Homework 4 Calculus 3

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(1) $f(x,y,z) = \sqrt{36 - 9x^2 + 4y^2 - z^2}$

(a) Describe and sketch the domain of f

(b) Find the linear approximation $L(x,y,z)$ to f at point $P(-2,5,-6)$

(c) Use the linear approximation from (b) to estimate $f(-2.05, 4.9, -6.1)$

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(2) Suppose that a function $f(x,y,z)$ is differentiable at $(3,5,8)$ and $L(x,y,z) = x - y + 2z - 2$ is the local linear approximation at $(3,5,8)$. Find $f(3,5,8)$, $f_x(3,5,8)$, $f_y(3,5,8)$, $f_z(3,5,8)$

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(3) Find an equation for the tangent plane and parametric equations for the normal line to the surface $z = e^{3y}\sin(3x)$ at $P(\pi/6, 0, 1)$

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(4) $f(x,y,z) = \sqrt{2x - 4y + 5z}$; Find a unit vector in the direction in which f increases most rapidly at $P(1,-3,7)$ and find the rate of change of f at P in that direction.

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(5) Find the critical points and classify each as a relative max, relative min or saddle point

a) $f(x,y) = 2x^3 + y^3 + 3x^2 - 3y - 12x - 4$

b) $f(x,y) = 4xy - 2x^4 - y^2 + 4x - 2y$

c) $f(x,y) = xye^{-(16x^2+9y^2)/288}$

d) $f(x,y) = x^3 + 3xy^2 - 15x + y^3 - 15y + 9$

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(6) Find the absolute extremum of the given function on the indicated closed and bounded set R

a) $f(x,y) = x^2 - 3y^2 - 2x + 6y$; R is the region bounded by the square with vertices $(0,0)$, $(0,2)$, $(2,0)$, $(2,2)$

b) $f(x,y) = xe^y - x^2 - e^y$; R is the region bounded by the rectangle with vertices $(0,0)$, $(0,1)$, $(2,1)$, $(2,0)$

c) $f(x,y) = x^2 + 2y^2 - x$ where R is disk $x^2 + y^2 \le 4$

d) $f(x,y) = xy^2$ where R is disk $x^2 + y^2 \le 1$, $x \ge 0$, $y \ge 0$

e) $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0, y = 4, y = 2x$ in the first quadrant

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(7) Use Lagrange multipliers to find the maximum and minimum values of

a) $f(x,y,z) = 3x + 6y + 2z$ subject to $2x^2 + 4y^2 + z^2 = 70$

b) $f(x,y,z) = xyz$ subject to $x^2 + y^2 + z^2 = 1$

(8) Find all points on $x + y + z = 5$ in the 1st octant at which $f(x, y, z) = xy^2z^2$ has a maximum value

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(9) A company produces x units of commodity A and y units of commodity B. All the units can be sold for $p = 100 - x$ dollars per unit of A and $q = 100 - y$ dollars per unit of B. The cost (in dollars) of producing these units is given by the joint-cost function $C(x, y) = x^2 + xy + y^2$

a) Write the formula for the profit $P(x, y)$ b) What should x and y be to maximize profit?

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(10) $f(x, y, z) = \sqrt{\dfrac{x + z}{2z - y}}$; Find a unit vector in the direction in which f decreases most rapidly at $P(5, 7, 6)$ and find the rate of change of f at P in that direction.

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(11) $f(x, y, z) = \tan^{-1} \dfrac{x}{z + y}$; Find a unit vector in the direction in which f increases most rapidly at $P(4, 2, 2)$ and find the rate of change of f at P in that direction.

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(12) Find two vectors of length 4 that are normal to the surface $z = \sqrt{\dfrac{z + x}{y - 1}}$ at the point $P(3, 5, 1)$