Classical Physics
Ph.D. Qualifying Exam Fall 2021
Department of Physics at FIU

Instructions: There are nine problems on this exam. Four on Mechanics (**CM**), three on Electricity and Magnetism (**EM**), and two on Statistical Physics and Thermodynamics (**SP**). You must solve a total of six problems with at least **two** from **Section CM**, **two** from **Section EM**, and **one** from **Section SP**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and number)** on each page you turn in. Also, turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, write your panther ID on each page at the top left-hand corner and the question identifier on each page. **DO NOT WRITE your name** anywhere on anything you turn in.

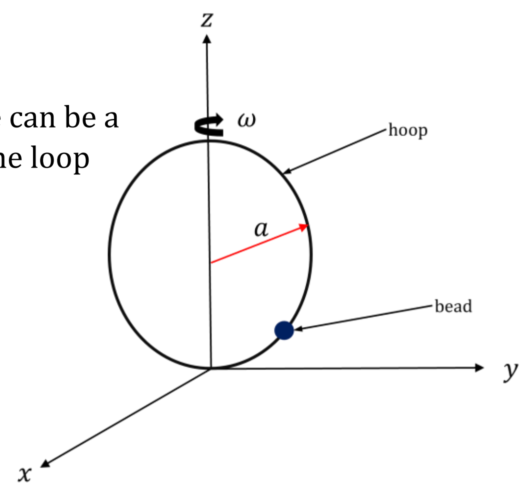
You may use a calculator and a math handbook as needed.

Section: Classical Mechanics

CM1: Two particles of mass m_1 and m_2 slide freely on a horizontal frictionless track and are connected by a spring with a force constant k. Derive the Lagrange equations of motion and find the oscillation frequency.

CM2: A particle of mass m is constrained to move on a frictionless circular hoop of radius a fixed in a vertical plane that rotates about its vertical symmetry axis with constant angular speed ω.

a) Find the equations of motion
b) Show that if ω is greater than a critical value ω_0, there can be a solution in which the particle remains stationary on the loop at a point other than the bottom. Find the value of ω_0.



CM3: A particle travels from position (x_1, y_1, z_1) to (x_2, y_2, z_2) during a (very short) time interval $\Delta t = t_2 - t_1$. The infinitesimal displacement vector for this particle in cylindrical coordinates is given by $d\hat{\boldsymbol{s}} = dr\hat{\boldsymbol{r}} + rd\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{z}}$

a) The cylindrical unit vectors $(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\phi}}, \hat{\boldsymbol{z}})$ are written in terms of cylindrical coordinates and the Cartesian unit vectors as $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}})$ and they are related by:

$$\hat{\boldsymbol{r}} = \cos\phi\,\hat{\boldsymbol{\imath}} + \sin\phi\,\hat{\boldsymbol{\jmath}}$$
$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\boldsymbol{\jmath}}$$
$$\hat{\boldsymbol{z}} = \hat{\boldsymbol{k}}$$

determine the time rate of change (derivative with respect to time) of the cylindrical unit vectors (*ie.*, $\dot{\hat{\boldsymbol{r}}}, \dot{\hat{\boldsymbol{\phi}}}$ and $\dot{\hat{\boldsymbol{z}}}$).

b) Determine the velocity vector $\vec{\boldsymbol{v}}$ of this particle in cylindrical coordinates.
c) Determine the acceleration vector $\vec{\boldsymbol{a}}$ of this particle in cylindrical coordinates.

CM4: A speedboat is at the equator and travels at a speed of 100 m/s. Find the apparent relative change in gravity $\left(\Delta g/g_0\right)$ for a boat traveling in all four directions:

a) straight east
b) straight west
c) straight north
d) straight south

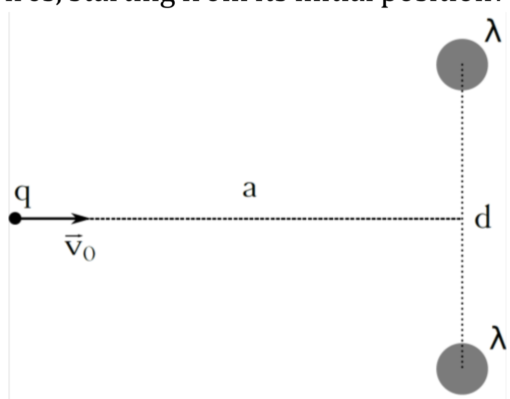
Note that there will potentially be contributions from both the centrifugal and Coriolis terms. Take the radius of the Earth to be $R_E = 6378$ km and use $g_0 = 9.81\ \mathrm{m/s^2}$.

Section: Electricity and Magnetism

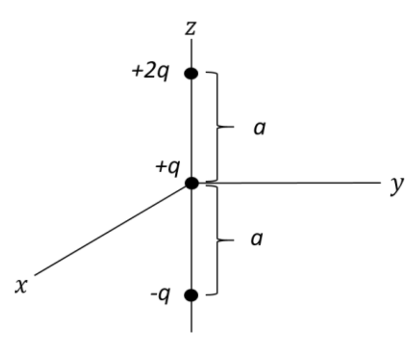
EM1: A sphere of radius a, has a volume charge density $\rho = Ar^2$ where A is a constant. It rotates about a diameter coinciding with the z-axis with constant angular velocity, ω . What is its magnetic dipole moment?

EM2: We consider the setup in the figure below. We have two infinite wires perpendicular to the plane of this sheet of paper, each with uniform linear charge density λ. The distance between the wires is d. We also have a positive point charge at a distance "a" along the perpendicular bisector of the line connecting the two wires. This point charge has mass m, charge $q > 0$ and an initial velocity v_0 along the perpendicular bisector towards the wires.

a) What is the magnitude and direction of the electrical force on the point charge as a function of the distance to the line connecting the two wires?
b) What is the work done by the electrical forces on the point charge as a function of the distance to the line connecting the two wires, starting from its initial position?
c) How large should v_0 minimally be so that the point charge passes the two wires?



EM3: For the system of 3-point charges shown in the figure below, determine the monopole moment, the dipole moment, and all components of the quadrupole moment tensor, relative to the origin.



Section: Statistical Physics & Thermodynamics

SP1: The thermodynamic properties of any system are completely given by its equation of state (EOS) and its internal energy function. The EOS of a certain gas is given as $(P + b)V = RT$, and its internal energy: $U = aT + bV + U_0$, where a, b, U_0, and R are constants.

a) Using the First Law: $\delta Q = dU + PdV$, calculate C_V, the heat capacity at constant volume.

b) Again, use the First Law: $\delta Q = dU + PdV$ to calculate the heat capacity at constant pressure, C_P, and show that $C_P - C_V$, is the constant R.

c) When combined with the Second Law ($\delta Q = TdS$), the First Law can be written in a differential form: $TdS = dU + PdV$. Prove that the internal energy function given above represents a real thermodynamic system. (*Hint: show that* $\left(\frac{\partial U}{\partial V}\right)_T$ *can be obtained directly from the EOS and it equals the constant* b).

SP2: The free-energy F for an ideal Fermi-Dirac gas is given by

$$F = -kTlnZ \; = \; -kT\left[\alpha N + \Sigma_r \ln\left(1 + e^{-(\alpha + \beta\epsilon_r)}\right)\right]$$

Using this expression, express F in terms of summations over $\bar{n}_r$ (the average number of particles in states r), the temperature T, and the parameter α.