Answer Review MAP2302 (Page 1)

1) a) (use double angle formula $\cos(2\theta) = 2\cos^2\theta - 1 \to \cos^2\theta = \frac{1+\cos 2\theta}{2}$)

$$\int \cos^2 x\,dx = \int \frac{1+\cos(2x)}{2}\,dx = \boxed{\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C} \text{ or } \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$$

b) long division first $\frac{y^3 - 3y^2 + 5y + 4}{y^2+1} = y - 3 + \frac{4y+7}{y^2+1}$

$$\int \frac{y^3-3y^2+5y+4}{y^2+1}\,dy = \int\left(y - 3 + \frac{4y+7}{y^2+1}\right)dy = \int (y-3)\,dy + 2\int \frac{2y}{y^2+1}\,dy + 7\int \frac{1}{y^2+1}\,dy$$

$$= \boxed{\frac{1}{2}y^2 - 3y + 2\ln(y^2+1) + 7\arctan y + C}$$

c) Note It's not inverse secant. $(\sec^{-1}x)' = \frac{1}{x\sqrt{x^2-1}}$

use trig sub $x = \sin\theta$, $dx = \cos\theta\,d\theta$

$$\int \frac{-4}{x\sqrt{1-x^2}}\,dx = \int \frac{-4\cos\theta\,d\theta}{(\sin\theta)\sqrt{1-\sin^2\theta}} = \int \frac{-4\cancel{\cos\theta}}{\sin\theta\,\cancel{\cos\theta}}\,d\theta = -4\int \csc\theta\,d\theta = 4\ln|\csc\theta + \cot\theta| + C$$

$$= \boxed{4\ln\left|\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right| + C}$$

d) $\int \csc^2 t\,dt - \int \sec t\tan t\,dt = \boxed{-\cot t - \sec t + C}$

e) $\int \sec\theta \cdot \frac{1}{\cos\theta}\,d\theta = \int \sec^2\theta\,d\theta = \tan\theta + C$

f) $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

g) (pythagorean identity $\cos^2 x + \sin^2 x = 1 \to \frac{\cos^2 x}{\sin^2 x} + 1 = \frac{1}{\sin^2 x} \to \cot^2 x + 1 = \csc^2 x$)

$$\int \cot^2 x\,dx = \int (\csc^2 x - 1)\,dx = \boxed{-\cot x - x + C}$$

h) exponent of sine is "odd" substitution $u = \cos x$ hence $du = -\sin x\,dx$

$$\int \sin^3 x\,dx = \int \sin^2 x\,(-du) = \int (1-\cos^2 x)(-du) = \int (1-u^2)(-du) = -u + \frac{u^3}{3} + C$$

$$= \boxed{-\cos x + \frac{1}{3}\cos^3 x + C}$$

i) (by parts) "LIATE" $u = \ln y$, $dv = y$; $du = \frac{1}{y}dy$, $v = \frac{1}{2}y^2$

by parts $\int u\,dv = uv - \int v\,du$

$$\int y\ln y\,dy = uv - \int v\,du = \frac{1}{2}y^2\ln y - \int \frac{1}{2}y\,dy = \boxed{\frac{1}{2}y^2\ln y - \frac{1}{4}y^2 + C}$$

Review MAP 2302 (page 2)

1)

(p) Trig Subs $x = 2\tan\theta$, $\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{2\sec^2\theta\, d\theta}{\sqrt{4(1+\tan^2\theta)}} = \int \frac{2\sec^2\theta\, d\theta}{2\sec\theta} = \int \sec\theta\, d\theta$

$= \ln|\sec\theta + \tan\theta| = \boxed{\ln\left|\sqrt{1+\frac{x^2}{4}} + \frac{x}{2}\right| + C}$

j) $\int \frac{x}{\sqrt{x^2-1}}\,dx$ subs $u = x^2-1$, $du = 2x\,dx$

$= \frac{1}{2}\int \frac{du}{\sqrt{u}} = \sqrt{u} + C = \boxed{\sqrt{x^2-1} + C}$

k) subs $u = \cot x$, $du = -\csc^2 x\,dx$, use identity $\cot^2 x + 1 = \csc^2 x$

$\int \csc^4 x\,dx = \int \csc^2 x\,(-du) = \int (\cot^2 x + 1)(-du) = \int (u^2+1)(-du) = -\frac{u^3}{3} - u + C$

$= \boxed{-\frac{\cot^3 x}{3} - \cot x + C}$

l) "odd" exponent for $\cos(2x)$, take $u = \sin(2x)$, $du = 2\cos 2x\,dx \to \cos 2x = \frac{du}{2}$

$\int \sin^4 2x \cos^3 2x\,dx = \int u^4 (\cos^2 2x)\frac{du}{2} = \int u^4(1-u^2)\frac{du}{2} = \frac{1}{2}\left(\frac{u^5}{5} - \frac{u^7}{7}\right) + C$

