

Classical Physics

Ph.D. Qualifying Exam 2025

Department of Physics at FIU

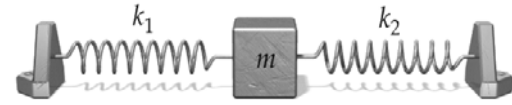
Instructions: There are nine problems on this exam. Three on Mechanics (**CM**), four on Electricity and Magnetism (**EM**), and two on Statistical Physics and Thermodynamics (**SP**). You must attempt a total of six problems with at least **two** from **Section CM**, **two** from **Section EM**, and **one** from **Section SP**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and numbers)** on each page you turn in. Also, turn in only those problems you want to have graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, make sure the pages you turn in have the arbitrary ID (AID) assigned to you by the proctor on each page at the top left-hand corner along with the question identifier. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

Section: Classical Mechanics

CM1: A block with mass m is attached to two springs with spring force constants k_1 and k_2 as shown in the figure to the right. The block is displaced from equilibrium and is free to oscillate from left to right. Assume the surface is frictionless.



- Use **Newton's 2nd Law to write the equations of motion** (i.e., $\ddot{x}$, $\ddot{y}$) for the mass. To receive full credit, you must **draw a free-body diagram**. Be sure to include a coordinate system and clearly label all forces acting on the block. What is the position as a function of time for the mass $x(t)$.
- Using the horizontal equation of motion from in part (a), determine the natural angular frequency ω_0 , the frequency f , and the period of motion τ for this oscillator in terms of the mass of the block m and spring constants k_1 and k_2 .
- Show that $x(t) = A \cos(\omega_0 t + \phi)$ is a possible solution to the equation of motion found in part (a).
- Using the solution in part (c), write an equation for the kinetic and potential energies. Show that the total energy is always a constant in time. Discuss when and where the kinetic and potential energies are a maximum.

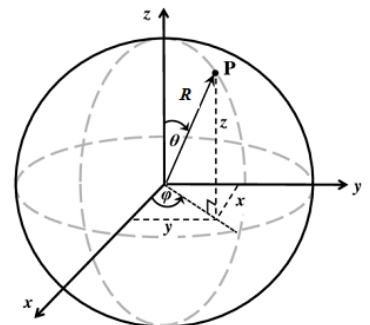
CM2: In the year 2400, humans have colonized a distant planet named Auroro. Due to the planet's harsh surface conditions, the preferred mode of transportation is via **gravity trains**—underground railways that follow straight-line paths between two arbitrary points on the surface (from point A to point B). The tunnels do not necessarily pass through the planet's center. Given that the planet's gravitational acceleration at the surface $g_0 = 10.0 \text{ m/s}^2$, its radius to be $R = 10,000 \text{ m}$, and spherical and of uniform mass density. Assume the mass of the train is m .

- Determine the magnitude of the gravitational acceleration as a function of the radial distance r , $g(r)$, from the center of the planet, for $r \in [0, \infty]$, and sketch $g(r)$ showing all essential information.
- Choose appropriate generalized coordinate(s), and derive the Lagrangian and the corresponding equation of motion for the train traveling along the tunnel.
- Estimate the travel time (in minutes, to two significant digits) it would take a train to move from point A to point B. Assume the train starts from rest and travels frictionlessly along the tunnel under the influence of gravity only.

CM3: A point particle of mass m is constrained to move on a sphere of radius R and is subject to a potential

$$V(\theta) = \frac{1}{2} V_0 \cos^2 \theta, \text{ where } V_0 > 0.$$

- Construct the Lagrangian for the point particle and find the constant(s) of the motion (i.e., the conserved quantities).
- Determine Lagrange's equations of motion for this point particle.



Section: Electricity and Magnetism

EM1: A long cylindrical tube of radius $s = a$ is maintained at a constant, ϕ dependent potential,

$$V(\phi) = V_0 \cos^2 \phi.$$

The region inside the tube is otherwise empty of charge.

- Find the potential $V(s, \phi)$ inside and outside the tube.
- Determine the electric field inside the tube.
- Find the electric field outside the tube and the charge distribution $\sigma(a, \phi)$ on the surface of the tube.

