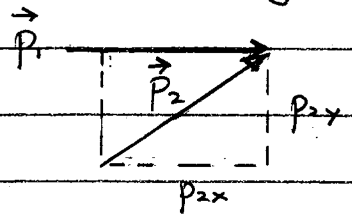
PHY 2048 — Test II — 27 March 2017

Solutions — V. I —

4 ① (a) $M_1 = 1500$ kg, $v_1 = 12.0$ m/s → $p_1 = M_1 v_1 = 1.80 \times 10^4$ kg-m/s

$M_2 = 2000$ kg, $v_2 = 8.0$ m/s → $p_2 = M_2 v_2 = 1.60 \times 10^4$ kg-m/s

5 (b)



$P_x = p_1 + p_2 \cos 36.9$

→ $P_x = 3.08 \times 10^4$ kg-m/s

$P_y = p_2 \sin 36.9 = 9.61 \times 10^3$ kg-m/s

6 (c) Momentum conserv. ⇒ $\vec{P}_f = \vec{P}_{in}$

$P_{in} = \sqrt{P_x^2 + P_y^2} = 3.226 \times 10^4$ kg-m/s ⇒ $v_f = P_{in}/(M_1 + M_2)$

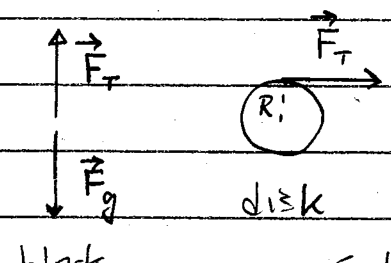
→ $v_f = 9.22$ m/s $\tan\phi = P_y/P_x = 0.3120$ → $\phi = 17.3°$

5 (d) $K_{in} = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2 = 1.720 \times 10^5$ J

$K_{fin} = \frac{1}{2}(M_1 + M_2) v_f^2 = 1.487 \times 10^5$ J

⇒ $K_{lost} = K_{in} - K_f = 2.33 \times 10^4$ J

8 ② (a)



block: $ma = F_g - F_T = mg - F_T$ (1)

disk: $I\alpha = F_T R$ ⇒ $F_T = I\alpha/R$ (2)

no slipping condition → $\alpha = a/R$ (3)

substitute Eqs (2) and (3) into Eq (1)

→ $ma = mg - Ia/R^2$ ⇒ $a = mg/(m + I/R^2)$

$I = 0.12$ kg-m², $m = 0.35$ kg, $R = 0.25$ m → $a = 1.51$ m/s²

5 (b) 5 revs → $\Delta\theta = 5 \times 2\pi = 10\pi$ radians, $\alpha = a/R = 6.044$ rad/s²

$\Delta\theta = \frac{1}{2}\alpha t^2$ ⇒ $t = \sqrt{2(\Delta\theta)/\alpha}$ → $t = 3.22$ sec

3 (c) Energy conserv ⇒ $K_{tot} = mgh = mg[5(2\pi R)] = (10\pi)mgR$

→ $K_{tot} = 26.9$ J

4 (d) $K_{trans} = \frac{1}{2} m v^2$ and $K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2} I v^2/R^2 = \frac{1}{2}(I/R^2) v^2$

→ $\frac{K_{trans}}{K_{tot}} = \frac{m}{m + I/R^2} = 0.15$ $\frac{K_{rot}}{K_{tot}} = \frac{I/R^2}{m + I/R^2} = 0.85$

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Solns, p.2

— V.I —

(3) (a) $m = 0.120$ kg, $r = 0.200$ m, $v = 4.0$ m/s

4/ $\Rightarrow$ $\boxed{L_m = mvr = 0.096 \text{ kg-m}^2/\text{s}}$

(b) Ang. mom. conserv $\Rightarrow L_d + L_m = 0 \Rightarrow I\omega = -L_m$

6/ $I = 0.150$ kg-m^2 $\Rightarrow \boxed{\omega = -L_m/I = -0.64 \text{ rad/s}}$

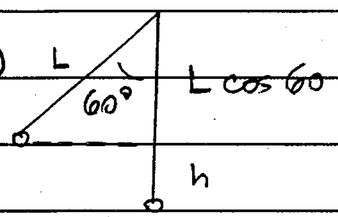
→ rotates opposite way mouse runs

5/ (c) $K_{total} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = 0.96 \text{ J} + 0.031 \rightarrow \boxed{K_{tot} = 0.99 \text{ J}}$

5/ (d) 3 revs $\rightarrow \Delta\theta = 3(2\pi) = 6\pi$ rads $\rightarrow \Delta\theta = |\omega| t$

$\Rightarrow \boxed{t = (\Delta\theta)/|\omega| = 29.5 \text{ sec}}$

(4) (a)



3/ $L = 1.80$ m $\rightarrow h = L - L\cos 60 = L/2$

$\rightarrow \boxed{h = 0.90 \text{ m}}$

4/ (b) $E_{bottom} = E_{top} \Rightarrow \frac{1}{2} m_B v^2 = m_B g h \Rightarrow v = \sqrt{2gh}$

$\Rightarrow \boxed{v = 4.20 \text{ m/s}}$

8 (c) momentum conserv $\rightarrow m_A v_A + m_B v_B = m_B v$ (1)

elastic collision $\rightarrow v_B - v_A = -v$ (2)

Eq(1) + $m_A \times$ Eq(2) $\rightarrow (m_A + m_B) v_B = (m_B - m_A) v$

$m_A = 0.90$ kg, $m_B = 1.50$ kg $\Rightarrow \boxed{v_B = 0.25\, v = 1.05 \text{ m/s}}$

$\rightarrow \boxed{v_A = v + v_B = 5.25 \text{ m/s}}$

5/ (d) Energy conserv $\rightarrow m_A g h_A = \frac{1}{2} m_A v_A^2 \Rightarrow h_A = v_A^2/2g = 1.406$ m

$h_A = L - L\cos\theta \Rightarrow \cos\theta = (L - h_A)/L = 0.2188$

$\rightarrow \boxed{\theta = 77°}$