$= \boxed{\frac{1}{10}\sin^5(2x) - \frac{1}{14}\sin^7(2x) + C}$

m) $\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{d(e^x)}{1+(e^x)^2} = \int \frac{du}{1+u^2}$ if $u = e^x$ $= \arctan u + C = \boxed{\arctan e^x + C}$

γ) (similar to k)) $u = \tan x$, $du = \sec^2 x\,dx$ and use $\sec^2 x = \tan^2 x + 1$

$\int \sec^4 x\,dx = \int \sec^2 x \underbrace{\boxed{\sec^2 x\,dx}}_{du} = \int (u^2+1)\,du = \frac{u^3}{3} + u + C = \boxed{\frac{\tan^3 x}{3} + \tan x + C}$

n) (Partial Fractions decomposition) $f(x) = \frac{x^2}{(x-1)(x+1)(x^2+1)} = \frac{a}{x-1} + \frac{b}{x+1} + \frac{cx+d}{x^2+1}$

$(x-1)f(x)\Big]_{\text{plug in 1 for } x} = \frac{1}{(2)(2)} = a + 0 + 0 \to a = \frac{1}{4}$

$(x+1)f(x)\Big]_{\text{plug in } -1 \text{ for } x} = \frac{1}{(-2)(2)} = 0 + b + 0 \quad b = -\frac{1}{4}$

$f(0) = 0 = -a + b + d \to d = a - b = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \quad d = \frac{1}{2}$

$\lim_{x\to\infty} x f(x) = 0 = a + b + c \to c = 0$

$\int \frac{x^2}{(x-1)(x+1)(x^2+1)}\,dx = \frac{1}{4}\int \frac{1}{x-1}\,dx - \frac{1}{4}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x^2+1}\,dx = \boxed{\frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{2}\tan^{-1}x + C}$

Review MAP 2302 (page 3)

1) p) (by parts) $\int u\,dv = uv - \int v\,du$,

$u = \sec x \qquad dv = \sec^2 x\,dx$

$du = \sec x \tan x\,dx \qquad v = \tan x$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec x \tan^2 x\,dx = \sec x \tan x - \int \sec x\,(\sec^2 x - 1)\,dx$$

use $= \sec^2 x - 1$ identity

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx \to 2\int \sec^3 x\,dx = \sec x \tan x + \int \sec x\,dx$$

add

$$\to 2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

Hence $\boxed{\int \sec^3 x\,dx = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C}$

(q) (Integration by parts)

u	dv
$\cos(3x)$	e^{2x}
$-3\sin 3x$	$\frac{1}{2}e^{2x}$
$-9\cos 3x$	$\frac{1}{4}e^{2x}$

$+$, $-$, $+$

$$\int e^{2x}\cos(3x)\,dx = \frac{1}{2}e^{2x}\cos(3x) + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}\int e^{2x}\cos 3x\,dx + C$$

add

$$\underbrace{\left(1 + \frac{9}{4}\right)}_{\frac{13}{4}}\int e^{2x}\cos(3x)\,dx = \frac{1}{2}e^{2x}\cos(3x) + \frac{3}{4}e^{2x}\sin(3x) + C$$

$$\boxed{\int e^{2x}\cos(3x)\,dx = \frac{4}{13}\left(\frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x\right) + C}$$

or $\frac{2}{13}e^{2x}\cos 3x + \frac{3}{13}e^{2x}\sin 3x + C$

λ) (similar to p) (by parts)

$u = \csc(2\theta) \qquad dv = \csc^2(2\theta)\,d\theta$

$du = -2\csc(2\theta)\cot(2\theta) \qquad v = -\frac{1}{2}\cot(2\theta)$

$$\int \csc^3(2\theta)\,d\theta = -\frac{1}{2}\csc(2\theta)\cot(2\theta) - \int \csc(2\theta)\underbrace{\cot^2(2\theta)}_{=\csc^2(2\theta)-1}\,d\theta = -\int \csc^3(2\theta)\,d\theta + \int \csc(2\theta)\,d\theta + C$$

$$\int \csc^3(2\theta)\,d\theta + \int \csc^3(2\theta)\,d\theta = -\frac{1}{2}\csc(2\theta)\cot(2\theta) + \int \csc(2\theta)\,d\theta + C$$

$$2\int \csc^3(2\theta)\,d\theta = -\frac{1}{2}\csc(2\theta)\cot(2\theta) + \frac{1}{2}\ln|\csc 2\theta - \cot(2\theta)| + C$$

divide by 2

$$\boxed{\int \csc^3(2\theta)\,d\theta = -\frac{1}{4}\csc(2\theta)\cot(2\theta) + \frac{1}{4}\ln|\csc(2\theta) - \cot(2\theta)| + C}$$

