EXERCISE SET

Graphing Utility

1–8 Find dy/dx.

1. $y = 4x^7$
2. $y = -3x^{12}$
3. $y = 3x^8 + 2x + 1$
4. $y = \frac{1}{2}(x^4 + 7)$
5. $y = \pi^3$
6. $y = \sqrt{2}x + (1/\sqrt{2})$
7. $y = -\frac{1}{3}(x^7 + 2x - 9)$
8. $y = \dfrac{x^2+1}{5}$

9–16 Find $f'(x)$.

9. $f(x) = x^{-3} + \dfrac{1}{x^7}$
10. $f(x) = \sqrt{x} + \dfrac{1}{x}$
11. $f(x) = -3x^{-8} + 2\sqrt{x}$
12. $f(x) = 7x^{-6} - 5\sqrt{x}$
13. $f(x) = x^e + \dfrac{1}{x^{\sqrt{10}}}$
14. $f(x) = \sqrt[3]{\dfrac{8}{x}}$
15. $f(x) = (3x^2 + 1)^2$
16. $f(x) = ax^3 + bx^2 + cx + d$ $(a, b, c, d$ constant)

17–18 Find $y'(1)$.

17. $y = 5x^2 - 3x + 1$
18. $y = \dfrac{x^{3/2}+2}{x}$

19–20 Find dx/dt.

19. $x = t^2 - t$
20. $x = \dfrac{t^2+1}{3t}$

21–24 Find $dy/dx|_{x=1}$.

21. $y = 1 + x + x^2 + x^3 + x^4 + x^5$
22. $y = \dfrac{1 + x + x^2 + x^3 + x^4 + x^5 + x^6}{x^3}$
23. $y = (1 - x)(1 + x)(1 + x^2)(1 + x^4)$
24. $y = x^{24} + 2x^{12} + 3x^8 + 4x^6$

25–26 Approximate $f'(1)$ by considering the difference quotient

$$\frac{f(1+h) - f(1)}{h}$$

for values of h near 0, and then find the exact value of $f'(1)$ by differentiating.

25. $f(x) = x^3 - 3x + 1$
26. $f(x) = \dfrac{1}{x^2}$

27–28 Use a graphing utility to estimate the value of $f'(1)$ by zooming in on the graph of f, and then compare your estimate to the exact value obtained by differentiating.

27. $f(x) = \dfrac{x^2+1}{x}$
28. $f(x) = \dfrac{x + 2x^{3/2}}{\sqrt{x}}$

29–32 Find the indicated derivative.

29. $\dfrac{d}{dt}[16t^2]$
30. $\dfrac{dC}{dr}$, where $C = 2\pi r$
31. $V'(r)$, where $V = \pi r^3$
32. $\dfrac{d}{d\alpha}[2\alpha^{-1} + \alpha]$

33–36 True–False Determine whether the statement is true or false. Explain your answer.

33. If f and g are differentiable at $x = 2$, then
$$\frac{d}{dx}[f(x) - 8g(x)]\bigg|_{x=2} = f'(2) - 8g'(2)$$

34. If $f(x)$ is a cubic polynomial, then $f'(x)$ is a quadratic polynomial.

35. If $f'(2) = 5$, then
$$\frac{d}{dx}[4f(x) + x^3]\bigg|_{x=2} = \frac{d}{dx}[4f(x) + 8]\bigg|_{x=2} = 4f'(2) = 20$$

36. If $f(x) = x^2(x^4 - x)$, then
$$f''(x) = \frac{d}{dx}[x^2] \cdot \frac{d}{dx}[x^4 - x] = 2x(4x^3 - 1)$$

37. A spherical balloon is being inflated.
    (a) Find a general formula for the instantaneous rate of change of the volume V with respect to the radius r, given that $V = \frac{4}{3}\pi r^3$.
    (b) Find the rate of change of V with respect to r at the instant when the radius is $r = 5$.

38. Find $\dfrac{d}{d\lambda}\left[\dfrac{\lambda\lambda_0 + \lambda^6}{2 - \lambda_0}\right]$ (λ_0 is constant).

39. Find an equation of the tangent line to the graph of $y = f(x)$ at $x = -3$ if $f(-3) = 2$ and $f'(-3) = 5$.

40. Find an equation of the tangent line to the graph of $y = f(x)$ at $x = 2$ if $f(2) = -2$ and $f'(2) = -1$.

41–42 Find d^2y/dx^2.

