Classical Physics
Ph.D. Qualifying Exam 2022
Department of Physics at FIU

Instructions: There are nine problems on this exam. Three on Mechanics (**CM**), four on Electricity and Magnetism (**EM**), and two on Statistical Physics and Thermodynamics (**SP**). You must attempt a total of six problems with at least **two** from **Section CM**, **two** from **Section EM**, and **one** from **Section SP**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and numbers)** on each page on the **TOP LEFT CORNER** of each page. Turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, write your panther ID on each page at the **TOP RIGHT CORNER** opposite to the problem identifier. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

Section: Classical Mechanics

CM 1: A projectile is launched straight up with an initial velocity of v_0. Assume that the air resistance varies quadratically with the speed (i.e., $\vec{\boldsymbol{F}}_r = -mkv^2\,\hat{\boldsymbol{v}}$), where m is the mass of the projectile, k is a constant, v is the magnitude of the velocity, and $\hat{\boldsymbol{v}}$ is a unit vector pointing in the direction of motion.

a) Show that the projectile's speed while traveling upward is given by

$$v^2 = \left(v_0^2 + \frac{g}{k}\right) e^{-2k(y-y_0)} - \frac{g}{k},$$

where g is the acceleration due to gravity, y_0 is the launch height, and y is the vertical position with upward being positive.

b) Using the expression from part a), determine the projectile's maximum height.

c) Show that the expression in part a) reduces to the standard kinematic relationship in the absence of air resistance (i.e., limit as $k \to 0$)

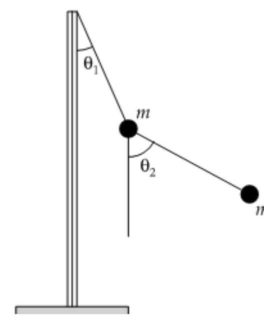
$$v^2 = v_0^2 - 2g(y - y_0).$$

CM 2: Consider the double pendulum as shown in the figure to the right. Each pendulum has length l and mass m.

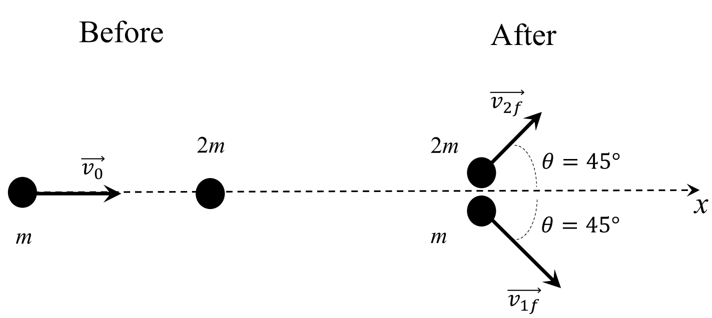
a) Determine the Lagrangian for this double pendulum in terms of the angles θ_1 and θ_2.

b) Determine Lagrange's equations of motion for the double pendulum.

c) Consider the case of small oscillations. Determine the characteristic frequencies and derive the normal modes of oscillation.

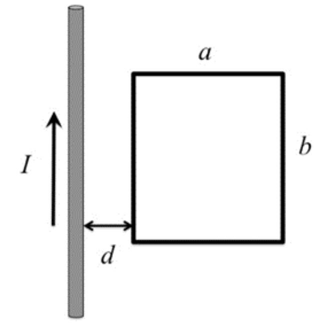


CM 3: A particle of mass m with an initial velocity $\vec{\boldsymbol{v}}_c = v_0\hat{\boldsymbol{x}}$ strikes a second particle of mass $2m$ at rest. After the collision, the two particles move away from each other as shown in the figure below. Determine the velocities of the two particles in terms of the magnitude of the initial velocity v_0. Is the collision elastic or inelastic? If the collision is inelastic, how much energy is lost?



Section: Electricity and Magnetism

EM 1: A rectangular loop with dimensions a, b is placed a distance d from a long, straight wire carrying current I_0.



a) Determine the total flux through the rectangular loop.
b) If the current in the long wire is increased steadily at a rate of $dI/dt = +k$, determine the induced emf in the loop.
c) If the loop has total resistance of R, determine the magnitude and direction of the induced current.

EM 2: A hollow sphere of radius R has an electric potential defined on its surface to be given by the following function,

$$V(\theta) = V_0 \sin^2\left(\frac{\theta}{2}\right),$$

assuming that there are no additional free charges inside or outside of the sphere.

a) Find the potential inside the sphere.
b) Find the electric field inside the sphere.

$$\text{a useful identity: } \sin^2\frac{\theta}{2} = \frac{1}{2}(1 - \cos\theta) = \frac{1}{2}[P_0(cos\theta) - P_1(cos\theta)]$$

EM 3: Consider a solid insulating sphere with total charge Q and radius R. The volume charge density for the sphere is given by,

$$\rho(r) = \begin{cases} A\left(1 - \dfrac{r^2}{R^2}\right) & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

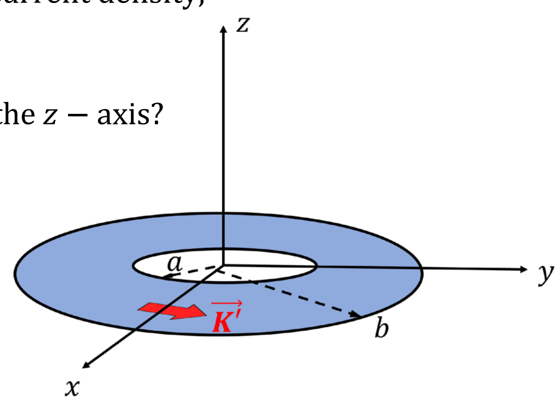
where A is a constant with the units of $\left[\frac{\mathrm{C}}{\mathrm{m}^3}\right]$.

a) Determine the constant A in terms of total charge Q and radius R.
b) Using Gauss's law, determine the electric field $\vec{\boldsymbol{E}}(r)$ for all points in the regions $0 < r < R$ and $r > R$
c) Determine the electric potential $V(r)$ for all points in the regions $0 < r < R$ and $r > R$.
d) Examine the continuity of the electric field and electric potential at the boundary of $r = R$. Are they continuous? Why or why not?

EM 4: An annular disk of inner radius a and outer radius b has its axis coinciding with the z axis and its center at the origin. The disk carries a surface current density,

$$\vec{\boldsymbol{K}}' = \frac{A}{\rho'}\hat{\boldsymbol{\phi}}'$$

where A is a constant. What is the $\vec{\boldsymbol{B}}(z)$ at an arbitrary point on the $z-$axis?



Section: Statistical Physics & Thermodynamics

SP 1: One mole of an ideal gas with a constant C_p is heated, isobarically, from T_1 to T_2. It expands further, adiabatically, until its temperature returns to T_1. Finally, the gas is compressed, isothermally, back to the starting volume of V_1.

a) Calculate all T, P, and V values at the end of each process in terms of R, C_p, V_1, T_1 and T_2.
b) Calculate the amount of heat Q that is released during the isothermal process.
c) Re-calculate the amount of heat Q in part (b), for a substance that is *not* an ideal gas (but still has a constant heat capacity C_p).

SP 2: A non-ideal gas has the following equation of state,

$$P = kT\frac{N}{V}\left[1 + \frac{N}{V}\left(a - be^{d/T}\right)\right]$$

Find an expression for how the mean internal energy of the gas depends on its volume in a process in which the temperature remains constant, i.e., find $\left(\frac{\partial E}{\partial V}\right)_T$.