-axis. After rotating at a constant speed for a long time, the disk is accelerated with a constant angular acceleration α. If the circular wire has internal resistance R, determine the **magnitude and direction of the induced current in the small circular wire due to the accelerating disk.** You may assume that the loop is extremely small (*i.e.*, the field passing through it is uniform) and very far away from the disk (*i.e.*, $z \gg a$).



Section C: Statistical Physics & Thermodynamics

8) A system is composed of N identical non-interacting particles. Each particle can be in one of two states with energies such that $\varepsilon_1 < \varepsilon_2$. The maximum for the heat capacity C_V occurs at a temperature

$$T_{max} = \frac{(\varepsilon_2 - \varepsilon_1)}{2.4\, k_B}.$$

What is the maximum value of the molar heat capacity?

9) For any quasi-static process, the First Law can be written as:

$$TdS = dU + PdV.$$

Consider water as an *incompressible* liquid and water vapor as an ideal gas, both with constant heat capacities:

a. For one mole of water, $dU = CdT$, show that entropy of one mole of water is: $S(T) = S(T_0) + C \ln \frac{T}{T_0}$ (molar heat capacity $C = 75.2$ J/K)

b. For one mole of water vapor, $dU = C_v dT$, show that its entropy is: $S(T,P) = S(T_0, P_0) + (C_v + R) \ln \frac{T}{T_0} - R \ln \frac{P}{P_0}$ (molar heat capacity $C_v =$ 24.9 J/K, $R = 8.31$ J/K).

c. Assuming a constant heat of evaporation (40.6 KJ/mol), derive the boiling temperature under a pressure of 1.013×10^4Pa (1/10 of atm).