PHY 2048
Test III – 24 April 2017

Solutions
V.II

(1) (a) $m = 2.00\ kg$, $k = 450\ N/m \Rightarrow \omega = \sqrt{k/m} = 15.0\ rad/s$ (4)

$\rightarrow \boxed{T = 2\pi/\omega = 0.419\ s} \rightarrow \boxed{n = \Delta t/T = 31\ osc \text{ in } \Delta t = 13.0\ s}$

(b) $\boxed{v_{max} = \omega x_0 = 0.90\ m/s}$ for $x_0 = 6.00\ cm = 0.06\ m$ (4)

(c) $K = U_{elastic} \Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} k x^2 \Rightarrow v^2 = \omega^2 x^2$ (6)

Now $v^2 = \omega^2(x_0^2 - x^2) \Rightarrow x_0^2 - x^2 = x^2 \Rightarrow \boxed{x = x_0/\sqrt{2} = 4.24\ cm}$

(d) $x = x_0 \cos(\omega t) \Rightarrow \omega t = \cos^{-1}(x/x_0) = \pi/4$ (5)

$\Rightarrow \boxed{t = \pi/4\omega = 0.0524\ sec} \rightarrow \boxed{v = \omega x = 0.636\ m/s}$

(2) (a) $M = 7.36 \times 10^{22}\ kg$ and $R = 1.74 \times 10^6\ m$ (4)

$\boxed{g_{surf} = GM/R^2 = 1.61\ m/s^2} \rightarrow \boxed{v_{esc} = \sqrt{2 g_s R} = 2.37\ km/s}$

(b) $\frac{1}{2} m v_0^2 - \frac{GMm}{R} = -\frac{GMm}{r} \Rightarrow v_0 = \sqrt{2\left(\frac{GM}{R} - \frac{GM}{r}\right)}$ (6)

$r = R + h = R + \frac{3}{4}R = \frac{7}{4}R \Rightarrow \frac{1}{R} - \frac{1}{r} = \frac{1}{R}\left(1 - \frac{4}{7}\right) = \frac{3}{7}\frac{1}{R}$

$\Rightarrow \boxed{v_0 = \sqrt{\frac{3}{7}}\, v_{esc} = 0.6547\, v_{esc} = 1.55\ km/s}$

(c) $v_{orb} = \sqrt{GM/r}$, $r = 3R \Rightarrow \boxed{v_{orb} = 9.67 \times 10^2\ m/s}$ (5)

$\Rightarrow \boxed{\Delta t = 6T = 12\pi r/v_{orb} = 2.034 \times 10^5\ s = 56.5\ hrs}$

(d) $m = 600\ kg \rightarrow \boxed{U = -GMm/r = -5.61 \times 10^8\ J}$ (1½)

$\rightarrow \boxed{E = \frac{1}{2} U = -2.81 \times 10^8\ J}$

(3) (a) $a = 0.250\ m \Rightarrow V = a^3 = 1.563 \times 10^{-2}\ m^3$ (5)

$\rho = 750\ kg/m^3 \Rightarrow m = \rho V = 11.72\ kg \Rightarrow \boxed{F_g = mg = 115\ N}$

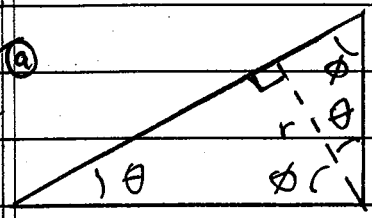
(b) $\rho_{fl} = 1000\ kg/m^3$ and $V_{disp} = V \Rightarrow \boxed{F_B = \rho_{fl} V_{disp} g = 153\ N}$ (5½)

(c) $F_T + F_g = F_B \Rightarrow \boxed{F_T = F_B - F_g = 38.3\ N}$ (4)

(d) Now $F_B = F_g$ and $V_{disp} = V_{imm}$ (6)

$\Rightarrow \rho_{fl} V_{imm} g = \rho V g \Rightarrow V_{imm} = (\rho/\rho_{fl}) V = 0.750 V$

$\rightarrow \boxed{V_{imm} = 1.17 \times 10^{-2}\ m^3}$

(4) (a) 

$\theta = 36.9°$ and $\phi = 90° - 36.9° = 53.1°$

$L = 2.40\ m \Rightarrow \boxed{r = L\cos\theta = L\sin\phi = 1.92\ m}$

(b) Torques about point P $\rightarrow F_T r = F_{app} y \Rightarrow F_T = F_{app}(y/r)$

$F_{app} = 50.0\ N$ and $y = 1.80\ m \Rightarrow \boxed{F_T = 46.9\ N}$

(c) $\boxed{F_V = F_T \sin\theta + Mg = 146\ N}$ for $M = 12.0\ kg$

(d) $F_H + F_T\cos\theta = F_{app} \Rightarrow \boxed{F_H = F_{app} - F_T\cos\theta = 12.5\ N}$

$\rightarrow$ acts to the left