Modern Physics
Ph.D. Qualifying Exam 2024
Department of Physics at FIU

Instructions: There are nine problems on this exam. Five on Quantum Mechanics (**QM**) and four on General Modern Physics (**MP**). You must attempt a total of six problems with at least **two** from each section.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and numbers)** on each page. Also, turn in only those problems you want to have graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, make sure the pages you turn in have the arbitrary ID (AID) assigned to you by the proctor on each page at the top left-hand corner along with the question identifier. Do this on each page, please. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator, and a math handbook as needed.

Section: Quantum Mechanics

QM1: Using the 3-dimensional Schrödinger wave equation with potential energy being a real function,

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right)\psi(\mathbf{r},t) = i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}$$

a. derive the continuity equation.
b. show that if the wave function is normalized to unity for the given volume V, then the net flux of the probability current through the surface that encloses the volume V is 0.

QM2: For the given one- dimensional infinite well potential

$$V(x) = \begin{cases} 0, & \text{if } x < |a| \\ \infty, & \text{if } x \geq |a| \end{cases}$$

a. Calculate the energy spectrum.
b. Calculate Normalized wave functions for ground and excited states.
c. Calculate the discontinuity of wave function at the boundary.
d. Calculate the probability current inside the well and give its interpretation.

QM3: Consider the one-dimensional wave mechanics problem

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

where,

$$V(x) = -\frac{\hbar^2}{m}\sum_{n=0}^{N-1}\delta(x - nd).$$

a. Show that all energy eigenstates, E, are obtained from solutions to the equation

$$[T^N]_{22} = 0; \qquad T = \begin{pmatrix} \left(1+\frac{1}{\alpha}\right)e^{-\alpha d} & +\frac{1}{\alpha} \\ -\frac{1}{\alpha} & \left(1-\frac{1}{\alpha}\right)e^{+\alpha d} \end{pmatrix}; \qquad E = -\frac{\hbar^2\alpha^2}{2m}.$$

b. How many eigenstates exist for $N = 1, 2$? How do they depend on d?
c. Provide a physical explanation for the behavior as a function of d for $N = 2$. Does the system lose energy or gain energy as d is increased? Does the answer depend on d. Use proper mathematical analysis to arrive at your answers (heuristic arguments are NOT allowed).

QM4: Start with the algebra of the angular momentum operators

$$[L_1, L_2] = iL_3; \qquad [L_2, L_3] = iL_1; \qquad [L_3, L_1] = iL_2.$$

You can assume the orthonormal basis vectors can be denoted by

$$|j, m\rangle, m = -j, -j+1, \cdots, j-1, j$$

and

$$L_3|j,m\rangle = \hbar m|j,m\rangle; \;\; L^2|j,m\rangle = \hbar^2 j(j+1)|j,m\rangle; \;\; L_\pm|j,m\rangle \propto |j, m \pm 1\rangle; \;\; L_\pm = L_1 \pm iL_2.$$

You are expected to obtain the positive real constants that convert the proportionality to an equality. Let $|\theta\rangle = \cos\theta\,|1,0\rangle + \sin\theta\,|1,1\rangle$ where $\theta \in [0, 2\pi]$ is a free parameter.

a. Prove that $|\theta\rangle$ is normalized for all values of θ.
b. Evaluate $l_1 = \langle\theta|L_1|\theta\rangle$.
c. Evaluate $l_2 = \langle\theta|L_2|\theta\rangle$.
d. Evaluate $l_3 = \langle\theta|L_3|\theta\rangle$.
e. Evaluate $l^2 = \langle\theta|L^2|\theta\rangle$.
f. Evaluate $v = l^2 - l_1^2 - l_2^2 - l_3^2$
g. Prove that $v > 0$ for all values of θ. Provide a physical explanation for this result.
h. What value of θ results in the lowest value of v. What is the value of v?

