HW Set 2

— Solutions —

(1) van der Waals EOS $\rightarrow (P + aN^2/V^2)(V - bN) = Nk_BT$

Now $V_c = 3bN \Rightarrow V = vV_c = 3bNv$

$P_c = a/(27b^2) \Rightarrow P = pP_c = ap/(27b^2)$

$k_BT_c = 8a/(27b) \Rightarrow k_BT = (k_BT_c)t = 8at/(27b)$

$$\Rightarrow \left(\frac{ap}{27b^2} + \frac{a}{(3bv)^2}\right)(3bNv - bN) = N\frac{8at}{27b}$$

$$\Rightarrow \cancel{a}\left(\frac{p}{\cancel{b}} + \frac{3}{\cancel{b}v^2}\right)(3v-1)\cancel{bN} = 8\cancel{a}\cancel{N}t \Rightarrow \boxed{\left(p + \frac{3}{v^2}\right)\left(v - \frac{1}{3}\right) = \frac{8}{3}t}$$

(2) (a) Monatomic gas $\rightarrow \gamma = 5/3$

Adiabatic process $P_fV_f^\gamma = P_{in}V_{in}^\gamma \Rightarrow P_f = P_{in}(V_{in}/V_f)^\gamma$

Now $V_f = 2V_{in} \Rightarrow \boxed{P_f = P_0/2^{5/3} = 0.315\,P_0}$

(b) Enthalpy $\rightarrow H = U + PV = 3/2\, nRT + PV = 5/2\, PV$

$\Rightarrow \Delta H = 5/2(P_fV_f - P_{in}V_{in}) = 5/2[2(0.315) - 1]P_0V_0$

$\rightarrow \boxed{\Delta H = -0.925\,P_0V_0}$

[Text 1.36] (3) (a) $P_{in} = 1.013\times10^5$ Pa, $P_f = 7P_{in}$, $V_{in} = 1.0\,\ell = 10^{-3}\,m^3$

diatomic gas $\rightarrow \gamma = 7/5$

$P_fV_f^\gamma = P_{in}V_{in}^\gamma \Rightarrow V_f = (P_{in}/P_f)^{1/\gamma}V_{in} = V_{in}/7^{5/7}$

$\rightarrow \boxed{V_f = 0.249\,\ell}$

(b) Adiabatic process $\rightarrow W = (P_fV_f - P_{in}V_{in})/(\gamma - 1)$

$\rightarrow \boxed{W = 5/2(P_fV_f - P_{in}V_{in}) = 188\text{ J}}$

(c) $T_{in} = 300$ K

$T_fV_f^{\gamma-1} = T_{in}V_{in}^{\gamma-1} \Rightarrow \boxed{T_f = (V_{in}/V_f)^{\gamma-1}T_{in} = 523\text{ K}}$

(4) (a) $(P+b)V = nRT \rightarrow P = nRT/V - b$

$\Rightarrow \boxed{dW = -PdV = (b - nRT/V)\,dV}$

(b) $U = naT + bV + U_0 \Rightarrow \underline{dU = nadT + bdV}$

First law $\Rightarrow dQ = dU - dW = nadT + bdV - bdV + (nRT/V)dV$

$\Rightarrow \boxed{dQ = nadT + (nRT/V)dV}$

(c) For constant volume, $dV = 0 \Rightarrow \boxed{C_V = \partial Q/\partial T|_V = na}$

(d) For constant pressure, $\underline{dV = [nR/(P+b)]\,dT}$

$\Rightarrow dW = -PdV = -\frac{nRP}{P+b}dT$ and $dU = na + \frac{nRb}{P+b}]\,dT$

$\Rightarrow dQ = dU - dW = n(a+R)\,dT \Rightarrow C_P = \partial Q/\partial T|_P = n(a+R)$

$\Rightarrow \boxed{C_P - C_V = nR}$

[Text 1.41] (5) (a) Water $\rightarrow C_{H_2O} = 4.186$ J/g-K and $m_{H_2O} = 250$ g

$\Delta T_{H_2O} = 24°C - 20°C \Rightarrow \boxed{Q_{H_2O} = m_{H_2O} C_{H_2O} \Delta T_{H_2O} = 4190 \text{ J}}$

(b) Energy conserv. $\rightarrow -Q_{metal} = Q_{H_2O}$

(c) $\Delta T_{metal} = 24°C - 100°C = -76°C \Rightarrow C_{metal} = -Q_{metal}/\Delta T_{metal}$

