Modern Physics
Ph.D. Qualifying Exam Fall 2021
Department of Physics at FIU

Instructions: There are nine problems on this exam. Four on Quantum Mechanics (**Section QM**), five on general Modern Physics (**Section MP**), You must solve a total of six problems with at least **two** from **each section**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and number)** on each page you turn in. Also, turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, write your panther ID on each page at the top left-hand corner and the question identifier on each page. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

Section: Quantum Mechanics

QM1: A beam of spin-1/2 particles are prepared in the state represented by

$$|\psi\rangle = C\big(\sqrt{2}|+\rangle - \sqrt{3}i|-\rangle\big),$$

where $+\rangle, -\rangle$ are S_z eigenstates.

a) Find the normalization constant C. (Use this normalized state for the rest of the problem)
b) Assume another state can be represented by $|\phi\rangle = a|+\rangle + b|-\rangle$, find a and b such that states $|\psi\rangle$ and $|\phi\rangle$ are orthogonal. (Choose a to be real and positive)
c) If the spin component S_x is measured after the system is prepared in the state $|\psi\rangle$, what is the probability of measuring spin up for the S_x component?
d) Assume the system is initially prepared in the state $|\psi\rangle$, what is the probability that the system is measured to be in the final state $|\psi_f\rangle = \frac{1}{\sqrt{5}}(2|+\rangle - |-\rangle)$?

QM2: Find the energy eigenstates and eigenvalues of a particle confined to a delta function potential $V(x) = -\beta\delta(x)$, where β is a positive real constant. You may find the following definition of Dirac delta function $\delta(x)$ useful:

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$$

QM3: An electron spin is in a uniform magnetic field $\vec{B} = B_0\,\hat{j}$ (where $\hat{j}$ is the unit vector in the $+y$ direction). At $t = 0$, the electron spin is aligned in the $+z$ direction. Find the spin wavefunction at late time t. Express your answer in terms of the $-\rangle$ and $+\rangle$ eigenstates in the z direction.

QM4: The Hamiltonian of a harmonic oscillator is given by

$$H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right).$$

a) Prove that $[H, a^\dagger] = \hbar\omega a^\dagger$. (Show your work, step by step!)
b) Assume $|E\rangle$ is an eigenstate with the eigenenergy E. Use the result from (a) and show that the eigenenergy of state $(a^\dagger|E\rangle)$ is given by $E + \hbar\omega$. (Show your work, step by step!)
c) Calculate the following: $\langle 4|a|5\rangle$, $\langle 6|a^\dagger|5\rangle$, $\langle 2|a^\dagger a|2\rangle$, $\langle 4|\hat{x}|3\rangle$

Section: Modern Physics

MP1: Two rockets that are coasting (engines off) are heading towards each other on a collision course. As measured by Liz, a stationary Earth observer, Rocket 1 has a speed of 0.800c, Rocket 2 has a speed of 0.600c, both rockets are 50.0 m in length, and are initially 2.52 billion kilometers apart.

a) What are their respective proper lengths?
b) What is the length of each rocket as observed by a stationary observer in the other rocket?
c) According to Liz, how long before the rockets collide?
d) According to Rocket 1, how long before they collide?
e) According to Rocket 2, how long before they collide?
f) If the crews can evacuate their rockets safely within 50 min (their own time), will they be able to do so before the collision?

MP2: The CANDU (Canada Deuterium Uranium) reactor is a Canadian pressurized heavy water reactor design used to generate electrical power. Unlike an American reactor, the CANDU uses deuterium oxide (i.e., heavy water) as its moderator and natural uranium (238U) as its fuel.

