Modern Physics
Ph.D. Qualifying Exam Fall 2020
Department of Physics at FIU

Instructions: There are nine problems on this exam. Five on Quantum Mechanics (**Section QM**), Four on general Modern Physics (**Section MP**), You must solve a total of six problems with at least **two** from **each section**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and number)** on each page you turn in. Also turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Finally, write your panther ID on each page at top left-hand corner and the question identifier on each page. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

Section: Quantum Mechanics

Problem QM1: Consider a particle of mass m in an infinite square well of width L. The eigenenergies are

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

and the eigenstates are

$$\varphi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L},$$

where $n = 1, 2, 3, \cdots$. The wave function of the particle at $t = 0$ is a coherent mixture of the second and third energy eigenstates:

$$\psi(x,0) = A(\varphi_2(x) + 2i\varphi_3(x)).$$

You might use $\int \sin^2 ax = \frac{x}{2} - \frac{\sin 2ax}{4a}$

a) Normalize $\psi(x,0)$ and find A.
b) Find the time dependent wave function $\psi(x,t)$ for $t \geq 0$. For convenience, you may use $\omega = \frac{\pi^2\hbar}{2mL^2}$.
c) At some time $t > 0$, what is the probability of measuring the particle to have energy $9\pi^2\hbar^2/(2mL^2)$? Does it depend on time?

Problem QM2:

1. Consider an electron (a spin ½ particle) in the spin state $|\psi\rangle = \frac{1}{\sqrt{25}}(3|+\rangle + 4|-\rangle)$, where $|+\rangle$ and $|-\rangle$ are the spin up and spin down state, respectively, in the z-direction. The electron passes through an ***x*-direction** Stern-Gerlach analyzer, what is the probability that it will be measured in the $|-\rangle_x$ state?
2. After the electron passes through the Stern-Gerlach analyzer, at time $t = 0$ the particle is measured to be in the the $|-\rangle_x$ state (i.e. for the following problem, the initial state is $|\psi(t=0)\rangle = |-\rangle_x$), also at this moment ($t = 0$), the electron enters a uniform magnetic field in the z direction: $\vec{B} = B_0\hat{z}$. What is the state $|\psi(t)\rangle$ at some time t later (in the S_z basis)?
   Hint: the Hamiltonian can be written as $H \doteq \frac{\hbar\omega_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$
3. What is the probability of measuring the particle in state $|\phi\rangle = \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$ at some time t later?

Section: Quantum Mechanics (cont.)

Problem QM3: A particle with positive energy ($E > 0$) moves in the $+x$ direction in a region where the potential is given by

$$\begin{aligned} V(x) &= 0 \qquad \text{for } x \le 0 \\ &= -V_0 \quad \text{for } x \ge 0, \end{aligned}$$

where V_0 is a positive, real constant.

a) Solve the Schrödinger equation for these two regions. If you make an ansatz, you must verify your answer to receive full credit.
b) State the appropriate boundary conditions for this problem. Use these boundary conditions to relate the incident wave amplitude to reflected wave amplitude.
c) Determine the reflection and transmission coefficients when $V_0 = 2E$. Your final answer should be in numeric form. Discuss the physical significance of your answer in terms of incident, reflected, and transmitted waves.

Problem QM4: Consider a four-state system with the Hamiltonian $H = H_0 + H'$, where

$$H_0 = \begin{pmatrix} V & 0 & 0 & 0 \\ 0 & 4V & 0 & 0 \\ 0 & 0 & 5V & 0 \\ 0 & 0 & 0 & 5V \end{pmatrix}, \quad \text{and} \quad H' = \begin{pmatrix} 0 & \delta V & 0 & 0 \\ \delta V & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\delta V \\ 0 & 0 & 2\delta V & 0 \end{pmatrix}.$$

The value $\delta \ll 1$ is a model of a small perturbation on the system.

a) Write down the eigenvalues and eigenvectors of the unperturbed system.
b) Use the perturbation theory to find the first non-zero corrections to the energy of the two **non-degenerate** eigenstates (the first two eigenstates).
c) Use the perturbation theory to find the first non-zero corrections to the energy of the two **degenerate** eigenstates.

Problem QM5: Consider a system of two angular momenta with $j_1 = 1$ and $j_2 = \frac{1}{2}$.

a) Write down all the possible states of this system in the uncoupled basis $|j_1 j_2 m_1 m_2\rangle$.
b) What are the allowed values of the coupled angular momentum quantum numbers J and M for this system?
c) Write down all the possible states of this system in the coupled basis $|JM\rangle$.
d) Use the Clebsch-Gordan coefficients (next page) in the attached table to express the coupled basis states $|JM\rangle$ in terms of the uncoupled basis states $|j_1 j_2 m_1 m_2\rangle$.