In cylindrical coordinates a solution to Laplace's equation is:

$$V(s, \phi) = A_0 + B_0 \ln s + \sum_{n=1}^{\infty} s^n (A_n \cos n\phi + B_n \sin n\phi) + s^{-n} (C_n \cos n\phi + D_n \sin n\phi)$$

EM2:

- Start with Maxwell equations in vacuum in the absence of electrical charges or current densities, derive the wave equation for both the electric and magnetic fields.
- Prove that solutions of these wave equations which only depend on one spatial coordinate (for example z) are plane wave solutions i.e., $\mathbf{E}(\mathbf{r}, t) = \mathbf{f}(z \pm ct)$, $\mathbf{B}(\mathbf{r}, t) = \mathbf{g}(z \pm ct)$ where c is a constant and the functions f, g are completely general. (*Note: showing that they satisfy the wave equation is not sufficient*)
- Prove that for plane waves propagating in the z -direction the electric and magnetic fields $\mathbf{E}, \mathbf{B}$ are perpendicular to the z -direction and each other.
- Consider a monochromatic (single frequency/wavelength) EM plane wave polarization linearly along the y -axis. This wave is superimposed on a second monochromatic EM plane wave also polarization linearly along the y -axis but traveling in the $+x$ direction. Both waves have the same angular frequency $\omega = 2\pi f$, but are 180 degrees out of phase at the origin, and have the same amplitude.
 - Write down the expression for the total electric field.
 - What is the expression for the total magnetic field
 - At $t = 0$, where is the magnetic field zero?

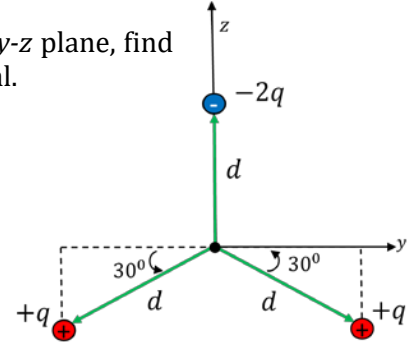
Section: Electricity and Magnetism

EM3: An infinite slab of magnetic material oriented parallel to the xy -plane but is of finite thickness of $2d$. The slab is made of linear material with permeability μ . The slab is placed in a uniform external field $\mathbf{B} = B_0 \hat{x}$

- Show why the $\mathbf{B}$ field outside the infinite slab is unaffected by the induced currents in the material. And determine the total magnetic field everywhere (direction and magnitude).
- Determine the direction and magnitude of the any bound magnetization currents in the slab.
- Determine the force density (force per unit of area) on one of the edges of the slab caused by the surface currents interacting with the combined external and induced magnetic fields. How would the resulting force density differ with material that is either paramagnetic or diamagnetic?

EM4: Consider three point charges separated by d as shown in the figure below. The charges are arranged so that a charge $-2q$ is aligned with the z -axis while the two other charges $+q$ are located 120° to the right and left so that the tripole system is on the y - z plane. Consider the electric potential at some point $\mathbf{r}$ far from the origin where that is $r \gg d$.

- Show that the monopole term vanishes.
- With the tri-charge system centered at the origin and aligned along the y - z plane, find the dipole $V(\mathbf{r})_{dip}$, and quadrupole $V(\mathbf{r})_{quad}$, terms of the electric potential.



Section: Statistical Physics & Thermodynamics

SP1: A particle has the energy level scheme:

$$\begin{array}{l} \epsilon_3 \text{ ———} \\ \epsilon_2 \text{ ———} \\ \epsilon_1 \text{ ———} \end{array}$$

- In the limit of $T \rightarrow 0$ what is the average energy $\langle E \rangle$ and free energy F of the particle where $F = -kT \ln Z$.
- In the limit of $T \rightarrow \infty$ what is the average energy $\langle E \rangle$ and free energy F of the particle where $F = -kT \ln Z$.

SP2: Let an ideal gas with N elements be such that each element has f degrees of freedom. You can assume the ideal gas law $PV = NkT$ and you can also assume the total internal energy of the ideal gas is given by $U = N \left(\frac{f}{2} \right) kT$. We have denoted the pressure by P , volume by V , and the temperature by T . The Boltzmann's constant is denoted by k .

This ideal gas undergoes an isothermal expansion from (P_1, V_1) to (P_2, V_2) at T_h . The gas then undergoes adiabatic expansion from (P_2, V_2) to (P_3, V_3) and the new lower temperature is T_c . The gas then undergoes isothermal compression from (P_3, V_3) to (P_4, V_4) . Finally, the cycle is closed by an adiabatic compression from (P_4, V_4) to (P_1, V_1) .

- Plot the full cycle in a (P, V) diagram. Your axes have to be labelled, and the plot should label isothermal and adiabatic curves. It should also label T_h and T_c .
- Prove that PV^γ remains a constant along an adiabatic process where $\gamma = \frac{f}{f+2}$.
- Prove that

$$V_1 V_3 = V_2 V_4; \quad P_1 P_3 = P_2 P_4.$$

- Derive expressions for Q , the heat into the gas, W the work done on the gas and ΔU , the change in internal energy of the gas for all the four segments. You have to derive all expressions, and the results can only have f, N, T_c, T_h, V_1, V_2 and k .
- Provide a physical explanation for this process. No points will be awarded for guesses.