PHY 3513 – PROBLEM SET 5

1) Consider a system with 7 energy levels with energies 0, ε, 2ε, 3ε, 4ε, 5ε, and 6ε, for some energy ε. Suppose it is desired to fill these energy levels with four particles such that the *total* energy of the system of four particles is 6ε. For example, we could put three particles in the level with zero energy and one particle in the level with energy 6ε.

a) Define macrostates of the system as states with different numbers of particles in each level. Then for the system described above with four particles and total energy 6ε, there are nine possible macrostates. Make a diagram of these macrostates, which shows for each macrostate, how many particles occupy each of the seven energy levels.

b) Now suppose the particles are *distinguishable* so that we can determine which particles are in which state. Then for each macrostate, there exist several microstates corresponding to different choices for the particles in each level. For example, for a macrostate which has two particles in level 2 and two particles in level 3 (for a total energy of 2 X ε + 2 X 2ε = 6ε), there are 4 choices for the first particle in level 2 and 3 choices for the second particle. But the order of choosing doesn't make any difference, so we have to divide by 2. Once the first two particles are chosen, the remaining particles must go into the other level. Thus, for this macrostate, there are six different microstates and the multiplicity Ω=6 for that macrostate. Determine the multiplicity of each macrostate according to this definition and then show that the sum of all nine multiplicities is 84. This is the total number of microstates in the system.

c) Calculate the average occupation numbers of the lowest energy level, the middle energy level (with energy 3ε), and the highest energy level. To obtain the average occupation number of a level, multiply the number of particles in that level for a particular macrostate by the multiplicity of that macrostate. Then sum this product over the nine macrostates and divide by the total number of microstates, which is 84. If you add up the average occupation numbers of all 7 levels, the result should just be the number of particles in the system, which is 4.

2) **Text Problem 6.5**

3) A system has three non-degenerate energy levels with energies 0, ε, and 2ε.

a) Calculate the entropy of the system if the three levels are populated by *two* distinguishable particles such that the total energy is U=2ε.

b) Calculate the entropy of the system if the three levels are populated by *three* distinguishable particles such that the total energy is U=2ε.

4) **Text Problem 6.12**

5) **Text Problem 6.13**

6) Consider a system with two *non-degenerate* energy levels with energies $\varepsilon_1=0$ and $\varepsilon_2=\varepsilon$. Suppose that the system contains N *distinguishable* particles at temperature T.

a) Determine the partition function of the system and the occupancies N_1 and N_2 of the two levels.

b) Find the average energy per particle given by $\langle u\rangle=U/N$, where U is the total internal energy of the system and N the total number of particles.

c) Show that at very small temperatures, $\langle u\rangle \cong \varepsilon e^{-\alpha}$, where $\alpha=\varepsilon/k_BT$, and that as the temperature becomes very large, $\langle u\rangle \to \frac{1}{2}\varepsilon$.

d) Show that the volume heat capacity per particle is given by

$$C/N = k_B\alpha^2e^{-\alpha}/(1+e^{-\alpha})^2$$

7) **Text Problem 6.31 –** to perform the required integrals, make a change of variable and then carry out an integration by parts.

8) **Text Problem 6.33 –** evaluate the requested quantities at $T=20^oC$.

9) Calculate the average energy (in eV) and rms velocity of an electron at the the temperatures $T_1=10^3K$ and $T_2=10^5K$.

10) Suppose that instead of the Maxwell-Boltzmann distribution, the distribution of molecular speeds in a gas was given by the expression $N(v)=Av\exp(-v/v_o)$, where A and v_o are constants.

a) Determine the constant A so that the total number of molecules in the gas is N.

b) In terms of v_o, find the average speed, the rms speed, and the most likely speed of the molecules in the gas.