

# Modern Physics

## Ph.D. Qualifying Exam Fall 2021

### Department of Physics at FIU

**Instructions:** There are nine problems on this exam. Four on Quantum Mechanics (**Section QM**), five on general Modern Physics (**Section MP**), You must solve a total of six problems with at least **two** from **each section**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and number)** on each page you turn in. Also, turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, write your panther ID on each page at the top left-hand corner and the question identifier on each page. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

## Section: Quantum Mechanics

**QM1:** A beam of spin-1/2 particles are prepared in the state represented by

$$|\psi\rangle = C(\sqrt{2}|+\rangle - \sqrt{3}i|-\rangle),$$

where  $|+\rangle, |-\rangle$  are  $S_z$  eigenstates.

- Find the normalization constant  $C$ . (Use this normalized state for the rest of the problem)
- Assume another state can be represented by  $|\phi\rangle = a|+\rangle + b|-\rangle$ , find  $a$  and  $b$  such that states  $|\psi\rangle$  and  $|\phi\rangle$  are orthogonal. (Choose  $a$  to be real and positive)
- If the spin component  $S_x$  is measured after the system is prepared in the state  $|\psi\rangle$ , what is the probability of measuring spin up for the  $S_x$  component?
- Assume the system is initially prepared in the state  $|\psi\rangle$ , what is the probability that the system is measured to be in the final state  $|\psi_f\rangle = \frac{1}{\sqrt{5}}(2|+\rangle - |-\rangle)$ ?

**QM2:** Find the energy eigenstates and eigenvalues of a particle confined to a delta function potential  $V(x) = -\beta\delta(x)$ , where  $\beta$  is a positive real constant. You may find the following definition of Dirac delta function  $\delta(x)$  useful:

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$$

**QM3:** An electron spin is in a uniform magnetic field  $\vec{B} = B_0 \hat{j}$  (where  $\hat{j}$  is the unit vector in the  $+y$  direction). At  $t = 0$ , the electron spin is aligned in the  $+z$  direction. Find the spin wavefunction at late time  $t$ . Express your answer in terms of the  $|-\rangle$  and  $|+\rangle$  eigenstates in the  $z$  direction.

**QM4:** The Hamiltonian of a harmonic oscillator is given by

$$H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right).$$

- Prove that  $[H, a^\dagger] = \hbar\omega a^\dagger$ . (Show your work, step by step!)
- Assume  $|E\rangle$  is an eigenstate with the eigenenergy  $E$ . Use the result from (a) and show that the eigenenergy of state  $(a^\dagger|E\rangle)$  is given by  $E + \hbar\omega$ . (Show your work, step by step!)
- Calculate the following:  $\langle 4|a|5\rangle, \langle 6|a^\dagger|5\rangle, \langle 2|a^\dagger a|2\rangle, \langle 4|\hat{x}|3\rangle$

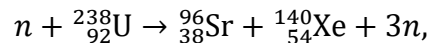
## Section: Modern Physics

**MP1:** Two rockets that are coasting (engines off) are heading towards each other on a collision course. As measured by Liz, a stationary Earth observer, Rocket 1 has a speed of  $0.800c$ , Rocket 2 has a speed of  $0.600c$ , both rockets are  $50.0\text{ m}$  in length, and are initially  $2.52$  billion kilometers apart.

- What are their respective proper lengths?
- What is the length of each rocket as observed by a stationary observer in the other rocket?
- According to Liz, how long before the rockets collide?
- According to Rocket 1, how long before they collide?
- According to Rocket 2, how long before they collide?
- If the crews can evacuate their rockets safely within  $50\text{ min}$  (their own time), will they be able to do so before the collision?

**MP2:** The CANDU (Canada Deuterium Uranium) reactor is a Canadian pressurized heavy water reactor design used to generate electrical power. Unlike an American reactor, the CANDU uses deuterium oxide (i.e., heavy water) as its moderator and natural uranium ( $^{238}\text{U}$ ) as its fuel.

