Classical Physics
Ph.D. Qualifying Exam Fall 2020
Department of Physics at FIU

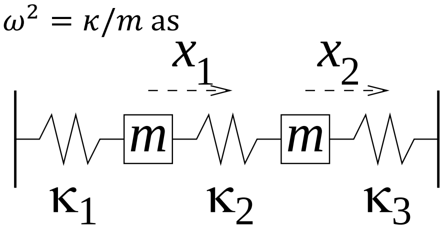
Instructions: There are nine problems on this exam. Three on Mechanics (**Section CM**), four on Electricity and Magnetism (**Section EM**), and two on Statistical Physics and Thermodynamics (**Section SP**). You must solve a total of six problems with at least **two** from **Section CM**, **two** from **Section EM**, and **one** from **Section SP**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and number)** on each page you turn in. Also turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Finally, write your panther ID on each page at top left-hand corner and the question identifier on each page. **DO NOT WRITE your name** anywhere on anything you turn in.

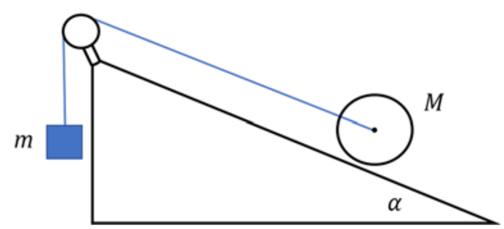
You may use a calculator and a math handbook as needed.

Section: Classical Mechanics

Problem CM1: Two identical masses are connected in series by three springs. The end springs each have one end fixed. The spring constants are each different, as shown. Determine the normal mode frequencies.
Hints: This can be done by finding the mass matrix and the $\boldsymbol{A}$ matrix. Also, if $\kappa_1 = \kappa_3 = \kappa_2 = \kappa$, the normal mode frequencies are: $\omega^2 = 3\kappa/m$ and $\omega^2 = \kappa/m$ as

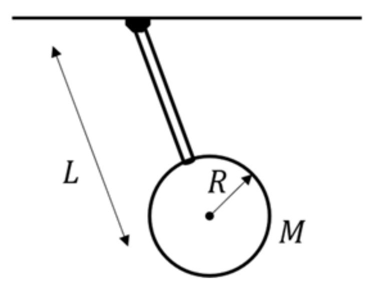


Problem CM2: A solid disk of mass M and radius R is rolling without slipping on a slope with an inclination angle α. The disk has a short weightless axle of negligible radius to which a string is attached. The string goes over a frictionless pulley and is attached to an object of mass m hanging vertically as shown in the figure below. Derive the acceleration of the object m from the Lagrange's equations of motion.



Problem CM3: A physical pendulum consists of a uniform disk of mass M and radius R rigidly attached to a massless rod such that the distance from the pivot to the center of the disk is L.

a) Find the period of small oscillation.
b) For what length of rod will the period be a minimum?



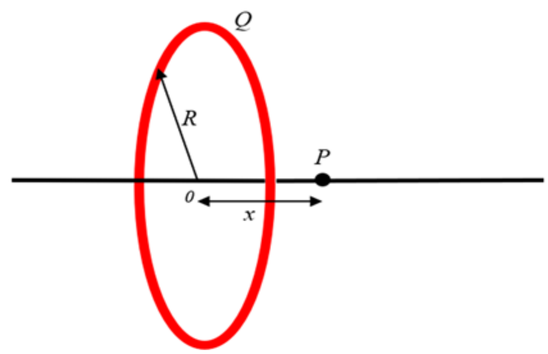
Section: Electricity and Magnetism

Problem EM1: An infinitely long cylinder of radius a has its axis along the z axis. Its magnetization is given in cylindrical coordinates by $\vec{\boldsymbol{M}} = M_0(\rho/a)^2\hat{\boldsymbol{\phi}}$.

a) Find the magnetization volume current density, $\vec{\boldsymbol{J}}_{\boldsymbol{m}}$, within the cylinder and the magnetization surface current density, $\vec{\boldsymbol{K}}_{\boldsymbol{m}}$ on the surface.
b) Show that the total charge transferred is zero. Find $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{H}}$ everywhere both inside and outside the cylinder.

