PHY 3513 – PROBLEM SET 7

1) **Text, problem 7.8**

2) A system has a series of evenly spaced, non-degenerate energy levels with energies $\varepsilon_o=0$, $\varepsilon_1=\varepsilon$, $\varepsilon_2=2\varepsilon$, $\varepsilon_3=3\varepsilon$, etc. It is desired to fill these levels with a total of 4 particles. Consider three different cases: (1) the particles are *distinguishable*, (2) the particles are *bosons*, and (3) the particles are *fermions*.

a) The ground state of a system is the multiparticle state which has the lowest possible energy. For each case above, what is the total energy of this ground state? For each case, how are the 4 particles distributed among the various energy levels in the ground state?

b) Now suppose that the system of 4 particles has a total energy ε above the ground state energy. For each case above, make a diagram that shows the possible distributions of the 4 particles in the various energy levels for this total energy and then determine how many microstates there are for each such distribution.

c) Repeat part b if the total energy above the ground state energy is 2ε, rather than ε.

3) For a temperature T=300 K, determine the equilibrium number of indistinguishable particles per state for an energy level with energy ε if

a) the particles are *bosons* and (i) $\varepsilon-\mu=0.001$ eV and (ii) $\varepsilon-\mu=0.1$ eV.

b) the particles are *fermions* and (i) $\varepsilon-\mu=-0.1$ eV and (ii) $\varepsilon-\mu=+0.1$ eV.

4) Suppose that at temperature $T=3T_F$, the chemical potential of a system of fermions has the value $\mu=-5.6\ \varepsilon_F$. For each of the energies below, calculate the Fermi function $f(\varepsilon)$.

a) $\varepsilon=0$

b) $\varepsilon=\varepsilon_F/2$

c) $\varepsilon=\varepsilon_F$

d) $\varepsilon=2\varepsilon_F$

5) **Text Problem 7.9 –** in the first part, compare the quantum volume with the volume per molecule of the gas at a temperature of 20ºC and a pressure of 1 atm. In the second part, you should assume that as the temperature is lowered, the pressure is also lowered so that the volume per molecule stays constant. Then find the temperature at which the volume per molecule and the quantum volume become equal.

6) **Text Problem 7.12**

7) **Text Problem 7.20 –** you will need to compare the quantum volume with the volume per electron in the sun and compare the electron Fermi temperature with sun's interior temperature.

8) **Text Problem 7.22** – Recall that in a degenerate Fermi gas at temperature T=0, the chemical potential is just the Fermi energy. Also, recall that the allowed wave numbers k_n in a box of sides of length L are given by $k_n=n\pi/L$ and that the density of states factor in three dimensions (including the factor 2 for the electron spin degeneracy) is given by $g(\varepsilon) = \pi n^2(dn/d\varepsilon)$.

9) A white dwarf star results when a low mass star like the sun exhausts all of its nuclear fuel and collapses under the influence of gravity. The gravitational collapse continues until the size of the star becomes comparable to that of Earth. The interior of the star is made up mostly of α particles, consisting of two protons and two neutrons, which are the end product of hydrogen fusion, and electrons. Since the star is electrically neutral, the number of electrons must equal the number of protons, and the electrons are highly degenerate.

a) Treating the white dwarf as a degenerate electron gas, show that the total internal energy of the electrons is given by $U_{elec}=AM^{5/3}/R^2$, where M and R are the mass and radius of the white dwarf, and determine the constant A. Assume that protons and neutrons have approximately the same mass and that the numbers of protons, neutrons, and electrons in the white dwarf are all equal.

b) Evaluate the electron Fermi energy and temperature for $M=2.5 \times 10^{30}$ kg and R=6500 km.