Modern Physics
Ph.D. Qualifying Exam 2023
Department of Physics at FIU

Instructions: There are nine problems on this exam. Five on Quantum Mechanics (**QM**) and four on General Modern Physics (**MP**). You must attempt a total of six problems with at least **two** from each section.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and numbers)** on each page. Also, turn in only those problems you want to have graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, write your panther ID on each page at the top left-hand corner along with the problem identifier. Do this on each page, please. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

Section: Quantum Mechanics

QM1: The Hamiltonian for a spin-½ particle in a uniform magnetic field that is oriented along the x direction, given by $H = \alpha S_x$, where α is a constant and

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

is the spin operator in the x direction. At $t = 0$, the spin is up along the z direction, i.e., $S_z = \frac{\hbar}{2}$. Find the expectation values of spin operators S_x, S_y, and S_z at some later time $t > 0$.

QM2: A particle is in a quantum state represented by the Dirac notation as

$$|\psi\rangle = \frac{1}{2}|\phi_1\rangle - \frac{i}{\sqrt{2}}|\phi_2\rangle - \frac{1}{2}|\phi_3\rangle.$$

Here $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ are three orthonormal eigenfunctions (eigenstates) of an operator $\hat{Q}$ such that $\hat{Q}|\phi_n\rangle = (n+1)|\phi_n\rangle$, where $n = 1, 2, 3 \ldots$. Find the expectation value of $\hat{Q}$ for the state $|\psi\rangle$ and its standard deviation $\sigma_Q = \sqrt{\langle \hat{Q}^2 \rangle - \langle \hat{Q} \rangle^2}$.

QM3: A free particle with kinetic energy E is incident from left to a delta-function potential barrier $V(x) = V_0\delta(x)$, where $\delta(x)$ is the Dirac delta function and V_0 is a positive constant. Find its transmission coefficient T and reflection coefficient R.

QM4: For a particle in an infinite potential well with the total width of a

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < a \\ \infty & x > a \end{cases}$$

a) Derive the energy spectrum (i.e., eigen energies) and the wave functions of the eigenstates.
b) Demonstrate that while the wave functions are continuous, their first derivatives have discontinuities.

QM5: For a Simple Harmonic Oscillator, the Hamiltonian can be written as

$$H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right),$$

where $a^\dagger$ and a are the raising operator and lowering operator, respectively. Derive the energy spectrum (i.e., eigen energies) of the Simple Harmonic Oscillator and demonstrate that it has a finite energy in the ground state.

Section: Modern Physics

MP1: Relativistic muons provided one of the first means to validate the effects of special relativity, specifically, time dilation and length contraction. A typical muon created high in the atmosphere (from the decay of pions) travels at 0.998c relative to the Earth. The typical lifetime of a muon as measured in its inertial reference frame is 2.20 μs.

a) Determine the mean lifetime of a typical muon as measured by an observer on Earth.
b) According to an observer on Earth, how far does a typical muon travel before it decays?
c) According to an observer in the muon's frame, how far does a typical muon travel before it decays?

MP2: In a particular Compton scattering experiment, an X-ray photon with a wavelength of 0.0200000 nm is scattered off a charged particle. The angle between the incident and scattered photon is given by q.

a) Sketch the difference between the wavelengths of the incident and scattered photons, as a function of q.
b) If the maximum change in the wavelength was measured to be 0.00000265 nm, determine what is the charged particle. The charged particle could be one of the following: an electron with rest mass energy $m_e c^2 = 0.511$ MeV, a muon with $m_\mu c^2 = 105.7$ MeV, a pion with $m_\pi c^2 = 139.6$ MeV, or a proton with $m_{proton} c^2 = 938.6$ MeV.
c) Determine the wavelength of an X-ray photon (in nm) that scattered at an angle of 90 degrees.

MP3: In a parallel universe, all physical laws and constants are the exactly the same as in our universe, except that Planck's constant h is different. To determine Planck's constant, one can perform photoelectric experiments by measuring the wavelengths of photons incident on a metal and the corresponding stopping voltages. Two precise measurements of photon wavelength (λ) and stopping potential (V^{stop}) were obtained: $\lambda_1 = 500$ nm with $V_1^{stop} = 1.50$ V and $\lambda_2 = 600$ nm with $V_2^{stop} = 1.00$ V.

a) Determine the quantity of hc in this parallel universe (in the units of $\text{eV} \cdot \text{nm}$)
b) Determine the maximum wavelength (in nm) that can produce a photocurrent on this material in this parallel universe.

Section: Modern Physics (continued)

MP4: A hydrogen atom in the ground state (n =1) absorbs a 93.7 nm photon, corresponding to a transition line in the Lyman series.

a) Determine how the absorption of this photon affects the hydrogen atom's energy, angular momentum, and the effective size of the atom (i.e., electron orbital radius). You may leave your answers in units of electron volts, $\hbar$ and Bohr radius a_0.
b) Determine how much energy is required to ionize the atom when it is in this excited state.
c) Discuss the significance of the J.J. Thomson, Millikan, Rutherford, and Franck-Hertz experiment and how the results of these experiments contribute to Bohr's model of the atom? Why does classical physics fail to correctly describe the emission spectrum for hydrogen atom?