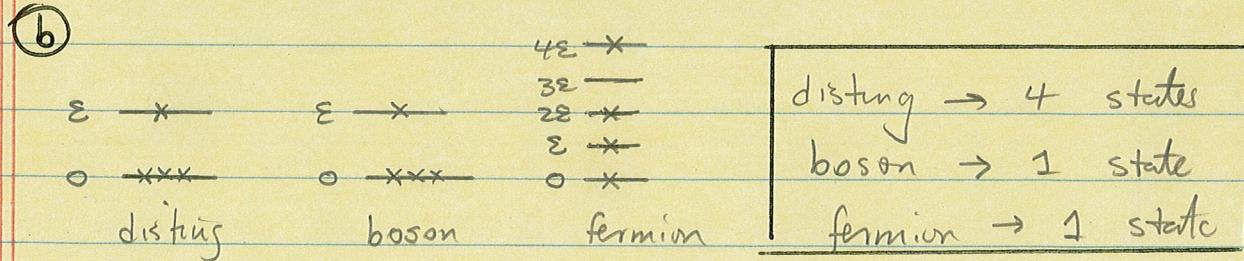


- [Text 7.8] (1) @ Particle can go into any state  $\Rightarrow \underline{z = 10}$
- (b) Each particle can go into any state  $\Rightarrow \underline{z = 10 \times 10 = 100}$
- (c) Identical bosons  $\rightarrow$  both in same state = 10 possibilities  
Different states  $\rightarrow (10)(9)/2$  poss. bilities  $\Rightarrow \underline{z = 55}$
- (d) Identical fermions  $\rightarrow$  must go in different states  $\rightarrow \underline{z = 45}$
- (e)  $\underline{z = 1/2 \cdot 10^2 = 50}$
- (f)
- |   |
|---|
| Disting $\rightarrow P = 10/100 = 0.10$ |
| Bosons $\rightarrow P = 10/55 = 0.18$   |
| Fermions $\rightarrow P = 0/45 = 0$     |

- (2) (a) Disting and bosons  $\rightarrow$  all particles in lowest state  $\rightarrow \underline{E = 0}$   
Fermions  $\rightarrow$  1 particle in each of lowest states  
 $\rightarrow \underline{E = 0 + \varepsilon + 2\varepsilon + 3\varepsilon = 6\varepsilon}$



(c)

| I                             | II                         | I                             | II                         | I                       | II                      |
|-------------------------------|----------------------------|-------------------------------|----------------------------|-------------------------|-------------------------|
| $2\varepsilon -$              | $\varepsilon - \times$     | $\varepsilon -$               | $\varepsilon - \times$     | $6\varepsilon -$        | $6\varepsilon -$        |
| $\varepsilon - \times \times$ | $\varepsilon -$            | $\varepsilon - \times \times$ | $\varepsilon -$            | $5\varepsilon - \times$ | $5\varepsilon -$        |
| $0 - \times \times$           | $0 - \times \times \times$ | $0 - \times \times$           | $0 - \times \times \times$ | $4\varepsilon -$        | $4\varepsilon - \times$ |

distinguishable                                  bosons                                  fermions

|   |
|---|
| disting $\rightarrow \Omega_{\text{tot}} = \Omega_I + \Omega_{II} = 6 + 4 = 10$ |
| boson $\rightarrow \Omega_{\text{tot}} = \Omega_I + \Omega_{II} = 1 + 1 = 2$    |
| fermion $\rightarrow \Omega_{\text{tot}} = \Omega_I + \Omega_{II} = 1 + 1 = 2$  |

(3) a)  $T = 300 \text{ K} \Rightarrow k_B T = 4.143 \times 10^{-21} \text{ J} = 0.02586 \text{ eV}$

Bosons  $\rightarrow f(\varepsilon) = [e^{(\varepsilon-\mu)/k_B T} - 1]^{-1}$

$\varepsilon - \mu = 10^{-3} \text{ eV}$   $\Rightarrow (\varepsilon - \mu)/k_B T = 0.03867 \Rightarrow$