a) Calculate the energy released (in units of MeV) in the neutron-induced fission

$$n + {}^{238}_{92}\mathrm{U} \rightarrow {}^{96}_{38}\mathrm{Sr} + {}^{140}_{54}\mathrm{Xe} + 3n,$$

given $m(n) = 1.008665\mathrm{u}, m(^{238}\mathrm{U}) = 238.050784\mathrm{u}, m(^{96}\mathrm{Sr}) = 95.921750\mathrm{u}, m(^{140}\mathrm{Xe}) = 139.921647\mathrm{u}$. Recall that 1 u is equivalent to 931.5 MeV/c^2

b) Confirm that the total number of nucleons and total charge are conserved in this reaction.
c) Using the result of part (a), determine the energy released from the fission of 1.00 kg of ^{238}U. How many gallons of gasoline would it take to generate the same amount of energy? Assume that 1 gallon of gasoline contains 1.3×10^8 Joules of energy. Recall that 1 eV = 1.60218×10^{-19} J.
d) Explain why the fission of heavier nuclei into lighter nuclei releases energy.
e) Discuss some of the advantages and disadvantages of using nuclear fission to generate electrical power. Why don't we currently use nuclear fusion to generate electrical power?

MP3: Let $\mathbf{u} = d\mathbf{r}/dt$ be the velocity of a particle observed in an inertial frame K. The same quantity observed in an inertial frame K' moving with velocity $\mathbf{v}$ with respect to K is $\mathbf{u}' = d\mathbf{r}'/dt$.

a) Use the transformation properties of dt, $\mathbf{r}_{\parallel}$ and $\mathbf{r}_{\perp}$(the direction along the velocity $\mathbf{v}$ or perpendicular to the moving K' frame, respectively), to directly derive the velocity addition rule,

$$\mathbf{u}_{\parallel} = \frac{d\mathbf{r}_{\parallel}}{dt} = \frac{\mathbf{u}'_{\parallel} + \mathbf{v}}{1 + \frac{\mathbf{v} \cdot \mathbf{u}'_{\parallel}}{c^2}} \qquad \text{and} \qquad \mathbf{u}_{\perp} = \frac{d\mathbf{r}_{\perp}}{dt} = \frac{\mathbf{u}'_{\perp}}{\gamma(v)\left[1 + \frac{\mathbf{v} \cdot \mathbf{u}'_{\parallel}}{c^2}\right]}.$$

b) Let $\mathbf{v}$ define a polar axis with polar coordinates $\mathbf{u} = (u, \theta)$, and $\mathbf{u}' = (u', \theta')$ for the particle velocities as measured in K and K'. Write the transformation laws in part (a) in the form of $u = u(u', \theta')$ and $\theta = \theta(u', \theta')$

c) Use the results of (b) to show that $u \to c$ when $v \to c$.

MP4: A muonic atom consists of a nucleus of charge Ze^+ and a single negative muon μ^-, found in a quantum mechanical orbit about said nucleus. Muons are elementary particles with the same spin and electric charge as an electron but have a mass that is 207 times larger. The following mass ratios will be necessary: $m_p = 1836 m_e, m_\mu = 9 m_e$. Also, note that the muon is a non-negligible fraction of the mass of a proton.

a) Using the Bohr angular momentum quantization condition $L = n\hbar$ and Coulomb's Law derive the equation for the Bohr radius r_{Bohr} . Express your answer in terms of the atomic number Z, the electric charge constant e, the reduced Planck's constant $\hbar$, the mass of the electron m_e and quantum number n.

b) Since the Bohr radius of a hydrogen atom in its ground state, in Angstroms is $r_{Bohr} = 0.529$ Å, find the muon nucleus separation, in Angstroms, for muonic hydrogen also in its ground state (Z = 1, n = 1).

c) If the binding energy of the electron in a normal hydrogen atom in its ground state is 13.6 eV, find the binding energy of the muonic hydrogen atom also in its ground state.

MP5: An electron e^- with a kinetic energy of 1.000 MeV and mass of 0.511 MeV makes a head-on collision with a positron e^+, its anti-particle, at rest. In the collision, the particle anti-particle pair annihilate and produce two photons of equal energy and momenta each traveling at an angle θ in the lab frame with respect to the electron's original direction. (Since photons are massless their energy is given by the relativist equation $E = pc$.) The reaction is:

$$e^- + e^+ \to 2\gamma.$$

Determine the energy E, and momentum p and angle of emission θ of each photon in the lab frame.