Classical Physics
Ph.D. Qualifying Exam 2023
Department of Physics at FIU

Instructions: There are nine problems on this exam. Three on Mechanics (**CM**), four on Electricity and Magnetism (**EM**), and two on Statistical Physics and Thermodynamics (**SP**). You shall attempt a total of **six problems** with at least **two** from **Section CM**, **two** from **Section EM**, and **one** from **Section SP**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and numbers, for example, CM.1)** on each page you turn in. Also, turn in only those problems you want to have graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, write your **panther ID** on each page at the top left-hand corner. The question identifier shall be placed on the right top corner of each page. **DO NOT WRITE your name** anywhere or on anything you turn in.

You may use a calculator and a math handbook as needed.

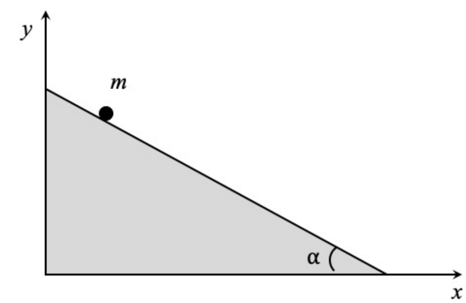
Section: Classical Mechanics

CM1: A block with mass m is launched along a horizontal surface with initial velocity v_0. As the block travels along the horizontal surface, it experiences a drag force that is proportional to the square root of the instantaneous velocity $F_{drag} = mk\sqrt{v}$ where k is constant with units of $\left[\frac{\mathrm{m}^{1/2}}{\mathrm{s}^{3/2}}\right]$

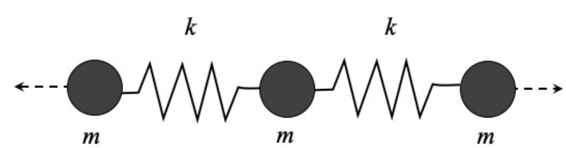
a) Use Newton's 2nd Law to determine the horizontal velocity $v(t)$ and position $x(t)$ of the block as a function of time.
b) Determine the maximum horizontal range x_{max} that the block travels. *Hint:* The maximum horizontal range x_{max} occurs when the block comes to rest.

CM2: A particle of mass m slides down a smooth slope with an inclination angle α. With the $x - y$ coordinates shown in the figure, write down the equation of constraint, and derive the Lagrange equations of motion with undetermined multipliers.

a) Find the particle acceleration in both the x and y directions.
b) Find the constraining forces.



CM3: Three particles of the same mass m are coupled by two springs of the same force constant k and move without friction in 1-D oscillation as shown in the figure to the right. Determine the characteristic frequencies of the coupled harmonic oscillation and the normal modes.



Section: Electricity and Magnetism

EM1: A slowly varying time-dependent infinite surface current $\mathbf{K}(t) = k(t)\hat{\mathbf{x}}$ is flowing in the xy-plane.

a) What is the time-dependent magnetic field? Give both magnitude and direction in all regions of space. Clearly motivate any assumptions you make.
b) What is the induced electric field, both magnitude and direction in all regions of space? Clearly motivate any assumptions you make.
c) What is the Poynting vector, both magnitude and direction in all regions of space?

EM2: A point charge q, located at the center of a thin spherical shell of radius R. The spherical shell is also charged but that charge is distributed non-uniformly on its surface or boundary, as

$$\sigma = \sigma_0 \sin\theta$$

(Should have been cos not sin!!!)

where σ_0 is constant and θ is the polar angle.

a) Find the electric potential inside and outside the shell.
b) Find the electric field inside and outside the shell.

EM3: Suppose a small sphere of radius a has a total charge Q_1 uniformly distributed over its surface. Surrounding this sphere is a thin spherical shell of radius b with a total charge Q_2. If the region between the two spheres is filled with a simple linear dielectric material of dielectric constant ϵ:

a) Find the potential difference between the two spheres.
b) If $Q_1 = -Q_2$, what is the capacitance of the system?

EM4: A solid cylinder of radius a and infinite length has its axis coinciding with the z-axis. Its magnetization is given by

$$\mathbf{M} = \mathrm{M}_0 \left(\frac{\rho}{a}\right)^3 \hat{\boldsymbol{\phi}}$$

where M_0 is constant and $\hat{\boldsymbol{\phi}}$ is the usual azimuthal direction in cylindrical coordinates (ρ, ϕ, z).

a) Find the volume and surface magnetization current densities $\mathbf{J}_\mathrm{M}$ and $\mathbf{K}_\mathrm{M}$.
b) Find the magnetic induction, $\mathbf{B}$, and the magnetic field, $\mathbf{H}$, inside and outside the cylinder. (Hint: use Ampere's Law to find $\mathbf{H}$)

Section: Statistical Physics & Thermodynamics

SP1: A system consists of N distinguishable, independent, identical particles, each fixed in place. Each particle has one of two possible energy states; the lower state has energy 0 and the upper state has energy ε. Therefore, the total energy of the system is $E = n\varepsilon$, where n is the number of particles in the upper state, and thus $(N - n)$ particles in the lower state. Calculate the energy of the system E as a function of temperature. Assume that N, n, and $N - n$ are all large so that Stirling's Approximation can be used.

SP2: A heat engine operates between two heat reservoirs (T_L and T_H) by going through two isochoric and two isobaric processes (a rectangle cycle on a $P - V$ diagram). The working media is one mole of monoatomic ideal gas with constant C_V. Here $P_H = 3P_L$ and $V_H = 2V_L$ (therefore $T_H = 6T_L$).

a) Calculate temperatures for all four corners of the cycle.
b) Calculate the efficiency of this heat engine.