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v) Tabular integration by parts

x^4	e^{2x}	$+$
$4x^3$	$\frac{1}{2}e^{2x}$	$-$
$12x^2$	$\frac{1}{4}e^{2x}$	$+$
$24x$	$\frac{1}{8}e^{2x}$	$-$
24	$\frac{1}{16}e^{2x}$	$+$
0	$\frac{1}{32}e^{2x}$	

$$\boxed{\int x^4 e^{2x}\,dx = \left(\frac{1}{2}x^4 - x^3 + \frac{3}{2}x^2 - \frac{3}{2}x + \frac{3}{4}\right)e^{2x} + C}$$

r) $\int (2x - 3y - \sqrt{2})\,dx = x^2 - 3xy - \sqrt{2}x + g(y)$ where $g(y)$ is a function of y

s) $\int (3x^2y^3 + 4)\,dy = \frac{3}{4}x^2y^4 + 4y + f(x)$ where $f(x)$ is a function of x

t) $\int x\cos y\,dx = \frac{x^2}{2}\cos y + h(y)$ where $h(y)$ is a function y

u) $\int x\cos y\,dy = x\sin y + C(x)$ where $C(x)$ is a function of x

w) $\int x^3 e^{x^2}\,dx$ (by parts) $u = x^2$, $dv = xe^{x^2}dx$, $du = 2x\,dx$, $v = \frac{1}{2}e^{x^2}$ $\quad \int u\,dv = uv - \int v\,du$

$$\boxed{\int x^3 e^{x^2}\,dx = \frac{1}{2}x^2e^{x^2} - \int xe^{x^2}\,dx = \frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C}$$

(2) $f'(x) = (3x^5)' + 3(\cos^4 x)' - (\ln(x^2+1))' - 4(\tan 2x)' + (\text{------})'$

$$f'(x) = 15x^4 - 12\cos^3 x\sin x - \frac{2x}{x^2+1} - 8\sec^2(2x) + \frac{\sec(\sqrt{x})\tan\sqrt{x}}{2\sqrt{x}} + \arctan x^2 + \frac{2x^2}{1+x^4} - 3e^{3x} + \sqrt{2}x^{\sqrt{2}-1} + \arcsin 2x + \frac{2x}{\sqrt{1-4x^2}}$$

(3) a) $D = \begin{vmatrix} 9 & -4 \\ 7 & -10 \end{vmatrix} = -90 + 28 = -62$, $D_x = \begin{vmatrix} 5 & -4 \\ -8 & -10 \end{vmatrix} = -50 - 32 = -82$

$D_y = \begin{vmatrix} 9 & 5 \\ 7 & -8 \end{vmatrix} = -72 - 35 = -107$ $\quad x = \frac{D_x}{D} = \frac{-82}{-62} = \frac{41}{31}$, $y = \frac{D_y}{D} = \frac{107}{62}$

Answer $(x, y) = \left\{\frac{41}{31}, \frac{107}{62}\right\}$

b) $D = \begin{vmatrix} 2 & -3 & 7 \\ -3 & 4 & -2 \\ 4 & -5 & 3 \end{vmatrix} = 2(12-10) + 3(-9+8) + 7(15-16) = 4 - 3 - 7 = -6$

$D_x = \begin{vmatrix} -9 & -3 & 7 \\ -8 & 4 & -2 \\ 8 & -5 & 3 \end{vmatrix} = 14$, $D_y = \begin{vmatrix} 2 & -9 & 7 \\ -3 & -8 & -2 \\ 4 & 8 & 3 \end{vmatrix} = 31$, $D_z = 17$ Answer $\boxed{x, y, z = \left\{-\frac{7}{3}, -\frac{31}{6}, -\frac{17}{6}\right\}}$

Answer review MAP 2302

(4) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ or $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \quad -\infty < x < \infty$

$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad -1 < x \leq 1$

$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \quad -\infty < x < \infty$

$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\arctan x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$

(5) by parts or from trigonometry

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

add: $\boxed{\cos(a+b) + \cos(a-b) = 2\cos a \cos b}$

or subtract: $\boxed{\cos(a-b) - \cos(a+b) = 2\sin a \sin b}$

a) $\int \cos(3x)\cos(7x)\,dx = \int \frac{\cos(10x) + \cos(4x)}{2}\,dx = \boxed{\frac{1}{20}\sin(10x) + \frac{1}{8}\sin(4x) + C}$ ← answer a)

b) $\int \sin(4y)\sin(6y)\,dy = \int \frac{\cos(4y-6y) - \cos(4y+6y)}{2}\,dy = \frac{1}{2}\left(\frac{\sin(2y)}{2} - \frac{\sin(10y)}{10}\right) + C$

Answer 5b) $\boxed{\frac{1}{4}\sin 2y - \frac{1}{20}\sin(10y) + C}$

c) Use $\sin(a+b) = \sin a \cos b + \sin b \cos a$

$\sin(a-b) = \sin a \cos b - \sin b \cos a$

add: $\sin(a+b) + \sin(a-b) = 2\sin a \cos b$

$\int \sin(2\theta)\cos(5\theta)\,d\theta = \int \frac{\sin(7\theta) + \sin(-3\theta)}{2}\,d\theta = \boxed{-\frac{1}{14}\cos(7\theta) + \frac{1}{6}\cos(3\theta) + C}$