- Calculate the energy released (in units of MeV) in the neutron-induced fission



given  $m(n) = 1.008665\text{u}$ ,  $m({}^{238}\text{U}) = 238.050784\text{u}$ ,  $m({}^{96}\text{Sr}) = 95.921750\text{u}$ ,  $m({}^{140}\text{Xe}) = 139.921647\text{u}$ . Recall that  $1\text{ u}$  is equivalent to  $931.5\text{ MeV}/c^2$

- Confirm that the total number of nucleons and total charge are conserved in this reaction.
- Using the result of part (a), determine the energy released from the fission of  $1.00\text{ kg}$  of  $^{238}\text{U}$ . How many gallons of gasoline would it take to generate the same amount of energy? Assume that  $1\text{ gallon}$  of gasoline contains  $1.3 \times 10^8\text{ Joules}$  of energy. Recall that  $1\text{ eV} = 1.60218 \times 10^{-19}\text{ J}$ .
- Explain why the fission of heavier nuclei into lighter nuclei releases energy.
- Discuss some of the advantages and disadvantages of using nuclear fission to generate electrical power. Why don't we currently use nuclear fusion to generate electrical power?

**MP3:** Let  $\mathbf{u} = d\mathbf{r}/dt$  be the velocity of a particle observed in an inertial frame  $K$ . The same quantity observed in an inertial frame  $K'$  moving with velocity  $\mathbf{v}$  with respect to  $K$  is  $\mathbf{u}' = d\mathbf{r}'/dt'$ .

- a) Use the transformation properties of  $dt$ ,  $\mathbf{r}_{\parallel}$  and  $\mathbf{r}_{\perp}$  (the direction along the velocity  $\mathbf{v}$  or perpendicular to the moving  $K'$  frame, respectively), to directly derive the velocity addition rule,

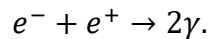
$$\mathbf{u}_{\parallel} = \frac{d\mathbf{r}_{\parallel}}{dt} = \frac{\mathbf{u}'_{\parallel} + \mathbf{v}}{1 + \frac{\mathbf{v} \cdot \mathbf{u}'_{\parallel}}{c^2}} \quad \text{and} \quad \mathbf{u}_{\perp} = \frac{d\mathbf{r}_{\perp}}{dt} = \frac{\mathbf{u}'_{\perp}}{\gamma(v) \left[ 1 + \frac{\mathbf{v} \cdot \mathbf{u}'_{\parallel}}{c^2} \right]}.$$

- b) Let  $\mathbf{v}$  define a polar axis with polar coordinates  $\mathbf{u} = (u, \theta)$ , and  $\mathbf{u}' = (u', \theta')$  for the particle velocities as measured in  $K$  and  $K'$ . Write the transformation laws in part (a) in the form of  $u = u(u', \theta')$  and  $\theta = \theta(u', \theta')$
- c) Use the results of (b) to show that  $u \rightarrow c$  when  $v \rightarrow c$ .

**MP4:** A muonic atom consists of a nucleus of charge  $Ze^+$  and a single negative muon  $\mu^-$ , found in a quantum mechanical orbit about said nucleus. Muons are elementary particles with the same spin and electric charge as an electron but have a mass that is 207 times larger. The following mass ratios will be necessary:  $m_p = 1836m_e$ ,  $m_{\mu} = 9m_e$ . Also, note that the muon is a non-negligible fraction of the mass of a proton.

- a) Using the Bohr angular momentum quantization condition  $L = n\hbar$  and Coulomb's Law derive the equation for the Bohr radius  $r_{Bohr}$ . Express your answer in terms of the atomic number  $Z$ , the electric charge constant  $e$ , the reduced Planck's constant  $\hbar$ , the mass of the electron  $m_e$  and quantum number  $n$ .
- b) Since the Bohr radius of a hydrogen atom in its ground state, in Angstroms is  $r_{Bohr} = 0.529 \text{ \AA}$ , find the muon nucleus separation, in Angstroms, for muonic hydrogen also in its ground state ( $Z = 1$ ,  $n = 1$ ).
- c) If the binding energy of the electron in a normal hydrogen atom in its ground state is 13.6 eV, find the binding energy of the muonic hydrogen atom also in its ground state.

**MP5:** An electron  $e^-$  with a kinetic energy of 1.000 MeV and mass of 0.511 MeV makes a head-on collision with a positron  $e^+$ , its anti-particle, at rest. In the collision, the particle anti-particle pair annihilate and produce two photons of equal energy and momenta each traveling at an angle  $\theta$  in the lab frame with respect to the electron's original direction. (Since photons are massless their energy is given by the relativist equation  $E = pc$ .) The reaction is:



Determine the energy  $E$ , and momentum  $p$  and angle of emission  $\theta$  of each photon in the lab frame.