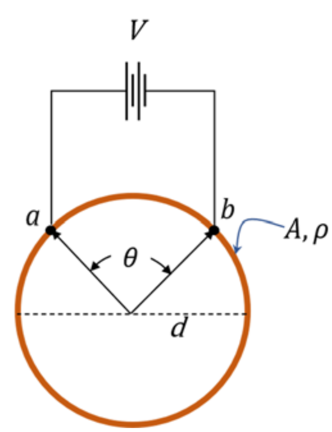
Problem EM2: Consider a thin, non-conducting circular loop of radius R that has a total charge $Q > 0$ uniformly distributed over it as shown below

a) Find the electric potential at point P on the symmetry axis a distance x away from the center of the ring.
b) Find the Electric field at the same point P.
c) A small particle of mass m and charge $q < 0$ is placed at point P and released. If $R \gg x$, the particle will undergo oscillations along the axis of symmetry. Find the angular frequency of the oscillations.

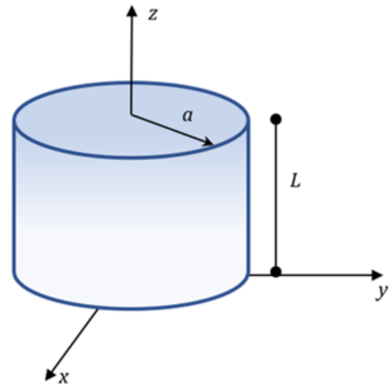


Problem EM3: A thin copper wire with cross-section A and resistivity ρ is fashioned into a circle of diameter d. Consider two points on the circumference of the circle that define an arc subtended by angle θ.

a) What is the resistance between the two points a and b on the circle?
b) Find the total or equivalent resistance of the ring.
c) The two points a and b on the circle are then connected to positive negative terminals of a battery of Voltage V, show that the magnetic field in the center of the circle is identically zero. You can neglect the magnetic field originating from the wires connecting the points on the circle to the battery.



Problem EM4: A solid cylinder of length L and radius a is aligned with its axis along the z axis and its lower end at the origin as shown. It has a magnetization given by $\vec{\boldsymbol{M}} = A_0 z\hat{\mathbf{z}}$ where A_0 is a constant and is situated in a magnetic induction given by $\vec{\boldsymbol{B}} = 3\hat{\mathbf{y}} + 4\hat{\mathbf{z}}$. Calculate the torque acting on the cylinder and its potential energy.



Section: Statistical Physics & Thermodynamics

Problem SP1: The essence of a fuel cell battery is to separate a chemical reaction, such as $O_2 + 2H_2 \rightarrow 2H_2O$, into two reactions (in aqueous solution):

1. $2H_2 + 4(OH^-) \rightarrow 4H_2O + 4e^-$ which occurs at the anode (negative terminal)
2. $O_2 + 2H_2O \rightarrow 4(OH^-) - 4e^-$ which occurs at the cathode and therefore captures the chemical energy as electric energy.

Using the following thermodynamic per mole data: **Enthalpy (in KJ):** $\mathbf{-285.8(H_2O)}$, $\mathbf{-230.0\ (OH^-)}$, **Entropy (in J/K):** $\mathbf{130.7(H_2), 205.1(O_2), -10.8(OH^-), 69.9(H_2O)}$ **and that the enthalpy of elemental gas (*O_2* and *H_2*) is zero.**

a) Calculate the amount of heat released for directly burning one mole of hydrogen gas in pure oxygen environment (under standard condition, and assume water formed is in the liquid phase).
b) Verify that a fuel cell based on the two electro-chemical reactions described above is thermodynamically feasible.
c) Calculate the electric energy released when one mole of oxygen gas (and two moles of hydrogen gas) is consumed in a fuel cell.
d) What is the voltage that such a fuel cell battery can produce?

Problem SP2: An unknown substance with a constant heat capacity under constant pressure $C_p = 34$ J/mol/K undergoes the following quasi-static cycle: from $T_1 = 298$ K and $P_1 = 1.0$ atm through an isobaric expansion to $T_2 = 600$ K, it expands further adiabatically and cools back to $T_3 =$ 298 K, and finally returns, isothermally, to pressure of 1.0atm.

a) Sketch qualitatively this three-legged cycle on a P vs. V and T vs. S diagram.
b) Calculate the amount of heat absorbed during isobaric expansion.
c) What is entropy change, ΔS, for the isobaric expansion?
d) Calculate the amount of heat released during the isothermal compression. Find the efficiency of this heat engine and compare to the theoretical Carnot cycle efficiency of $(1 - 298/600) = 50\%$.