# Classical Physics Ph.D. Qualifying Exam Fall 2019 Department of Physics at FIU

**Instructions:** There are nine problems on this exam. Four on Mechanics (**Section A**), three on Electricity and Magnetism (**Section B**), and two on Statistical Physics and Thermodynamics (**Section C**). You must solve a total of six problems with at least **two** from **Section A**, **two** from **Section B**, and **one** from **Section C**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem number** on each page you turn in. Also turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Finally, write your panther ID on each page at top left-hand corner and number each page. **DO NOT WRITE your name** anywhere on the page.

You may use a calculator and the math handbook as needed.

#### **Section A: Mechanics**

**Problem A1:** An empty truck of mass *M* starting at rest moves under the influence of a constant net force *F*. As it moves, coal drops into the truck bed at a constant rate of

$$\frac{dm}{dt} = b$$

a. Show that the force *F*, the truck speed *v*, and the mass *m* of the coal in the truck at a particular moment are related by the equation

$$\frac{F}{b} = (m+M)\frac{dv}{dm} + v.$$

b. Solve the equation above to determine the velocity of the truck as a function of the mass *m*.

**Problem A2:** Two particles of mass  $m_1$  and  $m_2$  slide freely on a horizontal frictionless table and are connected by a spring with a force constant *k*. Derive Lagrange's equation of motion and find the oscillation frequency.

# **Classical Physics 2019 Cont.**

**Problem A3:** A particle of mass *m* moves in a central force with potential energy  $U(r) = Kr^4$ .

- a. For the force to be attractive, what is the sign of *K*? Don't guess. Prove it.
- b. What is the effective potential energy, V(r)?
- c. For a given value of angular momentum, *l*, find *r* for a circular orbit.
- d. What is the total energy of the circular orbit? Express your answer in terms of K and  $r_0$ , where  $r_0$  is the radius of the circular orbit.
- e. What is the period of the circular orbit? Express your answer in terms of *K*, *m*, and *r*<sub>0</sub>.

**Problem A4:** A particle of mass *m* is moving inside a vertical circular track of radius *R* (neglect friction). At the lowest track position, the particle speed is  $V_0$ . Use the Lagrange method to find the minimum value of  $V_0$  with which the particle will go completely around the circle without losing contact with the track.



### **Classical Physics 2019 Cont.**

### **Section B: Electricity and Magnetism**

**Problem B1:** A conical funnel-shaped surface is centered on the *z* axis as shown with its open ends at  $z = Z_1$  and  $z = Z_2$ . If it has a uniform surface charge density,  $\sigma$  what is the electric field at the origin?



**Problem B2:** A spherical capacitor consists of thin, spherical shells of radii *a* (inner) and *b* (outer), and the space between them contains a dielectric with a dielectric constant of  $\kappa_e$ .

a. If the potential difference between the shells is

$$\Delta \phi = (\phi_a - \phi_b) = A(1 - e^{-\alpha t}),$$

where *A* and  $\alpha$  are constants, calculate the  $\vec{D}$  field between the shells and the total displacement current between the shells.

b. Calculate the conduction current flowing to and from the shells and verify that it is the same as the total displacement current. (The capacitance of a spherical capacitor is  $C = 4\pi\epsilon ab/(b-a)$ ).

**Problem B3:** Starting from the Lorentz Force equation  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$  and using Maxwell's equations, in SI units for a vacuum, derive Coulomb's Law,

$$ec{m{F}}=1/(4\pi\epsilon_0\,)~(qq')/R^2\,\hat{m{r}}$$
 ,

take *R* as the separation between the charges q and q'.

## **Classical Physics 2019 Cont.**

### **Section C: Statistical Physics & Thermodynamics**

**Problem C1:** A system consists of *N* distinguishable, independent, identical particles, each fixed in place. Each particle has one of two possible energy states; the lower state has energy E = 0 and the upper state has energy  $E = \varepsilon$ . Calculate the energy of the system *E* as a function of temperature. Assume that *N* is large, and the temperature is high enough that a significant fraction of the particles are in their upper state.

**Problem C2:** An ideal gas with adiabatic exponent  $\gamma$  is taken around a complete thermodynamic cycle consisting of three steps. Staring at point *A*, the pressure is increased at constant volume  $V_1$  from  $P_1$  to  $P_2$  at point *B*. From point *B* to point *C*, the gas is allowed to expand adiabatically from volume  $V_1$  and pressure  $P_2$  to volume  $V_2$  and original pressure  $P_1$ . Finally, from point *C* to point *A*, the volume of the gas is decreased at constant pressure  $P_1$  back to the original volume  $V_1$ .

- a. Make a *PV* diagram of the complete cycle.
- b. For each step of the cycle, determine the change in the entropy of the gas. Sum your results to find the total entropy change for the whole cycle. Note that every step is reversible.
- c. Use the adiabatic relations to eliminate the pressure from your result for part b. Show that the resulting expression gives a total entropy change for the complete cycle equal to zero.