41. (a) $y = 7x^3 - 5x^2 + x$ (b) $y = 12x^2 - 2x + 3$
    (c) $y = \dfrac{x+1}{x}$ (d) $y = (5x^2 - 3)(7x^3 + x)$

42. (a) $y = 4x^7 - 5x^3 + 2x$ (b) $y = 3x + 2$
    (c) $y = \dfrac{3x-2}{5x}$ (d) $y = (x^3 - 5)(2x + 3)$

43–44 Find y'''.

43. (a) $y = x^{-5} + x^5$ (b) $y = 1/x$
    (c) $y = ax^3 + bx + c$ $(a, b, c$ constant)

44. (a) $y = 5x^2 - 4x + 7$ (b) $y = 3x^{-2} + 4x^{-1} + x$
    (c) $y = ax^4 + bx^2 + c$ $(a, b, c$ constant)

45. Find
    (a) $f'''(2)$, where $f(x) = 3x^2 - 2$
    (b) $\dfrac{d^2y}{dx^2}\bigg|_{x=1}$, where $y = 6x^5 - 4x^2$
    (c) $\dfrac{d^4}{dx^4}[x^{-3}]\bigg|_{x=1}$.

EXERCISE SET : Graphing Utility

1–4 Compute the derivative of the given function $f(x)$ by (a) multiplying and then differentiating and (b) using the product rule. Verify that (a) and (b) yield the same result.

1. $f(x) = (x+1)(2x-1)$ **2.** $f(x) = (3x^2-1)(x^2+2)$

3. $f(x) = (x^2+1)(x^2-1)$

4. $f(x) = (x+1)(x^2-x+1)$

5–20 Find $f'(x)$.

5. $f(x) = (3x^2+6)\left(2x-\frac{1}{4}\right)$

6. $f(x) = (2-x-3x^3)(7+x^5)$

7. $f(x) = (x^3+7x^2-8)(2x^{-3}+x^{-4})$

8. $f(x) = \left(\dfrac{1}{x}+\dfrac{1}{x^2}\right)(3x^3+27)$

9. $f(x) = (x-2)(x^2+2x+4)$

10. $f(x) = (x^2+x)(x^2-x)$

11. $f(x) = \dfrac{3x+4}{x^2+1}$ **12.** $f(x) = \dfrac{x-2}{x^4+x+1}$

13. $f(x) = \dfrac{x^2}{3x-4}$ **14.** $f(x) = \dfrac{2x^2+5}{3x-4}$

15. $f(x) = \dfrac{(2\sqrt{x}+1)(x-1)}{x+3}$

16. $f(x) = (2\sqrt{x}+1)\left(\dfrac{2-x}{x^2+3x}\right)$

17. $f(x) = (2x+1)\left(1+\dfrac{1}{x}\right)(x^{-3}+7)$

18. $f(x) = x^{-5}(x^2+2x)(4-3x)(2x^9+1)$

19. $f(x) = (x^7+2x-3)^3$ **20.** $f(x) = (x^2+1)^4$

21–24 Find $dy/dx|_{x=1}$.

21. $y = \dfrac{2x-1}{x+3}$ **22.** $y = \dfrac{4x+1}{x^2-5}$

23. $y = \left(\dfrac{3x+2}{x}\right)(x^{-5}+1)$ **24.** $y = (2x^7-x^2)\left(\dfrac{x-1}{x+1}\right)$

25–26 Use a graphing utility to estimate the value of $f'(1)$ by zooming in on the graph of f, and then compare your estimate to the exact value obtained by differentiating.

25. $f(x) = \dfrac{x}{x^2+1}$ **26.** $f(x) = \dfrac{x^2-1}{x^2+1}$

27. Find $g'(4)$ given that $f(4) = 3$ and $f'(4) = -5$.
(a) $g(x) = \sqrt{x}f(x)$ (b) $g(x) = \dfrac{f(x)}{x}$

28. Find $g'(3)$ given that $f(3) = -2$ and $f'(3) = 4$.
(a) $g(x) = 3x^2 - 5f(x)$ (b) $g(x) = \dfrac{2x+1}{f(x)}$

29. In parts (a)–(d), $F(x)$ is expressed in terms of $f(x)$ and $g(x)$. Find $F'(2)$ given that $f(2) = -1$, $f'(2) = 4$, $g(2) = 1$, and $g'(2) = -5$.
(a) $F(x) = 5f(x) + 2g(x)$ (b) $F(x) = f(x) - 3g(x)$
(c) $F(x) = f(x)g(x)$ (d) $F(x) = f(x)/g(x)$

30. Find $F'(\pi)$ given that $f(\pi) = 10$, $f'(\pi) = -1$, $g(\pi) = -3$, and $g'(\pi) = 2$.
(a) $F(x) = 6f(x) - 5g(x)$ (b) $F(x) = x(f(x)+g(x))$
(c) $F(x) = 2f(x)g(x)$ (d) $F(x) = \dfrac{f(x)}{4+g(x)}$

31–36 Find all values of x at which the tangent line to the given curve satisfies the stated property.

