density $\delta(x,y,z)$

Mass $= \iiint \delta(x,y,z)\,dV$

$$\bar{x} = \frac{\iiint x\,(\text{density})\,dV}{M}$$

$$\bar{y} = \frac{\iiint y\,(\text{density})\,dV}{M}$$

$$\bar{z} = \frac{\iiint z\,(\text{density})\,dV}{M}$$

One-d 1D.

Hw 16.6 (11) Mass $= \frac{20}{3}$

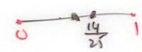
center of mass $= \frac{9}{5}$

section 16.6

(10) density $\delta(x) = 1 + x^3$, $0 \le x \le 1$

M mass $= \int_0^1 (1+x^3)\,dx = x + \frac{x^4}{4}\Big]_0^1 = 1 + \frac{1}{4} = \frac{5}{4}$

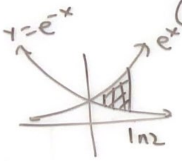
Center of mass $= \dfrac{\int_0^1 x(1+x^3)\,dx}{\frac{5}{4}}$



$$= \frac{\frac{x^2}{2} + \frac{x^5}{5}\Big]_0^1}{\frac{5}{4}} = \frac{\frac{1}{2} + \frac{1}{5}}{\frac{5}{4}} = \frac{\frac{7}{10}}{\frac{5}{4}} = \frac{7}{\cancel{10}} \cdot \frac{\cancel{4}^{\,2}}{5} = \frac{14}{25}$$

Centroid = center of mass for a constant density.

Section 16.6 , Ex 18 (Hwk 6)



Centroid: density = ~~constant~~ = 1

region R: $y=e^x$, $y=e^{-x}$, $x=0$, $x=\ln 2$

$$\text{Mass } M=\int_0^{\ln 2}\int_{e^{-x}}^{e^x} 1\,dy\,dx=\int_0^{\ln 2}(e^x-e^{-x})\,dx$$

$$=e^x+e^{-x}\Big]_0^{\ln 2}=e^{\ln 2}+e^{-\ln 2}-e^0-e^0$$

$$=2+\frac{1}{2}-2=\frac{1}{2}$$

$$\bar{x}=\frac{\int_0^{\ln 2}\int_{e^{-x}}^{e^x}x\,dy\,dx}{1/2}=2\int_0^{\ln 2}x(e^x-e^{-x})\,dx=\text{by parts}=2\Big[x(e^x+e^{-x})\Big]_0^{\ln 2}-\int_0^{\ln 2}(e^x+e^{-x})\,dx=2\left(2\ln 2+\frac{1}{2}\ln 2-\left(2-\frac{1}{2}-\cancel{1}+\cancel{1}\right)\right)$$

$$\int_0^{\ln 2}(e^x+e^{-x})\,dx=(e^x-e^{-x})\Big]_0^{\ln 2}$$

$$=5\ln 2-3$$

$$\bar{y}=\frac{\int_0^{\ln 2}\int_{e^{-x}}^{e^x}y\,dy\,dx}{1/2}=\cancel{2}\int_0^{\ln 2}\frac{\overbrace{(e^x)^2}^{=e^{2x}}-\overbrace{(e^{-x})^2}^{=e^{-2x}}}{\cancel{2}}\,dx=\frac{e^{2x}}{2}+\frac{e^{-2x}}{2}\Big]_0^{\ln 2}=\frac{e^{\ln 2^2}+e^{-\ln 2^2}-e^0-e^0}{2}=\frac{4+\frac{1}{4}-2}{2}=\frac{2+\frac{1}{4}}{2}=\frac{9}{8}$$

$\int_{e^{-x}}^{e^x} y\,dy = \frac{y^2}{2}$

Center $\left(5\ln 2-3,\ \frac{9}{8}\right)$

Hwk6

(33) density $=1+\frac{x}{2}$, R: $0\le x\le 4$, $0\le y\le 1$, $0\le z\le 1$

Mass

Center of mass