Modern Physics
Ph.D. Qualifying Exam Fall 2018
Florida International University Department of Physics

Instructions: There are nine problems on this exam. Five on quantum mechanics (Section A), four on general modern physics (Section B). You must solve a total of six problems with at least two from each section.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Write your student ID on each page and the question number **but DO NOT WRITE your name**.

You may use a calculator and the math handbook as needed.

Section A: Quantum Mechanics

1) Consider a particle with the Hamiltonian:

$$H = E_0 \begin{pmatrix} 4 & 3i \\ -3i & 4 \end{pmatrix}$$

   a. If the energy of the particle is measured what are their possible energy values?
   b. What is (are) the possible state (states) that the particle can be in after the energy of the system is measured?
   c. If the system is prepared in an initial state $|\psi(t=0)\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, find the time-evolved state $|\psi(t)\rangle$ at some later time, in matrix form.

2) Two spin ½ particles form a composite system. Spin 1 is in the eigenstate of S_{1x} with the eigenvalue $+\frac{\hbar}{2}$ and spin 2 is in the eigenstate of S_{2y} with the eigenvalue $-\frac{\hbar}{2}$. What is the probability that the measurement of the total spin will give the value $\hbar$?

3) The Hamiltonian of a quantum system in the 3-D Hilbert space is given by:

$$\begin{pmatrix} 1 & \varepsilon & 0 \\ \varepsilon & 1 & \varepsilon \\ 0 & \varepsilon & 2 \end{pmatrix}$$

   a. Find the energy eigenvalues and eigenfunctions of the system.
   b. With $\varepsilon \ll 1$, treat ε as a small perturbation, find the energy eigenvalues and the wavefunctions of the system up to first order corrections. Compare with your results in (a).

4) A particle of mass m is stuck in a 2-D rectangular box of length a and width b. Derive the normalized wavefunctions and allowed energies for the ground and first excited states using the Schrodinger equation.

5) A quantum system consists of two particles with three single-particle states represented by the orthonormal wavefunctions Ψ_0 (the ground state with energy E_0), Ψ_a and Ψ_b (the degenerate excited states with same energy E_1). Find all different orthonormal wavefunctions of the **first excited states** of the two-particle system if
   a. the two particles are distinguishable,
   b. the two particles are identical fermions, and
   c. the two particles are identical bosons.

Section B: Modern Physics

6) Two protons are each accelerated to a kinetic energy of 2-GeV and are then collided head-on.
   a. Determine the total energy (in GeV) and momentum (in GeV/c) of each proton as measured in the laboratory frame. Recall that the rest mass energy of a proton is 0.938 GeV.
   b. Determine the speed (in units of c) of each proton just before the collision as measured in the lab frame.
   c. Determine the speed (in units of c) of each proton just before the collision as measured in the rest frame of the other proton.
   d. Determine the momentum (in GeV/c) that would be measured for one of the protons by an observer in the rest frame of the other proton?

7) The first ever interstellar flight takes off from Earth traveling at 0.75c. The flight plan requires the crew to send a signal back to Earth 30 days after departure as measured in the ship's frame. How long after departure is the signal received on Earth?

8) The initial activity N_0 and the mean life τ of a radioactive source are known with uncertainties of 1% each. The activity follows the exponential distribution
$$N = N_0 e^{-t/\tau}.$$
The uncertainty in the initial activity N_0 dominates at small t; the uncertainty in the mean life τ dominates at large t ($t \gg \tau$). For what value of t/τ do the uncertainties in N_0 and τ contribute equally to the uncertainty in N ? What is the resulting uncertainty in N

9) Consider the relativistic form of Newton's Second Law of motion. Show that when a force $\vec{\boldsymbol{F}}$ is applied along the direction of relative motion $\vec{\boldsymbol{v}}$
$$\vec{\boldsymbol{F}} = m\left(1 - \frac{v^2}{c^2}\right)^{-3/2} \frac{d\vec{\boldsymbol{v}}}{dt}$$