Modern Physics
Ph.D. Qualifying Exam 2025
Department of Physics at FIU

Instructions: There are nine problems on this exam. Five on Quantum Mechanics (**QM**) and four on General Modern Physics (**MP**). You must attempt a total of six problems with at least **two** from each section.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and numbers)** on each page. Also, turn in only those problems you want to have graded (**Do NOT submit for grading more than 6 problems altogether**). Finally, make sure the pages you turn in have the arbitrary ID (AID) assigned to you by the proctor on each page at the top left-hand corner along with the question identifier. Do this on each page, please. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator, and a math handbook as needed.

Section: Quantum Mechanics

QM1 (Misak): Use Schrodinger's equation wave equation to

a. Derive the continuity equation
b. Calculate the probability current for a free particle.

QM2 (Rajamani): Use $\hbar$ and not h for all questions in the problem. You can use the integral

$$\int_{-\infty+i\beta}^{\infty+i\beta} e^{-\alpha x^2}\, dx = \sqrt{\frac{\pi}{\alpha}}$$

for all complex α with $\mathrm{Re}(\alpha) > 0$ and for all real values of β . If we write $\alpha = |\alpha| e^{i\theta}$ with $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

a. Consider a plane wave in one dimension,
$$\psi(x) = e^{ikx}.$$
What is its momentum?
b. Assum $\psi(x)$ is the initial condition for a free wave describing a non-relativistic particle mass m. Derive an expression for the time dependence of the wave function, $\psi(x,t)$ for all $t \in (-\infty, \infty)$.
c. Now assume
$$\psi(x,0) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} e^{-\frac{x^2}{4\sigma^2}}$$
is the initial condition for a free wave function describing a non-relativistic particle of mass m. Starting from $\psi(x,0)$, derive an expression for the time dependence of the wave, $\psi(x,t)$ for all $t \in (-\infty, \infty)$.
d. Derive an expression for
$$p(x,t) = \psi^*(x,t)\psi(x,t)$$
and discuss its experimental relevance in detail for different choices of m and σ. Just stating that $p(x,t)$ is the probability distribution will not result in any points for this part.

QM3 (Jorge): Consider a system of two particles, each with spin $s_1 = 1$, and $s_2 = \frac{1}{2}$

a. Determine the possible total spin quantum numbers S for the system.
b. For each total spin S, determine the possible values of the magnetic quantum number M_s.
c. Express the combined state $|S = \frac{3}{2}, M_s = \frac{1}{2}\rangle$ as a linear combination of product states $|s_1 = 1, m_1\rangle \otimes |s_2 = \frac{1}{2}, m_2\rangle$.
d. Verify the normalization of your combined state.

Section: Quantum Mechanics

QM4 (Misak): For symmetric infinite well potential:

$$V(x) = 0, \quad \text{at} - a \leq x \leq +a$$
$$V(x) = \infty, \quad \text{otherwise}$$

a. Calculate corresponding wave functions with given parity, where parity operator is defined as $\mathcal{R}\psi_{\pm} = \pm\psi_{\pm}$
b. Calculate the energy spectrum
c. Determine the parity of the ground and first excited states

QM5 (Rajamani): Let O be an operator acting on functions of (r, θ) in two dimensions with polar coordinates. Define its adjoint, $O^{\dagger}$, using the definition

$$\int_0^{\infty} dr\, r \int_0^{2\pi} d\theta [\xi(r,\theta)]^* [O\chi](r,\theta) = \int_0^{\infty} dr\, r \int_0^{2\pi} d\theta \left[[O^{\dagger}\xi](r,\theta)\right]^* \chi(r,\theta).$$

While performing integration by parts, you can assume that all wave-functions are well behaved such that boundary terms at $r = 0$ and $r = \infty$ do not contribute. Derive the adjoints of the following operators:

a. $O = \frac{1}{r}\frac{\partial}{\partial r}$
b. $O = \frac{\partial^2}{\partial r^2}$
c. $O = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$

Section: Modern Physics

MP3 (Lei):

a. Determine the Lorentz transformation matrix that relates the lab frame to a reference a frame moving with velocity $\vec{v} = +\beta c\hat{y}$ that is, a boost in the positive y-direction.
b. Calculate the **eigenvalues** and the corresponding **eigenvectors** of this Lorentz transformation matrix.
c. Discuss the physical significance and meaning of the eigenvalues and eigenvectors you obtained in part b.

MP1 (Kamel): A proton has a non-relativistic kinetic energy of 1.4 MeV. If its momentum is measured with an uncertainty of 4 %.

a. What is the minimum uncertainty in its position?
b. What is the group velocity of this proton?

MP4 (Lei): Photoproduction of the charmed baryons known as the Λ_c^+ are possible when high energy photons (gamma rays) scatter off the nucleus of a hydrogen atom. Assume that the incident photon energy is fixed at 20 GeV,

a. Find the maximum possible mass for the hypothetical Λ_c^+ that can be produced via the reaction
$$\gamma + p \rightarrow \Lambda_c^+ + D^0.$$
Where D^0 created along with the charmed baryon is also charmed and known as the charmed meson. Use the following particle masses:
$$m_{D^0} = 1.864\ [\mathrm{Gev}/c^2]$$
$$m_{p^+} = 0.938\ [\mathrm{Gev}/c^2]$$
b. Using the mass of the of Λ_c^+ obtained in part a. calculate the **speeds** of the Λ_c^+ and D^0 in the lab frame (where the proton is initially at rest). Express your answers as fractions of the speed of light c (i.e., in units of c), not in meters per second.

MP2 (Kamal): This problem relates to the Bohr model of the atom at rest. Assume that the mass of the atom is M.

a. If the atom is in its $n = 4$ state, calculate the atomic radius and the linear and angular momentum of its electron.
b. If the electron is initially in the $n = 4$ state and undergoes a transition to the $n = 1$ state. Show that the recoil speed of the atom from the emission of a photon is given approximately as:
$$v = \frac{15hR_H}{16M},$$
where, R_H and h are Rydberg and Plank's constant respectively.