Classical Physics
Ph.D. Qualifying Exam 2024
Department of Physics at FIU

Instructions: There are nine problems on this exam. Three on Mechanics (**CM**), four on Electricity and Magnetism (**EM**), and two on Statistical Physics and Thermodynamics (**SP**). You must attempt a total of six problems with at least **two** from **Section CM**, **two** from **Section EM**, and **one** from **Section SP**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and numbers)** on each page you turn in. Also, turn in only those problems you want to have graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, make sure the pages you turn in have the arbitrary ID (AID) assigned to you by the proctor on each page at the top left-hand corner along with the question identifier. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

Section: Classical Mechanics

CM1: A small ball is fastened to a long rubber band and twirled around in such a way that the ball moves in an elliptical path given by the equation,

$$\vec{\boldsymbol{r}}(t) = \mathrm{b}\cos\omega t\ \hat{\mathbf{i}} + 2\mathrm{b}\sin\omega t\ \hat{\mathbf{j}}$$

where b and ω are constants.

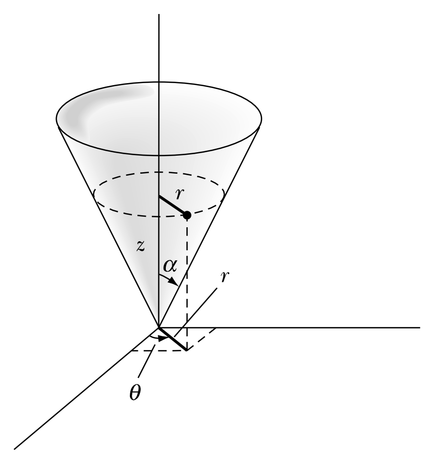
a. Determine the velocity, acceleration, and speed of the ball as a function of time.
b. Determine the times at which the ball is at its maximum and minimum distance from the origin. Determine the ball's speed at these times.
c. Determine the angle between the velocity and acceleration when t = 0. Explain.

CM2: When a light spring supports a block of mass m in a vertical position, the spring is found to stretch by an amount D_1 over its unstretched length. The block is then pulled further downward a distant D_2 from the equilibrium position and released.

a. Determine the natural frequency of oscillation ω and the position of the block as a function of time x(t).
b. Determine the velocity of the block as it passes back upward through the equilibrium position.
c. Determine the acceleration of the block at the top of its oscillatory motion.

CM3: A particle of mass m is constrained to move on the inside surface of a smooth cone of half-angle α (*i.e.*, $z = r\cot\alpha$) as shown in the figure to a gravitational force. The particle is subject to a gravitational force (i.e., $\vec{g} = -g\hat{k}$).

a. Determine the Lagrangian for this particle in terms of the generalized coordinates r and θ.
b. Determine Lagrange's equations of motion for this particle.



Section: Electricity and Magnetism

EM1: Assume that Coulomb's Law is not quite correct. Instead, careful measurements reveal that the electric force between two ***stationary*** charged point charges $q_1$and q_2 is:

$$\vec{\mathbf{F}}_{\mathrm{c}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\boldsymbol{\mathscr{r}}^2}\left(1 + \frac{\boldsymbol{\mathscr{r}}}{\lambda}\right)e^{-\boldsymbol{\mathscr{r}}/\lambda}\,\hat{\boldsymbol{\mathscr{r}}}\,,$$

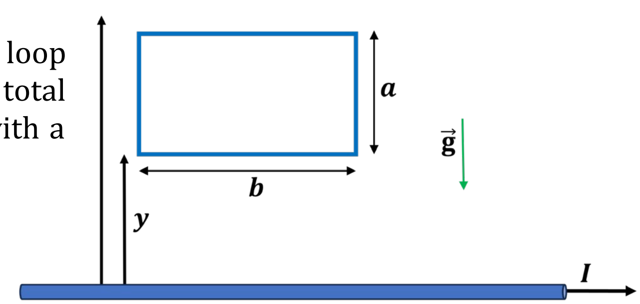
where $\boldsymbol{\mathscr{r}} = |\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1|$ is the separation between the two charges and λ is a new constant of nature with units of length and a large numerical value.

Hint: Assume that the principle of superposition still holds.

a. What is the electric field $\vec{\mathbf{E}}(\mathbf{r})$ at some point arbitrary point $\mathbf{r}$ from a non-point-like charge distribution $\rho(\mathbf{r}')$?
b. For the updated electric field above can a scalar potential $V(\mathbf{r})$, be defined? Explain your reasoning.
c. Find the new electric potential $V(\mathbf{r})$ for a point charge, assume the potential at infinity is zero.