$\varepsilon - \mu = 0.10 \text{ eV}$   $\Rightarrow " = 3.867 \Rightarrow$

$$f(\varepsilon) = 25.4$$

$$f(\varepsilon) = 0.0214$$

Fermions  $\rightarrow f(\varepsilon) = [e^{(\varepsilon+\mu)/k_B T} + 1]^{-1}$

$\varepsilon - \mu = -0.10 \text{ eV}$   $\Rightarrow f(\varepsilon) = 0.980$

$\varepsilon - \mu = +0.10 \text{ eV}$   $\Rightarrow f(\varepsilon) = 0.0205$

(4)  $f(\varepsilon) = [e^{(\varepsilon-\mu)/k_B T} + 1]^{-1} \rightarrow T = 3T_F \Rightarrow k_B T = 3\varepsilon_F$

$\mu = -5.6 \varepsilon_F$

a)  $\varepsilon = 0$   $\Rightarrow (\varepsilon - \mu)/k_B T = (5.6/3) \Rightarrow f(\varepsilon) = 0.134$

b)  $\varepsilon = \frac{1}{2} \varepsilon_F$   $\Rightarrow (\varepsilon - \mu)/k_B T = (6.1/3) \Rightarrow f(\varepsilon) = 0.116$

c)  $\varepsilon = \varepsilon_F$   $\Rightarrow (\varepsilon - \mu)/k_B T = (6.6/3) \Rightarrow f(\varepsilon) = 0.0998$

d)  $\varepsilon = 2\varepsilon_F$   $\Rightarrow \varepsilon - \mu/k_B T = (7.6/3) \Rightarrow f(\varepsilon) = 0.0736$

[Text 7.9] (5)  $T = 20^\circ\text{C} = 293 \text{ K} \Rightarrow k_B T = 4.046 \times 10^{-21} \text{ J}$

$N_2 \rightarrow m = 28 \times 10^{-3} \text{ kg} / N_A = 4.651 \times 10^{-26} \text{ kg}$

quantum volume  $\rightarrow V_Q = [h / \sqrt{2\pi mk_B T}]^3 = 7.15 \times 10^{-23} \text{ m}^3$

For 1 molecule of an ideal gas  $V = k_B T / P$

$P = 1.013 \times 10^5 \text{ Pa} \Rightarrow V = 3.99 \times 10^{-26} \text{ m}^3 \gg V_Q$

$\Rightarrow$  Boltzmann statistics applicable

Constant density  $\Rightarrow$  temp at which  $V_Q = V$

$\Rightarrow 2\pi m k_B T = h^2 / V^{2/3} \Rightarrow k_B T = h^2 / (2\pi m V^{2/3}) = 1.286 \times 10^{-25} \text{ J}$

$\Rightarrow T \approx 9.3 \times 10^{-3} \text{ K}$

[Text 7.12] (6)  $\varepsilon_A = \mu - x$  and  $\varepsilon_B = \mu + x$   
 Define  $y = e^{(\varepsilon_B - \mu)/k_B T} = e^{-x/k_B T} \Rightarrow e^{(\varepsilon_A - \mu)/k_B T} = e^{-x/k_B T} = y^{-1}$   
 Prob. of B occupied =  $f_B(\varepsilon_B) = (y+1)^{-1}$   
 Prob. of A unoccupied =  $1 - f_A(\varepsilon_A) = 1 - (y+1)^{-1}$   
 $= 1 - y/(y+1) = 1/(y+1) \Rightarrow \boxed{1 - f_A(\varepsilon_A) = f_B(\varepsilon_B)}$