Table 11.3 Clebsch-Gordan Coefficients for $j_1 = 1$ and $j_2 = \frac{1}{2}$

$j_1 = 1$	$j_2 = \frac{1}{2}$	j	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
		m	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
m_1	m_2							
1	$\frac{1}{2}$		1	0	0	0	0	0
1	$-\frac{1}{2}$		0	$\frac{1}{\sqrt{3}}$	0	0	$\sqrt{\frac{2}{3}}$	0
0	$\frac{1}{2}$		0	$\sqrt{\frac{2}{3}}$	0	0	$-\frac{1}{\sqrt{3}}$	0
0	$-\frac{1}{2}$		0	0	$\sqrt{\frac{2}{3}}$	0	0	$\frac{1}{\sqrt{3}}$
−1	$\frac{1}{2}$		0	0	$\frac{1}{\sqrt{3}}$	0	0	$-\sqrt{\frac{2}{3}}$
−1	$-\frac{1}{2}$		0	0	0	1	0	0

Section: Modern Physics

Problem MP1: An electron and a positron (i.e., an anti-election) are created in a nuclear reaction and travel in opposite directions. The speed of the electron as measured in the laboratory frame is 0.724c. The speed of each particle relative to other is 0.950c.

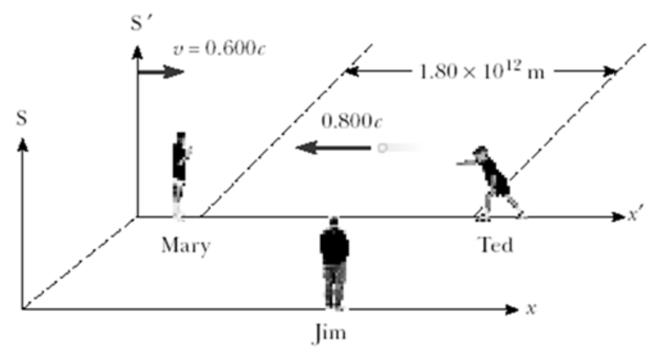
a) Determine the speed of the positron as measured in the laboratory frame.
b) Determine the kinetic energy and total energy (in MeV) of each particle, as measured in the laboratory frame. Recall that the rest mass energy of an electron and positron is 0.511 MeV, each
c) Determine the kinetic energy and total energy (in MeV) of each particle, as measured in the frame of the other particle.
d) Determine the magnitude of the momentum (in MeV/c) of each particle, as measured in the frame of the other particle.

Problem MP2: Consider an x-ray beam with $\lambda_x = 1.0 \times 10^{-10}$m and a γ-rays from a sample of Cs^{137}, with $\lambda_\gamma = 1.88 \times 10^{-12}$m. If the radiation scattered from free electrons is viewed at 90^0 to the incident beam. Constants:
$h = 4.1347 \times 10^{-12}\text{eV} \cdot \text{s}$, $c = 3.00 \times 10^8$ m/s, and $m_e = 0.511$ MeV/c^2

a) Derive the Compton wavelength shift equation.
b) Find the Compton wavelength shift for each case.
c) What kinetic energy is given to a recoiling electron in each case?
d) What percentage of the incident photon energy is lost in the collision for each case?

Problem MP3: Ted and Mary are playing a friendly game of relativistic catch in frame S', which is moving at 0.600c with respect to frame S. Jim, at rest in frame S, watches the action. Ted throws the ball to Mary at 0.800c (according to Ted) and their separation (measured in S') is 1.80×10^{12} m.

a) According to Mary, how fast is the ball moving?
b) According to Mary, how long does it take the ball to reach her?
c) According to Jim, how far apart are Ted and Mary,
d) According to Jim, how fast is the ball moving?
e) According to Jim, how long does it take the ball to reach Mary?



Section: Modern Physics (cont.)

Problem MP4: Meson states are categorized according to their quantum numbers J^{PC}, where J is the total angular momentum, P is the parity and C is the charge conjugation. In a constituent quark model (CQM), mesons are made up of spin-half quark-antiquark ($q\bar{q}$) pairs, where the gluons (g) that do not contribute to the (J^{PC}) of the composite meson state. For such a $q\bar{q}$ two body system the total angular momentum $\vec{J} = \vec{L} + \vec{S}$, $P = (-1\ )^{L+1}$and $C = (-1\ )^{L+S}$, where L is the orbital angular momentum, S is the total spin, and J is the total angular momentum. If you have not learned about parity P and charge Conjugation C , the above definitions should suffice to solve this problem.

a) Determine what are the combinations of J^{PC} that cannot be allowed for a meson state that is purely made of a ($q\bar{q}$) pair, (these kind of mesons are called exotic mesons), for $J\ =\ 0, 1, 2$.
b) If such a meson is found, what do you think they are made of? There are different possibilities, and you only need to provide one example