31. $y = \dfrac{x^2-1}{x+2}$; horizontal **32.** $y = \dfrac{x^2+1}{x-1}$; horizontal

33. $y = \dfrac{x^2+1}{x+1}$; parallel to the line $y = x$

34. $y = \dfrac{x+3}{x+2}$; perpendicular to the line $y = x$

35. $y = \dfrac{1}{x+4}$; passes through the origin

36. $y = \dfrac{2x+5}{x+2}$; y-intercept 2

FOCUS ON CONCEPTS

37. (a) What should it mean to say that two curves intersect at right angles?
(b) Show that the curves $y = 1/x$ and $y = 1/(2-x)$ intersect at right angles.

38. Find all values of a such that the curves $y = a/(x-1)$ and $y = x^2 - 2x + 1$ intersect at right angles.

39. Find a general formula for $F''(x)$ if $F(x) = xf(x)$ and f and f' are differentiable at x.

40. Suppose that the function f is differentiable everywhere and $F(x) = xf(x)$.
(a) Express $F'''(x)$ in terms of x and derivatives of f.
(b) For $n \geq 2$, conjecture a formula for $F^{(n)}(x)$.

41. A manufacturer of athletic footwear finds that the sales of their ZipStride brand running shoes is a function $f(p)$ of the selling price p (in dollars) for a pair of shoes. Suppose that $f(120) = 9000$ pairs of shoes and $f'(120) = -60$ pairs of shoes per dollar. The revenue that the manufacturer will receive for selling $f(p)$ pairs of shoes at p dollars per pair is $R(p) = p \cdot f(p)$. Find $R'(120)$. What impact would a small increase in price have on the manufacturer's revenue?

42. Solve the problem in Exercise 41 under the assumption that $f(120) = 9000$ and $f'(120) = -80$.

43. Use the quotient rule (Theorem 2.4.2) to derive the formula for the derivative of $f(x) = x^{-n}$, where n is a positive integer.

✔ QUICK CHECK EXERCISES

(See page 174 for answers.)

1. Find dy/dx.
   (a) $y = \sin x$ (b) $y = \cos x$
   (c) $y = \tan x$ (d) $y = \sec x$
2. Find $f'(x)$ and $f'(\pi/3)$ if $f(x) = \sin x \cos x$.
3. Use a derivative to evaluate each limit.
   (a) $\lim_{h\to 0} \dfrac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$ (b) $\lim_{h\to 0} \dfrac{\csc(x+h)-\csc x}{h}$

EXERCISE SET

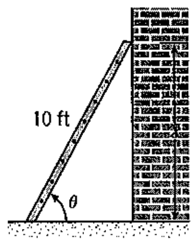
Graphing Utility

1–18 Find $f'(x)$.

1. $f(x) = 4\cos x + 2\sin x$
2. $f(x) = \dfrac{5}{x^2} + \sin x$
3. $f(x) = -4x^2 \cos x$
4. $f(x) = 2\sin^2 x$
5. $f(x) = \dfrac{5-\cos x}{5+\sin x}$
6. $f(x) = \dfrac{\sin x}{x^2+\sin x}$
7. $f(x) = \sec x - \sqrt{2}\tan x$
8. $f(x) = (x^2+1)\sec x$
9. $f(x) = 4\csc x - \cot x$
10. $f(x) = \cos x - x\csc x$
11. $f(x) = \sec x \tan x$
12. $f(x) = \csc x \cot x$
13. $f(x) = \dfrac{\cot x}{1+\csc x}$
14. $f(x) = \dfrac{\sec x}{1+\tan x}$
15. $f(x) = \sin^2 x + \cos^2 x$
16. $f(x) = \sec^2 x - \tan^2 x$
17. $f(x) = \dfrac{\sin x \sec x}{1 + x\tan x}$
18. $f(x) = \dfrac{(x^2+1)\cot x}{3-\cos x \csc x}$

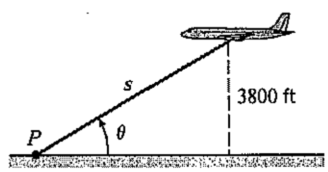
19–24 Find d^2y/dx^2.