$\Rightarrow \boxed{C_{metal} = 55.1 \text{ J/K}}$

(d) $m_{metal} = 100$ g $\Rightarrow \boxed{c_{metal} = C_{metal}/m_{metal} = 0.551 \text{ J/g-K}}$

[Text 1.47] (6) Tea $\rightarrow T_{in} = 100°C$, $T_{final} = 65°$, $m_{Tea} = 200$ g

and $C_{Tea} = 1.0$ cal/g-K $\rightarrow -Q_{Tea} = (-\Delta T_{Tea})\, m_{Tea} C_{Tea}$

$\Rightarrow -Q_{Tea} = 7000$ cal

Ice $\rightarrow T_{in} = -15°C$, $T_{final} = 0°$, $C_{ice} = 0.50$ cal/g-K

and $L_{ice} = 80$ cal/g

$\rightarrow Q_{ice} = m_{ice} C_{ice} \Delta T_{ice} + m_{ice} L_{ice} + m_{ice} \Delta T_{water} C_{water}$

$\Delta T_{ice} = 15°C$ and $\Delta T_{water} = 65° \Rightarrow C_{ice}\Delta T_{ice} + L_{ice} + C_{water}\Delta T_{water}$

$= 152.5$ cal/g $\Rightarrow \boxed{m_{ice} = -Q_{Tea}/(152.5 \text{ cal/g}) = 45.9 \text{ g}}$

(7)(a) $P = aT^4/3$ and $U = aT^4V$

Now $dQ = dU + PdV = aT^4dV + (aT^4/3)dV$ for $dT=0$

$\Rightarrow dQ = 4/3(aT^4)dV$ for an isothermal process

$\Rightarrow Q = 4/3\, aT^4(V_{fin} - V_{init}) \rightarrow \boxed{Q = 4/3\, aT^4V}$

(for $V_{fin} = 2V$ and $V_{init} = V$)

(b) Adiabatic process $\rightarrow dQ = 0 \Rightarrow dU = -PdV$

$\Rightarrow 4aT^3VdT + aT^4dV = -aT^4/3\, dV \Rightarrow 4/3\, TdV = -4VdT$

$\Rightarrow \underline{dV/V = -3\, dT/T} \Rightarrow \ln V = -3\ln T + C$

$\Rightarrow V = K/T^3 \Rightarrow \boxed{VT^3 = K}$ $(K = e^C)$

[Text 1.49] (8) Ice $\rightarrow L_{ice} = 334\ J/g = 334\ kJ/kg$, $\rho_{ice} = 0.92\ g/cm^3 = 920\ kg/m^3$

Over an area A, $m_{ice} = 1/2\ \rho_{ice} \times$ volume $= 1/2\ \rho_{ice}\, hA$

(factor 1/2 because snow is only half ice)

$\Rightarrow Q_{melt} = L_{ice}\, m_{ice} = 1/2\ \rho_{ice}\, hAL_{ice}$

Power supplied by sun $\rightarrow P_{sun} = (0.10)\mathcal{I}A$

where $\mathcal{I} = 1000\ W/m^2$ and only 1/10 is absorbed by ice

Now $Q_{melt} = P_{sun}(\Delta t) \Rightarrow \Delta t = 1/2\ \rho_{ice}\, hL_{ice}/(0.10)\mathcal{I}$

$h = 2.0\ m \rightarrow \boxed{\Delta t = 3.07 \times 10^6 s = 5.1 \text{ weeks}}$

Assuming 1/2 of each day is sunlight $\rightarrow$ <u>10 weeks</u>

[Text 1.49] (9) $\Delta H = -286\ kJ$ (text, p. 35)

At start, 1 mole H_2 and 1/2 mole O_2 = 1.5 moles of gas

At standard temp and pressure, $T = 298\ K$, $P = 1.0 \times 10^5\ Pa$

$\rightarrow V_{gas} = nRT/P = 3.717 \times 10^{-2}\ m^3$

$W_{atmosph} = -W_{gas} = +P(\Delta V) = P(0 - V_{gas}) = -P\, V_{gas}$

$\Rightarrow \underline{W_{atmosph} = -3720\ J}$

$\rightarrow$ <u>4 kJ from work and 282 kJ from ΔU</u>

(10) ΔH for one mole atomic H = 217.97 kJ

This corresponds to ½ mole H_2 = 3.01×10^{23} molecules

$\Rightarrow$ ΔH/molecule = 7.242×10^{-19} J/molecule

1 eV = 1.602×10^{-19} J $\Rightarrow$ $\boxed{\Delta H/\text{molecule} = 4.5 \text{ eV/molecule}}$