PHY 3513 – PROBLEM SET 8

1) The nucleus of a large atom can be treated approximately as a Fermi gas of *nucleons* (neutrons and protons) with spin ½. The system is like a Fermi gas of electrons, except that the nucleons have a mass equal to the proton mass and there are *four* states at each energy (either proton or neutron and spin projection either up or down), so that the density of states factor has an extra factor of 2 compared with the electron degeneracy of states factor.

a) Given that the inside of a large nucleus has mass density $\rho=3.00 \times 10^{17}$ kg/m³, calculate the number density (nucleons per unit volume) inside a large nucleus. Express your result in units of fm^{-3}, where 1 fm = 10^{-15} m.

b) Find the Fermi energy of the nucleus. Express your result in MeV (1 MeV=10^6 eV) and don't forget the extra factor 2 in the density of states.

c) Determine the degeneracy pressure inside the nucleus.

d) Calculate the temperature at which $\mu=0.99\varepsilon_F$, where μ is the nucleon chemical potential at temperature T.

2) Aluminum has a molar mass of 27 X 10^{-3} kg, a mass density of 2.69 X 10^3 kg/m³, and 3 conduction electrons per atom.

a) Calculate the electron Fermi energy in aluminum.

b) Determine the *fractional* difference between the Fermi energy and the chemical potential of aluminum at a temperature T=1000K.

3) The interior of an oven with volume V=1.0 m³ has pressure P=1.0 atm and temperature T=200ºC. The air inside the oven is mostly made of N_2.

a) Calculate the average number of photons in the oven interior assuming that the oven interior can be treated as a black body.

b) Determine the ratio of the photon number to the number of gas molecules in the oven.

c) At what interior temperature would the average number of photons in the oven and the number of gas molecules in the oven be equal?

d) At the temperature determined in part c, what fraction of the total energy in the oven is contained in the photons? Assume that despite the high temperature, only translational and rotational degrees of freedom are excited in the diatomic gas molecules.

4) **Text Problem 7.52 –** recall that the temperature of the human body is about 37ºC. Use the value 1.5 m^2 for the average surface area of a human body.

5) **Text Problem 7.45 –** in the first part of this problem, you are to obtain the radiation pressure by taking a derivative of the internal energy. Note that in this derivative, it is the *entropy* that is held constant, not the temperature. Therefore, in the expression for U, you have to express the temperature in terms of the entropy *before* taking the derivative with respect to V. With the entropy constant, you should then be able to show that $U = AV^{-4/3}$, where A is a constant. To find the gas pressure in the kiln, assume that the gas was originally at pressure P=1 atm and temperature T=20ºC and was then heated up to T=1500 K with N and V constant.

6) The spectral distribution function $u(\nu)$ is defined as the energy per unit frequency interval.

a) For a particular temperature T, express $u(\nu)$ as a function of $x=h\nu/k_BT$.

b) Derive a transcendental equation for the value of x at which $u(\nu)$ has its peak value. Confirm that $x_{peak}=2.82$ satisfies this equation.

c) Use the fact that $u(\nu)$ peaks at $\nu=3.55 \times 10^{14}$ Hz on the sun's surface to determine the temperature on the sun's surface.

7) **Text Problem 7.54 –** note that since the energy of a photon is proportional to its frequency, the peak in the energy spectrum is at the same frequency as the peak in the spectral distribution function $u(\nu)$ considered in the previous problem.