EM2 (Wim): A rectangular current loop with sides b and a (see figure) has resistance R and can move in the vertical plane. The loop falls towards the earth due to gravity $\vec{\mathbf{g}}$. On the ground, there is a long wire with a constant current I flowing in the direction of the arrow shown in the figure. Given that the speed of the loop depends on time, $v(t)$,

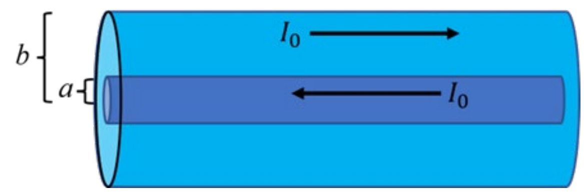
a. what is the current through the loop as a function of its speed $v(t)$ and its altitude $y(t)$ above the wire? In what direction, clockwise or counterclockwise, does the current in the loop flow?
b. Find the magnetic force on each wire segment in the loop from the infinite wire on the ground. What is the total force on the current loop? Explain the direction with a physical argument.



Section: Electricity and Magnetism

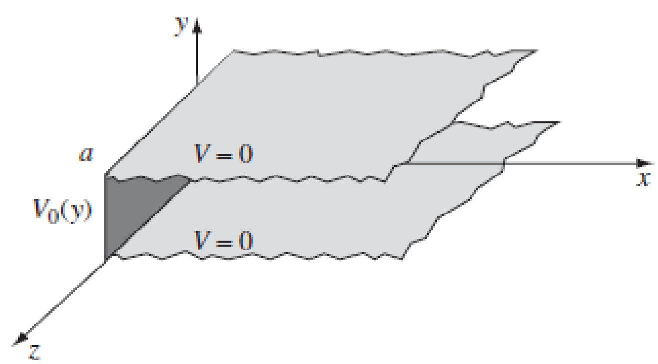
EM3: A coaxial cable consists of two very long cylindrical tubes, separated by linear insulating material of magnetic susceptibility χ_m. Free current, I_0 flows down the inner conductor and returns along the outer one; in each case, the current distributes itself uniformly over the surface.

a. Find the magnetic field $\vec{\boldsymbol{B}}$ and magnetization $\vec{\boldsymbol{M}}$ everywhere, inside, outside and between the two tubes, near the center far from the ends.
b. What are the bound currents, surface $\vec{\boldsymbol{K}}_{\boldsymbol{b}}$ and volume $\vec{\boldsymbol{J}}_{\boldsymbol{b}}$?



EM4: Two infinite grounded metal plates lie parallel to the xz plane, shown in the figure below, one at $y = 0$, the other at $y = a$. The left end, at $x = 0$, is closed off with an infinite strip insulated from the two plates and maintained at a specific potential $V_0(y)$.

a. Find the potential inside the slot between the grounded plates.
b. Find the surface charge density $\sigma(x)$ on the bottom plate at $y = 0$.



Section: Statistical Physics & Thermodynamics

SP1: The liquid-gas phase boundary is determined by the Clausius-Clapeyron relation

$$\frac{dP}{dT} = \frac{L}{T\left(V_g - V_l\right)}.$$

If one assumes that (i) latent heat is a constant, (ii) gas-phase volume $V_g = R_T/P$, and (iii) liquid-phase volume V_l is negligible, the Clapeyron relation becomes:

$$\frac{dP}{dT} = \frac{LP}{RT^2},$$

from which one can derive an approximate vapor pressure equation:

$$P = P^{\infty}\, e^{-L/RT},$$

where P^{∞} is a constant, and $L_0 = 40.7$ KJ/mol which compares well with experiments in water with a temperature range between 373K − 450K.

a. Remove the first approximation and consider $L = L_0 - \alpha(T - T_0)$, with positive constants L_0 and α, and 373K. Derive an 'improved' vapor pressure equation.
b. When applying this 'improved' equation to water, whose vaporization latent heat is fairly linear, the results are worse compared to experimental values than the original vapor pressure equation. Propose an explanation for this somewhat surprising outcome.

SP2: Determine an expression for the ratio K_S/K_T in terms of C_P and C_V, where,

$$K_s = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S, \qquad \text{and} \qquad K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T.$$

Hints:

– *C_P and C_V involve dQ, which is related to dS and dT.*
– *Write an expression for dS in terms of dV and dP and use the condition that S remains constant, i.e. $dS = 0$. Do the same for dT.*
– *Use Maxwell Relations and Chain Rules to convert to $\left(\frac{\partial S}{\partial T}\right)_{P(or\ V)}$*