# Modern Physics <br> Ph.D. Qualifying Exam Fall 2016 <br> Florida International University Department of Physics 

Instructions: There are nine problems on this exam. Five on quantum mechanics (Section A), and four on general modern physics (Section B). You must solve a total of six problems with at least two from each section. You must also do the problems marked Required.

Do each problem on its own sheet (or sheets) of paper. Turn in only those problems you want graded (Do NOT submit for grading more than 6 problems all together). Write your student ID number on each page but not your name.

You may use a calculator and the math handbook as needed.

## Section A

1. (Required) A particle in an infinite square well potential between $x=0$ and $x=L$ is described initially by a normalized wavefunction that represents a superposition of the ground and first excited states: $\Psi(x, 0)=\frac{1}{\sqrt{2}}\left[\psi_{1}(x)+\psi_{2}(x)\right]$, where $\psi_{1}(x)$ and $\psi_{2}(x)$ are themselves normalized.
a. What is $\Psi(x, t)$ at a later time $t$ ?
b. Show that the average energy is equal to $\left(E_{1}+E_{2}\right) / 2$, where $E_{1}$ and $E_{2}$ are energies of the ground and first excited states respectively.
c. Show that the average position of the particle oscillates with time as
$\langle x\rangle=\frac{L}{2}+A \cos (\Omega t)$, where $A=\int x \psi_{1}^{*} \psi_{2} d x$ and $\Omega=\left(E_{2}-E_{1}\right) / \hbar$
d. Show that $A=-\frac{16 L}{9 \pi^{2}}$
e. Briefly discuss the relevance of the expression for $\Omega$.
2. An electron is moving freely in one dimension in the region $-\frac{L}{2} \leq x \leq \frac{L}{2}$ and its wave function satisfies the periodic boundary condition $\psi(x)=\psi(x+L)$.
a. Find the eigenfunctions for both the momentum and Hamiltonian (neglect the electron spin).
b. A small perturbation $H^{\prime}=\varepsilon \cos (q x)$ (with $L q=4 \pi N$ and $N$ is an integer) is added. Find the energy and stationary wave function to order $\varepsilon$ if the electron momentum is $|p|=q \hbar / 2$.
3. An electron spin is in a uniform magnetic field $\vec{B}=B_{0} \hat{\jmath} \hat{\jmath}$ is the unit vector in the $+y$ direction). At $t=0$, the electron spin is aligned in the $+z$ direction. Find the spin wave function at a later time $t$.
4. Two spin $1 / 2$ particles form a composite system. Spin 1 is in the eigenstate of $S_{1 x}=-\hbar / 2$ and spin 2 is in the eigenstate of $S_{2 y}=+\hbar / 2$. What is the probability that the measurement of the total spin will give the value zero?
5. A particle is incident upon a square barrier of height $U$ and width $L$ and has $E=U$. What is the probability of transmission? You must show all work.

## Section B

6. The $J / \psi$ particle (mass $=3.097 \mathrm{GeV} / \mathrm{c}^{2}$ ) is composed of a charm and an anticharm quark and was first produced in $e^{+} e^{-}$collision experiments in the 1970's. It provides one of the most important bits of evidence for the quark model. It can also be produced using a photon beam, using various targets. Consider the simplest case where a proton $\left(M_{p} c^{2}=938 \mathrm{MeV}\right)$ target is used, and a photon is incident on a proton in the reaction: $\gamma+p \rightarrow p+J / \Psi$.
a. What is the minimum photon energy needed to produce the $J / \psi$ particle in the above reaction?
b. Determine the speeds $(\beta)$ of all four particles involved in this reaction in the laboratory frame at the threshold beam energy determined from part a.
7. A particle, moving at a velocity of $(0,0.4 c, 0.3 c)$ relative to the lab frame, will have a different velocity in another reference frame, moving at ( $0,0,0.5$ c) relative to the lab frame. Determine the speed of this particle in the new frame.
8. Two events occur at locations $x_{1}$ and $x_{2}$ and at times $t_{1}$ and $t_{2}$ in reference frame $S$.
a. What is the time difference, $\Delta t^{\prime}=t_{2}^{\prime}-t_{1}^{\prime}$ between these two events in a reference frame $S^{\prime}$ that is moving with speed $\beta c$ in the positive $x$ direction relative to $S$ ? Your answer should be in terms of $\Delta t, \Delta x, \beta$, and $\gamma$, where $\Delta t=t_{2}-t_{1}$ and $\Delta x=$ $x_{2}-x_{1}$
b. Describe the special case of $x_{1}=x_{2}$.
c. Find $\beta$ such that the two events occur simultaneously in $S^{\prime}$ and describe any limiting conditions.
9. A certain star has one hydrogen atom per million in the first excited state. Assuming that all the rest are in the ground state, what is the temperature of the star? What fraction of the hydrogen atoms in this star are in the $n=3$ state? You will need $k_{B}=8.617 \times 10^{-5}$ $\mathrm{eV} / \mathrm{K}$.
