# Modern Physics Ph.D. Qualifying Exam Fall 2019 Department of Physics at FIU 

Instructions: There are nine problems on this exam. Four on Quantum Mechanics (Section A), Five on general Modern Physics (Section B), You must solve a total of six problems with at least two from each section.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to write the problem number on each page you turn in. Also turn in only those problems you want graded (Do NOT submit for grading more than 6 problems all together). Finally, write your panther ID on each page at top left-hand corner. DO NOT WRITE your name anywhere on the page.

You may use a calculator and the math handbook as needed.

## Section A: Quantum Mechanics

Problem A2: Consider a quantum system with the Hamiltonian

$$
H=E_{0}\left(\begin{array}{cc}
5 & -2 \\
1 & 2
\end{array}\right)
$$

where $E_{0}$ is a positive real number.
a. What are the eigenvalues of $H$ ?
b. What are the eigenvectors (normalized, matrix form) of $H$ ?
c. What is the expectation value of $H$ when the system is in the state

$$
|\psi\rangle=\frac{1}{\sqrt{10}}\binom{3}{1} ?
$$

Problem A2: A hydrogen atom is in the superposition state of eigenstates $|n| m\rangle$,

$$
|\psi\rangle=\frac{4}{\sqrt{26}}|1,1,1\rangle-\frac{i}{\sqrt{26}}|3,3,-3\rangle+\frac{3}{\sqrt{26}}|3,2,1\rangle
$$

a. What are the possible results of a measurement of energy and with what probabilities would they occur? What is the expectation value of the energy?
b. What are the possible results of a measurement of the angular momentum operator $\mathbf{L}^{2}$ and with what probabilities would they occur? What is the expectation value of $\mathbf{L}^{2}$ ?
c. What are the possible results of a measurement of the angular momentum component operator $L_{z}$ and with what probabilities would they occur? What is the expectation value of $L_{z}$ ?

## Modern Physics 2019 cont.

Problem A3: A harmonic oscillator system is prepared in the initial state described by

$$
|\psi(t=0)\rangle=\frac{2}{\sqrt{14}}|1\rangle+\frac{3}{\sqrt{14}}|2\rangle-\frac{1}{\sqrt{14}}|3\rangle
$$

a. At $t=0$, what is the expectation value of position, i.e. $\langle\widehat{\boldsymbol{x}}\rangle$ ? (Part $a$. is worth $2 x$ the number of points that parts b. and c. are.)
b. At $t=0$, what are the possible results of a measurement of energy and with what probabilities would they occur?
c. Find the time evolved state $\psi(x, t)$ or $|\psi(t)\rangle$. (You may use either the wavefunction or Dirac notation)

Problem A4: Consider the Hamiltonian $H=H_{0}+H^{\prime}$ for a 3-state system where

$$
H_{0}=\left(\begin{array}{ccc}
\frac{1}{2} \hbar \omega_{0} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} \hbar \omega_{0}
\end{array}\right) \quad H^{\prime}=\left(\begin{array}{ccc}
0 & \frac{\sqrt{3}}{2} \hbar \omega_{1} & 0 \\
\frac{\sqrt{3}}{2} \hbar \omega_{1} & \frac{1}{2} \hbar \omega_{1} & \frac{\sqrt{3}}{2} \hbar \omega_{1} \\
0 & \frac{\sqrt{3}}{2} \hbar \omega_{1} & 0
\end{array}\right)
$$

where $H^{\prime}$ is considered the perturbation.
a. Find the first-order corrections to the energies of all three eigenstates?
b. Assume the zeroth-order eigenstates are $\left|1^{(0)}\right\rangle,\left|2^{(0)}\right\rangle$, and $\left|3^{(0)}\right\rangle$, find the first-order corrections to the eigenstate vectors.
c. Find the second-order corrections to the energies of all three eigenstates.

## Modern Physics 2019 cont.

## Section B: Modern Physics

Problem B1: Proxima $b$ is the closest known exoplanet orbiting in the habitable zone of the star Proxima Centauri, approximately 4.2 light-years from the Earth. NASA decides to send its fastest experimental spacecraft that will travel at 0.75 c to study this planet.
a. According to an observer on Earth, how long will this trip take?
b. According to the crew of the spacecraft, how long will this trip take?
c. According to the crew of the spacecraft, how far will they travel?
d. As the spacecraft approaches Proxima b, the crew sends a radio signal back to Earth informing Mission Control of their arrival. If Mission Control is expecting a radio signal with frequency of 2.0 GHz , at what frequency should the spacecraft broadcast? Assume that the spacecraft is still traveling at 0.75c when the signal is broadcast.

Problem B2: Answer each of the Hydrogen atom related questions below.
a. Determine the energy (in eV), angular momentum (in $\hbar$ ), and radius (in $a_{0}$ ) of the first four electron orbitals in the hydrogen atom. Display your results on an energy level diagram.
b. A hydrogen atom absorbs a UV photon and an electron is excited to the third excited state $(n=4)$. Determine how much energy is required for this electronic transition. What is the maximum wavelength of this UV photon?
c. An electron initially in the $n=4$ state decays to the ground state. Determine all of the possible decay pathways available to this electron. Determine the energy (in eV ) and wavelength (in nm ) for all of the emitted photons

Problem B3: X-rays with energy of 300 keV undergo Compton scattering from a carbon target. The scattered X-rays are detected at an angle of 30 degrees relative to the incident rays.
a. Determine the Compton shift in units of nm for this scattering angle.
b. Determine the energy of the scattered x-rays in units of keV .
c. Determine the kinetic energy of the recoiling electron in units of keV.
d. Qualitatively describe the Compton Effect. What physical quantities are conserved during this interaction?

## Modern Physics 2019 cont.

Problem B4: Suppose a particle is confined to an infinite square well with potential

$$
V=\left\{\begin{array}{lc}
0, & \text { if } 0 \leq x \leq a \\
\infty, & \text { otherwise }
\end{array}\right.
$$

a. Starting with the time-independent Schrödinger equation, show that the general solution for the infinite square well is given by

$$
\psi(x)=A \sin (k x)+B \cos (k x) \quad \text { where } k=\frac{\sqrt{2 m E}}{\hbar}
$$

b. State and apply the appropriate boundary conditions to determine the coefficients $A, B$ and $k$. Use your result for $k$ to determine the allowed energy levels. Lastly, normalize the wave function.
c. Suppose a particle is described by the following mixed-state wave function

$$
\psi=\frac{1}{5}\left[4 \psi_{2}+3 \psi_{3}\right]
$$

what are the possible results of the measurement of energy on the system and with what probability would they occur? What energies do you get?

Problem B5: A $\pi^{-}$(pion) incident upon a proton that is at rest producing a $K^{0}+\Lambda$ (a neutral Kaon and lambda baryon) in the final state. Using the measured rest masses of the particles:

$$
m_{\pi}=139.57[\mathrm{MeV}], m_{K^{0}}=497.27[\mathrm{MeV}], m_{p}=938.27[\mathrm{MeV}], m_{\Lambda}=1115.68[\mathrm{MeV}]
$$

a. What is the threshold energy and momentum for the incident pion?
b. At threshold find the total energy of the $\pi-p$ system in the lab frame?
c. At threshold find the total momentum of the $\pi-p$.
d. Find the speed $\beta$ and the Lorentz factor $\gamma$ of the $\pi-p$ system at threshold.
e. Find the energy and momentum, in the lab frame, of both the $K^{0}$ and the $\Lambda$ at threshold. Compare these results to what you found in parts $b$ and $c$, respectively.