19. $y = x\cos x$
20. $y = \csc x$
21. $y = x\sin x - 3\cos x$
22. $y = x^2\cos x + 4\sin x$
23. $y = \sin x \cos x$
24. $y = \tan x$
25. Find the equation of the line tangent to the graph of $\tan x$ at
    (a) $x = 0$ (b) $x = \pi/4$ (c) $x = -\pi/4$.
26. Find the equation of the line tangent to the graph of $\sin x$ at
    (a) $x = 0$ (b) $x = \pi$ (c) $x = \pi/4$.
27. (a) Show that $y = x\sin x$ is a solution to $y'' + y = 2\cos x$.
    (b) Show that $y = x\sin x$ is a solution of the equation $y^{(4)} + y'' = -2\cos x$.
28. (a) Show that $y = \cos x$ and $y = \sin x$ are solutions of the equation $y'' + y = 0$.
    (b) Show that $y = A\sin x + B\cos x$ is a solution of the equation $y'' + y = 0$ for all constants A and B.
29. Find all values in the interval $[-2\pi, 2\pi]$ at which the graph of f has a horizontal tangent line.
    (a) $f(x) = \sin x$ (b) $f(x) = x + \cos x$
    (c) $f(x) = \tan x$ (d) $f(x) = \sec x$
30. (a) Use a graphing utility to make rough estimates of the values in the interval $[0, 2\pi]$ at which the graph of $y = \sin x \cos x$ has a horizontal tangent line.
    (b) Find the exact locations of the points where the graph has a horizontal tangent line.
31. A 10 ft ladder leans against a wall at an angle θ with the horizontal, as shown in the accompanying figure. The top of the ladder is x feet above the ground. If the bottom of the ladder is pushed toward the wall, find the rate at which x changes with respect to θ when $\theta = 60°$. Express the answer in units of feet/degree.



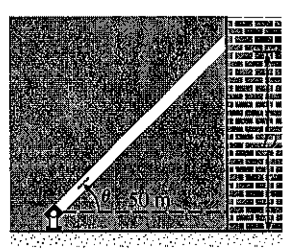
◄ **Figure Ex-31**

32. An airplane is flying on a horizontal path at a height of 3800 ft, as shown in the accompanying figure. At what rate is the distance s between the airplane and the fixed point P changing with respect to θ when $\theta = 30°$? Express the answer in units of feet/degree.



◄ **Figure Ex-32**

33. A searchlight is trained on the side of a tall building. As the light rotates, the spot it illuminates moves up and down the side of the building. That is, the distance D between ground level and the illuminated spot on the side of the building is a function of the angle θ formed by the light beam and the horizontal (see the accompanying figure). If the searchlight is located 50 m from the building, find the rate at which D is changing with respect to θ when $\theta = 45°$. Express your answer in units of meters/degree.



