Modern Physics Ph.D. Qualifying Exam 2022 Department of Physics at FIU

Instructions: There are nine problems on this exam. Five on Quantum Mechanics (**QM**) and four on general Modern Physics (**MP**). You must attempt a total of six problems with at least **two** from each section.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and numbers)** on each page on the **TOP LEFT CORNER** of each page. Turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, write your panther ID on each page at the **TOP RIGHT CORNER** opposite to the problem identifier. Do this on each page please. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

Section: Quantum Mechanics

QM 1: Consider two indistinguishable, non-interacting, spin-1/2 fermions in a onedimensional infinite square well potential of length *L*.

- a) What is the ground-state energy of the two-particle system? What is the groundstate wavefunction of the two-particle system? (full wavefunction including both the spin and spatial parts).
- b) For the first excited state in which one particle is in the ground state (n = 1) and other is in the first excited state (n = 2), what is the state energy? What are the wavefunctions of the first excited state? (Again, include both the spin and spatial parts).

Useful information: the eigen energies and eigenstates for a particle in the infinite square well potential are

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}, \ \varphi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, \ (n = 1, 2, 3, \dots)$$

QM 2: A beam of spin-1/2 particles is prepared in the state represented by

$$|\psi\rangle = C(\sqrt{2}|+\rangle - \sqrt{3}i|-\rangle)$$

- a) This state is not normalized. Please normalize the state and find *C*. (Use the normalized state for the rest of the problem)
- b) Assume another state can be represented by $|\phi\rangle = a|+\rangle + b|-\rangle$, find *a* and *b* such that states $|\psi\rangle$ and $|\phi\rangle$ are orthogonal. (Choose *a* to be real and positive)
- c) If the spin component S_x is measured after the system is prepared in the state $|\psi\rangle$, what is the probability of measuring spin up for the S_x component?
- d) Assume the system is initially prepared in the state $|\psi\rangle$, what is the probability that the system is measured to be in the final state $|\psi_f\rangle = \frac{1}{\sqrt{5}}(2|+\rangle |-\rangle)$?

QM 3: Consider the Hamiltonian $H = H_0 + H'$ for a 3-state system where

$$H_{0} = \begin{pmatrix} \frac{1}{2}\hbar\omega_{0} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\frac{1}{2}\hbar\omega_{0} \end{pmatrix} \qquad H' = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2}\hbar\omega_{1} & 0\\ \frac{\sqrt{3}}{2}\hbar\omega_{1} & \frac{1}{2}\hbar\omega_{1} & \frac{\sqrt{3}}{2}\hbar\omega_{1}\\ 0 & \frac{\sqrt{3}}{2}\hbar\omega_{1} & 0 \end{pmatrix}$$

H' is considered the perturbation.

- a) Find the first-order corrections to the energies of all three eigenstates?
- b) Assume the zeroth-order eigenstates are $|1^{(0)}\rangle$, $|2^{(0)}\rangle$, and $|3^{(0)}\rangle$, find the first-order corrections to the eigenstate vectors.
- c) Find the second-order corrections to the energies of all three eigenstates

Section: Quantum Mechanics, cont.

QM 4: A particle with mass m and charge q is moving in a central-force potential V(r). When $r \to \infty$, $V(r) \to 0$. The particle is in the energy eigenstate

$$\varphi_0 = Are^{-r/a}$$

(where *a* is a constant and *A* is the normalization constant). Find the energy eigenvalue E_0 and the potential energy V(r).

QM 5: Find the eigenfunction of the annihilation operator a_{-} as a superposition of the eigenfunctions of a 1-D harmonic oscillator.

Section: Modern Physics

MP 1: The starship *Enterprise* passes you at a speed of 0.90c. You very quickly measure its length to be 120 m.

- a) What would the length of the *Enterprise* be at rest with respect to your frame?
- b) If you are in a starship that is identical to the one passing (*i.e.*, the *Enterprise*), what length would an observer on the passing ship determine for your starship? Explain.
- c) As the *Enterprise* moves away, you notice that its engines glow bright red ($\lambda = 650$ nm). Determine the wavelength of light emitted by the engines that is measured by an observer on the *Enterprise*.
- d) The *Enterprise* travels from Earth to Jupiter, which are approximately 40 lightminutes apart. If the Enterprise is travelling at 0.90c for the entire trip, how long does the trip take according to crew of the Enterprise?

MP 2: A particle of rest mass m_0 moving with a speed of 0.95*c* makes a completely inelastic collision with a particle of rest mass $3m_0$ that is initially at rest. Determine the rest mass and velocity of the composite particle.

MP 3:

- a) Qualitatively describe the photoelectric effect. What experimental features can be explained by classical physics? What features cannot?
- b) Determine the work function (in eV) for aluminum if the largest wavelength for electron emission is 304 nm.
- c) Aluminum is illuminated with 250 nm ultraviolet light photons. Determine the maximum kinetic energy (in eV) and maximum speed (in units of c) of the emitted electrons.
- d) The 250 nm ultraviolet light has an intensity of 2 W/m². Determine the rate of electron emission (electrons/sec) if the aluminum photocathode has a surface area of 0.10 m^2 . **Hint:** 1 Watt = 1 Joule/second.

MP 4:

- a) Consider a particle of mass *m* confined to an infinite square well of width *L*. Determine the time-independent wave function and the energy value for a particle confined to an infinite square well potential in the second excited state. Be sure to show that your wave function is normalization and is a solution to the time-independent Schrödinger equation.
- b) Sketch the wave function and the probability density as a function of position for a particle in the second excited state. Determine the most probable location of this particle.
- **c)** Determine the expectation values $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$ for a particle in the second excited state of the infinite square well. Check that the Heisenberg uncertainty principle is satisfied. Qualitatively describe the meaning of the expectation value.