Modern Physics
Ph.D. Qualifying Exam Fall 2017
Florida International University Department of Physics

Instructions: There are nine problems on this exam. Five on quantum mechanics (Section A), four on general modern physics (Section B). You must solve a total of six problems with at least two from each section. You must also do the problems marked **Required**.

Do each problem on its own sheet (or sheets) of paper. Turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Write your student ID number on each page **but not your name**.

You may use a calculator and the math handbook as needed.

Section A: Quantum Mechanics

1) The Hamiltonian of a quantum system is given by $H = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} + \begin{pmatrix} 0 & -i\beta \\ i\beta & 0 \end{pmatrix}$ where α and β are real constants.

   a. Find the energy eigenvalues and eigenfunctions.
   b. With $\beta \ll \alpha$ treat the 2nd term as a small perturbation, find the first-order correction to the energy eigenvalues and the corresponding eigenfunctions. Compare with your results in (a).

2) Two particles move in a 2-D infinite potential well given by

$$V = 0 \qquad \text{for } -a \le x \le a \text{ and } -a \le y \le a$$
$$V = \infty \qquad \text{elsewhere.}$$

   Find the first excited states of the two-particle system in terms of explicit single-particle wavefunctions for
   a. When the two particles are identical bosons
   b. When the two particles are identical fermions (neglect particle spin).

3) A spin-1/2 particle with gyromagnetic ratio γ is at rest in a static magnetic field $B_0\widehat{\boldsymbol{k}}$ (along the z direction). Now you turn on a small transverse radio-frequency field $\vec{\boldsymbol{B}}(t) = B_{rf}\cos(\omega t)\,\hat{\boldsymbol{\imath}}$ (along the x direction) with $B_{rf} \ll B_0$. At $t = 0$, the spin is in the down state $\begin{pmatrix}0\\1\end{pmatrix}$. Using the first-order time-dependent perturbation theory, find the probability of the spin in the up state $\begin{pmatrix}1\\0\end{pmatrix}$ as a function of t.

4) **[Required]** The ground state of a particle is given by the time-dependent wave function

$$\Psi_0(x,t) = Ae^{-\alpha x^2 + i\beta t}$$

with an energy eigenvalue of $E_0 = \frac{\hbar^2\alpha}{m}$.

a. Determine the potential in which this particle exists. Does this potential resemble any that you have seen before?
b. Determine the normalization constant A for this wave function.
c. Determine the expectation values of x, x^2, p and p^2.
d. Check the uncertainty principle Δx and Δp. Is their product consistent with the uncertainty principle?

5) A particle with positive energy ($E > 0$) moves in the $+x$ direction in a region where the potential is given by

$$\begin{aligned} V(x) &= 0 && \text{for } x < 0 \\ &= -V_0 && \text{for } x > 0 \end{aligned}$$

where V_0 is a positive, real constant.

a. Solve the Schrödinger equation for the regions $x < 0$ and $x > 0$.
b. State the appropriate boundary conditions for this problem. Use these boundary conditions to relate the incident wave amplitude to the reflected wave amplitude.
c. Determine the reflection coefficient when $V_0 = 2E$. Your final answer should be in numeric form. Discuss the physical significance of your answer in terms of incident, reflected, and transmitted waves.

Section B: Modern Physics

6) The Rutherford scattering cross-section formula for charged particles makes two basic assumptions: 1) the interaction is mediated by long-range Coulomb forces and 2) the particles are point particles. Departure from the point-particle form is an indicator of nuclear structure for low energy projectiles that do not have sufficient energy to penetrate the Coulomb barrier. Consider an alpha particle that collides head-on with an Aluminum (Z = 13) target and thus scatters at large angle. With this information estimate the radius of the Aluminum nucleus for an alpha particle with a KE = 7.7 MeV initially very far away from the target.

7) In a photoproduction experiment a 5 GeV photon beam is incident on a fixed proton target. If in the reaction $\gamma + p \rightarrow p + \pi^0$, one wants to detect π^0s that are produced at 90^0 with respect to the photon beam in the center-of-momentum frame (Note $m_p = 0.938\ \mathrm{GeV}/c^2$ and $m_{\pi^0} = 0.135\ \mathrm{GeV}/c^2$)
    a. determine the angle between the produced π^0s and the photon beam in the lab frame.
    b. determine the energy of protons in the target in the center-of-momentum frame.

8) Given the stated measurements below each with independent, uncorrelated and random uncertainties, calculate the value and uncertainty for the results q for each of the scenarios. note: quote properly the units in your final answer
    a. When q depends on x and y as $q = x^2 y$, $x = 3.0 \pm 0.3$ cm, $y = 2.0 \pm 0.1$ m.
    b. The charge to mass ratio e/m of an electron is determined from accelerating voltages V, and currents I and I_e,

$$q = e/m = 1.6 \times 10^{10} \frac{V}{(I - I_e)^2}$$

where $V = 22.2 \pm 1.1$ [V], $I = 1.7 \pm 0.2$ [A] and $I_e = 0.2 \pm 0.1$ [A]

9) A plank at rest has a length L_0 and is inclined at an angle θ_0 to the x axis. The plank now moves at a speed v parallel to the x axis relative to Bob.

a. Show that, according to Bob, the plank has a length,

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}\cos^2\theta_0}\ ,$$

and the angle it makes with the x axis is:

$$\theta = \tan^{-1}\left(\frac{\tan\theta_0}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

b. Bob quickly constructs a wall slanted at an angle θ and with an opening of length L so that when the plank passes through the opening (at time $t = 0$), the plank completely fills the opening in the wall's rest frame. According to Anna, an observer moving along with the plank, the plank is of length L_0, which is greater than L. Also, because the wall is moving towards Anna, the length of the opening is less than L. Can the plank pass through the opening according to Anna? Give an explanation. If it can, what is the difference in the times that the top and bottom of the plank pass through the opening (according to Anna)?