QM5: In a region of space, a particle with a **time-independent** wave function

$$\psi(x) = Axe^{-x^2/L^2}$$

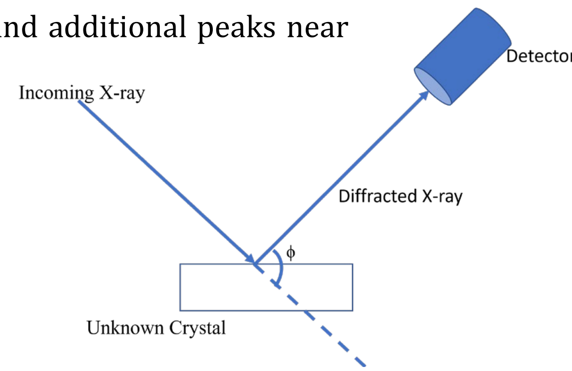
and zero total energy, with known constants A, and L.

a. What is the potential energy of the particle as a function of position x?
b. Sketch the position-dependent potential of the particle.

Section: Modern Physics

MP1: In an X-ray scattering experiment, a student recorded the scattered X-ray intensity as a function of the angle ϕ as shown below. There are two characteristic wavelengths (0.139 nm and 0.152 nm) from the copper target that are relevant. The student noticed two prominent peaks very close to each other at 28.0 degrees and 30.0 degrees and no peaks prior to that were observed.

a. Ignore measurement uncertainties/inaccuracies, determine the relevant atom spacing of the crystal used in this experiment.
b. Predict where else you would also observe peak intensities as the angle ϕ being increased to up to 120 degrees.
c. A student did the prediction in part b) and indeed found additional peaks near the predictions, but consistently slightly smaller than the predictions. Explain why this would be the case by a sketch using the provided two data points.



MP2: Antineutron ($\bar{n}$) can be produced via the reaction

$$\gamma + p \rightarrow p + p + \pi^- + \bar{n}$$

Use: $m_p = 0.938\ [\mathrm{GeV/c^2}], m_{\pi^-} = 0.139[\mathrm{GeV/c^2}]$, and $m_{\bar{n}} = 0.939[\mathrm{GeV/c^2}]$

a. What is the threshold energy or minimum photon energy that will allow the reaction to occur?
b. At this minimum photon energy, what is the speed of **each final state particle**, measured in the laboratory frame of reference?
c. In the center-of-momentum frame, what is the value of the photon's energy?

Section: Modern Physics cont.

MP3: In a Compton scattering experiment, the incident photon has an energy of 180 [keV], the recoiling electron is measured to have an 50 [keV].

a. What is the wavelength of the scattered photon?
b. What is the scattering angle θ of the photon?
c. What is the recoil angle ϕ of the electron?

MP4: At $t_0 = 0.000$ [ns], several unknown particles all with the same momentum enter a region with a magnetic field $|\vec{\boldsymbol{B}}| = 2.00$ [T] perpendicular to the direction of the initial velocity. They will be detected by various detectors at different locations. One of the particles (trajectory not shown), is identified as a proton and was observed to travel in the counterclockwise direction along a **half circle** with a radius $R = 3.00$ [m]. Two detectors A and B are placed along an arc of radius R at angles $\theta_A = \pi/3$ to the left and $\theta_B = \pi/2$ to the right. If two of the unknown particles (A and B) follow counterclockwise and clockwise paths as shown in the figure where particle A reached the detector at $t_A = 10.502$ [ns], and particle B reached the detector at $t_B = 16.285$ [ns].

Use: $m_{\pi^\pm} = 0.139$ [GeV/c^2], $m_{\pi^0} = 0.135$ [GeV/c^2], $m_{K^\pm} = 0.497$[GeV/c^2], $m_{\bar{p}} = 0.938$ [GeV/c^2], and $m_{\bar{n}} = 0.939$[GeV/c2.

a. Determine the momentum in units of [GeV/c].
b. Determine when the proton reached the detector.
c. Identify both particles A and B, as a: $\pi^+, \pi^-, \pi^0, K^+, K^-, \bar{p}$, or $\bar{n}$, assuming the timing and distance measurements are accurate to within 1%.