

PHY 3513 – PROBLEM SET 6

- 1) A closed, thermally isolated container contains 100 moles of argon gas at a pressure of 1 atm and temperature 300 K. Given that argon is a monatomic gas and that the mass of an argon atom is 6.63×10^{-26} kg, calculate the following quantities:
 - a) The average energy per argon atom in eV.
 - b) The partition function of the gas.
 - c) The entropy of the gas.
 - c) The average chemical potential per atom in eV.
 - d) The ratio $N(\epsilon)/g(\epsilon)$ for ϵ = the average atomic energy calculated in part a.
- 2) **Text Problem 2.35** – note that helium is monatomic and that for an ideal gas, the ratio N/V is given by the ideal gas law.
- 3) **Text Problem 6.10** – note that since the exact result for the partition function was derived in lecture, it is not necessary to approximate it.
- 4) According to the Einstein model, a crystal lattice of N atoms can be treated as a system of $3N$ *distinguishable* oscillators. At temperature T , the partition function for the lattice can be expressed as

$$Z = e^{-\frac{1}{2}\alpha} / (1 - e^{-\alpha})$$

with $\alpha = T_E/T$, where T_E is a constant with the dimensions of a temperature.

- a) Determine the internal energy U of the system.
- b) Determine the Helmholtz free energy of the system.
- c) Show that the entropy of the system is given by

$$S = 3Nk_B [\alpha/(e^\alpha - 1) - \ln(1 - e^{-\alpha})]$$

- d) Show that $S \rightarrow 0$ as $T \rightarrow 0$, and that at very high temperatures, $S \cong 3Nk_B [1 - \ln(\alpha)]$

- 5) **Text Problem 6.25**
- 6) **Text Problem 6.29** – determine the temperature at which $T = T_{\text{vib}}/10$.
- 7) **Text Problem 6.48** – in part a, note that $Z_{\text{rot}} = k_B T / (2\epsilon)$ with ϵ given in problem 6.24. Your numerical result in this part should be close to the value given on p. 405. In part b, recall that the chemical potential per molecule is just G/N , where G , the Gibbs free energy, is given by $G = U - TS + PV$.
- 8) Calculate the volume heat capacity per mole for a collection of H_2 molecules at temperature $T = 2000\text{K}$. Note that the heat capacity has translational, rotational, and vibrational contributions. At the given temperature, the equipartition theorem can be used for the first two contributions, but for the vibrational contribution, one has to use the exact expression. For H_2 molecules, $T_{\text{vib}} = 6140\text{K}$.
- 9) **Text Problem 3.20** – note that the ratios requested do not depend on the number of particles in the system. Also, since $U < 0$, the requested energy ratio involves the maximum of the magnitude of U , rather than the maximum of U itself. To determine the maximum entropy, you have to examine what happens to the hyperbolic tangent and the hyperbolic cosine in the limits of both very large argument and very small argument. In the last part, either the magnetic field has to be increased or the temperature lowered. Consider both possibilities.
- 10) **Text Problem 6.52** – note that in one dimension, there is only one energy per value of n , so the system is non-degenerate. First write E_n in terms of n , then determine Z by approximating the sum over n by an integral over n , as was done for the non-relativistic case. You should find that Z is directly proportional to T .