[Text 7.20] (7)  $T = 10^7 K \Rightarrow k_B T = 1.381 \times 10^{-16} J$   
 $m_e = 9.109 \times 10^{-31} kg \Rightarrow V_0 = [h/\sqrt{2\pi m_e k_B T}]^3 = 1.309 \times 10^{-32} m^3$   
 $N/V \approx 10^{32} m^{-3} \Rightarrow V/N \approx 10^{-32} m^3 \sim V_0$   
 $\Rightarrow$  cannot be treated as an ideal gas  
 $\varepsilon_F = (h^2/2mc)(3N/8\pi V)^{2/3} = 1.259 \times 10^{-16} J \approx k_B T \quad T_F \approx 9.1 \times 10^6 K$   
 $\Rightarrow$  cannot be treated as  $T=0$  degenerate Fermi gas

[Text 7.22] (8) a)  $\varepsilon_n = p_n c = \hbar k_n c = \hbar c (n\pi/L) = nhc/2V^{1/3} \quad (L = V^{1/3})$   
 Now  $g(\varepsilon) = 2(\pi/2) n^2 (dn/d\varepsilon) = \pi n^2 (2V^{1/3}/hc)$   
 $n^2 = 4V^{2/3}\varepsilon^2/(hc)^2 \Rightarrow g(\varepsilon) = [8\pi V/(hc)^3] \varepsilon^2$   
 $\Rightarrow N = \int_0^\infty g(\varepsilon) f(\varepsilon) d\varepsilon \xrightarrow{T=0} \int_0^{\varepsilon_F} g(\varepsilon) d\varepsilon = \frac{8\pi V}{(hc)^3} \frac{1}{3} \varepsilon_F^3$   
 $\Rightarrow \boxed{\varepsilon_F = (3N/8\pi V)^{1/3} hc}$   
 b)  $U = \int_0^\infty \varepsilon g(\varepsilon) f(\varepsilon) d\varepsilon \xrightarrow{T=0} \int_0^{\varepsilon_F} \varepsilon g(\varepsilon) d\varepsilon = \frac{8\pi V}{(hc)^3} \frac{1}{4} \varepsilon_F^4$   
 $\frac{8\pi V}{(hc)^3} = 3N/\varepsilon_F^3 \Rightarrow \boxed{U = 3/4 N \varepsilon_F}$

(9) @ Average electron energy  $\rightarrow \langle \varepsilon \rangle = \frac{3}{5} \varepsilon_F$   
 $\Rightarrow U = N \langle \varepsilon \rangle = \frac{3}{5} N \varepsilon_F = \frac{3}{5} N \left( \frac{h^2}{2m_e} \right) \left( \frac{3N}{8\pi r} \right)^{2/3}$

Now  $N = \text{number protons} = \frac{1}{2} (\text{number of protons} + \text{neutrons})$   
 $= M/2m_p \quad \text{and} \quad V = \frac{4}{3} \pi R^3$

$$\Rightarrow U = \frac{3}{10} \left( \frac{3}{8\pi} \right)^{2/3} \frac{h^2}{m_e} \frac{N^{5/3}}{V^{2/3}} \rightarrow U = A \frac{M^{5/3}}{R^2}$$

with  $A = \frac{3}{20} \left( \frac{9}{64\pi^2} \right)^{2/3} \left( \frac{h^2}{m_e m_p} \right)^{5/3} = 1.808 \times 10^6 \text{ J-m}^2/\text{kg}^{5/3}$

(b)  $M = 2.5 \times 10^{31} \text{ kg}, \quad R = 6.50 \times 10^{-6} \text{ m}$

$$\Rightarrow U = 1.970 \times 10^{43} \text{ J} \quad \text{and} \quad N = 7.485 \times 10^{56}$$

$$\Rightarrow \boxed{\varepsilon_F = \frac{5}{3}(U/N) = 4.387 \times 10^{-14} \text{ J} = 0.274 \text{ MeV}}$$

$$\Rightarrow \boxed{T_F = \varepsilon_F/k_B = 3.2 \times 10^9 \text{ K}}$$