◄ **Figure Ex-33**

FOCUS ON CONCEPTS

5. Given the following table of values, find the indicated derivatives in parts (a) and (b).

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
3	5	−2	5	7
5	3	−1	12	4

(a) $F'(3)$, where $F(x) = f(g(x))$
(b) $G'(3)$, where $G(x) = g(f(x))$

6. Given the following table of values, find the indicated derivatives in parts (a) and (b).

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
−1	2	3	2	−3
2	0	4	1	−5

(a) $F'(-1)$, where $F(x) = f(g(x))$
(b) $G'(-1)$, where $G(x) = g(f(x))$

7–26 Find $f'(x)$. ■

7. $f(x) = (x^3 + 2x)^{37}$
8. $f(x) = (3x^2 + 2x - 1)^6$
9. $f(x) = \left(x^3 - \dfrac{7}{x}\right)^{-2}$
10. $f(x) = \dfrac{1}{(x^5 - x + 1)^9}$
11. $f(x) = \dfrac{4}{(3x^2 - 2x + 1)^3}$
12. $f(x) = \sqrt{x^3 - 2x + 5}$
13. $f(x) = \sqrt{4 + \sqrt{3x}}$
14. $f(x) = \sqrt[3]{12 + \sqrt{x}}$
15. $f(x) = \sin\left(\dfrac{1}{x^2}\right)$
16. $f(x) = \tan\sqrt{x}$
17. $f(x) = 4\cos^5 x$
18. $f(x) = 4x + 5\sin^4 x$
19. $f(x) = \cos^2(3\sqrt{x})$
20. $f(x) = \tan^4(x^3)$
21. $f(x) = 2\sec^2(x^7)$
22. $f(x) = \cos^3\left(\dfrac{x}{x+1}\right)$
23. $f(x) = \sqrt{\cos(5x)}$
24. $f(x) = \sqrt{3x - \sin^2(4x)}$
25. $f(x) = [x + \csc(x^3 + 3)]^{-3}$
26. $f(x) = [x^4 - \sec(4x^2 - 2)]^{-4}$

27–40 Find dy/dx. ■

27. $y = x^3\sin^2(5x)$
28. $y = \sqrt{x}\tan^3(\sqrt{x})$
29. $y = x^5\sec(1/x)$
30. $y = \dfrac{\sin x}{\sec(3x + 1)}$
31. $y = \cos(\cos x)$
32. $y = \sin(\tan 3x)$
33. $y = \cos^3(\sin 2x)$
34. $y = \dfrac{1 + \csc(x^2)}{1 - \cot(x^2)}$
35. $y = (5x + 8)^7\left(1 - \sqrt{x}\right)^6$
36. $y = (x^2 + x)^5\sin^8 x$
37. $y = \left(\dfrac{x - 5}{2x + 1}\right)^3$
38. $y = \left(\dfrac{1 + x^2}{1 - x^2}\right)^{17}$
39. $y = \dfrac{(2x + 3)^3}{(4x^2 - 1)^8}$
40. $y = [1 + \sin^3(x^5)]^{12}$

41–42 Use a CAS to find dy/dx. ■

41. $y = [x\sin 2x + \tan^4(x^7)]^5$
42. $y = \tan^4\left(2 + \dfrac{(7 - x)\sqrt{3x^2 + 5}}{x^3 + \sin x}\right)$

43–50 Find an equation for the tangent line to the graph at the specified value of x. ■

43. $y = x\cos 3x,\ x = \pi$
44. $y = \sin(1 + x^3),\ x = -3$
45. $y = \sec^3\left(\dfrac{\pi}{2} - x\right),\ x = -\dfrac{\pi}{2}$
46. $y = \left(x - \dfrac{1}{x}\right)^3,\ x = 2$
47. $y = \tan(4x^2),\ x = \sqrt{\pi}$
48. $y = 3\cot^4 x,\ x = \dfrac{\pi}{4}$
49. $y = x^2\sqrt{5 - x^2},\ x = 1$
50. $y = \dfrac{x}{\sqrt{1 - x^2}},\ x = 0$

51–54 Find d^2y/dx^2. ■

51. $y = x\cos(5x) - \sin^2 x$
52. $y = \sin(3x^2)$
53. $y = \dfrac{1 + x}{1 - x}$
54. $y = x\tan\left(\dfrac{1}{x}\right)$

55–58 Find the indicated derivative. ■

55. $y = \cot^3(\pi - \theta)$; find $\dfrac{dy}{d\theta}$.
56. $\lambda = \left(\dfrac{au + b}{cu + d}\right)^6$; find $\dfrac{d\lambda}{du}$ (a, b, c, d constants).
57. $\dfrac{d}{d\omega}[a\cos^2\pi\omega + b\sin^2\pi\omega]$ (a, b constants)
58. $x = \csc^2\left(\dfrac{\pi}{3} - y\right)$; find $\dfrac{dx}{dy}$.

59. (a) Use a graphing utility to obtain the graph of the function $f(x) = x\sqrt{4 - x^2}$.
(b) Use the graph in part (a) to make a rough sketch of the graph of f'.
(c) Find $f'(x)$, and then check your work in part (b) by using the graphing utility to obtain the graph of f'.
(d) Find the equation of the tangent line to the graph of f at $x = 1$, and graph f and the tangent line together.

60. (a) Use a graphing utility to obtain the graph of the function $f(x) = \sin x^2\cos x$ over the interval $[-\pi/2, \pi/2]$.
(b) Use the graph in part (a) to make a rough sketch of the graph of f' over the interval.
(c) Find $f'(x)$, and then check your work in part (b) by using the graphing utility to obtain the graph of f' over the interval.
(d) Find the equation of the tangent line to the graph of f at $x = 1$, and graph f and the tangent line together over the interval.