Set 8 — Solutions —

(1)(a) $\rho = 3.00 \times 10^{17}\ kg/m^3$, $m = 1.67 \times 10^{-27}\ kg$

$\Rightarrow \boxed{N/V = \rho/m = 1.796 \times 10^{44}\ m^{-3} = 0.18\ fm^{-3}}$

(b) with extra factor 2, $\varepsilon_F = (h^2/2m)(3N/16\pi V)^{2/3}$

$\rightarrow \boxed{\varepsilon_F = 6.391 \times 10^{-12}\ J = 39.9\ MeV}$

(c) $\boxed{P = 2/5\,(N/V)\,\varepsilon_F = 4.59 \times 10^{32}\ Pa}$

(d) $\mu/\varepsilon_F = 0.99 \Rightarrow 1 - \pi^2/12\,(T/T_F)^2 = 0.99$

$\Rightarrow T/T_F = (0.1)\sqrt{12}/\pi = 0.1102$

$\Rightarrow \boxed{T = 0.1102\ \varepsilon_F/k_B = 5.10 \times 10^{10}\ K}$

(2)(a) $\rho = 2.69 \times 10^3\ kg/m^3$ and $m = 27 \times 10^{-3}\ kg/N_A = 4.485 \times 10^{-26}$

$\Rightarrow N/V = 3(\rho/m) = 1.799 \times 10^{29}\ m^{-3}$

$\Rightarrow \boxed{\varepsilon_F = (h^2/2m_e)(3N/8\pi V)^{2/3} = 1.862 \times 10^{-18}\ J = 11.6\ eV}$

(b) $T = 1000\ K \Rightarrow T/T_F = k_B T/\varepsilon_F = 7.417 \times 10^{-3}$

now $\mu/\varepsilon_F = 1 - \pi^2/12\,(T/T_F)^2 \Rightarrow \boxed{1 - \mu/\varepsilon_F = \pi^2/12\,(T/T_F)^2 = 4.5 \times 10^{-5}}$

(3)(a) $T = 200^\circ C = 473\ K \Rightarrow (k_B T/hc) = 3.286 \times 10^4\ m^{-1}$

$\rightarrow \boxed{N_{photon} = (2.404)(8\pi)(k_B T/hc)^3 = 2.14 \times 10^{15}}$

(b) $P = 1.013 \times 10^5\ Pa$ and $V = 1.0\ m^3 \Rightarrow N_{mol} = PV/k_B T = 1.551 \times 10^{25}$

$\Rightarrow \boxed{N_{phot}/N_{mol} = 1.38 \times 10^{-10}}$

(c) $N_{phot} = N_{mol} \Rightarrow k_B T = hc\left[N_{mol}/(2.404)(8\pi)\right]^{1/3} = 1.263 \times 10^{-17}\ J$

$\Rightarrow \boxed{T = 9.15 \times 10^5\ J}$

(d) $U_{phot} = (8\pi^5/15)(k_B T)^4/(hc)^3\,V = 5.292 \times 10^8\ J$

$U_{gas} = 5/2\ N_{gas}\ k_B T = 4.898 \times 10^8\ J \Rightarrow U_{total} = 1.019 \times 10^9\ J$

$\Rightarrow \boxed{U_{phot}/U_{total} = 0.52}$

[Text 7.52] ④ (a) $T = 37°C = 310\ K \Rightarrow \sigma T^4 = 523.6\ W/m^2$

area $= 1.5\ m^2 \Rightarrow \boxed{P = 785\ W}$

(b) 1 day $= 8.64 \times 10^4\ s \rightarrow \underline{U = 6.786 \times 10^7\ J = 1.62 \times 10^4\ kcal}$

$\rightarrow$ not produced from food

(c) Body mass $\rightarrow m = 75\ kg \Rightarrow \underline{P/m \sim 10\ W/kg}$

Sun mass $\rightarrow M = 2 \times 10^{30}\ kg$ and $P_{sun} = 3.9 \times 10^{26}\ W$

$\Rightarrow \underline{P_{sun}/M_{sun} \sim 2 \times 10^{-4}\ W/kg}$

[Text 7.45] ⑤ $U = (8\pi^5/15)(k_B T)^4/(hc)^3\ V$

Entropy $\rightarrow S = (32/45)\pi^5 k_B (k_B T/hc)^3 V$

$$\Rightarrow k_B T = \left(\frac{45S}{32 k_B \pi^5}\right)^{1/3} \frac{hc}{V^{1/3}} \Rightarrow (k_B T)^4 = \left(\frac{45S}{32 k_B \pi^5}\right)^{4/3} (hc)^4 V^{-4/3}$$

So for <u>constant entropy</u>, $U = AV^{-1/3}$, A = a constant

$$\Rightarrow P = -\left.\frac{\partial U}{\partial V}\right|_S = +\frac{1}{3} A V^{-4/3} \Rightarrow \underline{P = \frac{1}{3}\frac{U}{V}}$$

$T = 1500\ K \Rightarrow k_B T = 2.072 \times 10^{-20}\ J \rightarrow \boxed{P_{rad} = 1.28 \times 10^{-3}\ Pa}$

If $P_{gas} = 1$ atm at $T = 293\ K$, then

$P_{gas} = (1\ atm)(1500/293)$ at $T = 1500\ K \Rightarrow P_{gas} = 5.186 \times 10^5\ Pa$

$\Rightarrow \boxed{P_{rad}/P_{gas} = 2.46 \times 10^{-9}}$

In the sun at $T = 1.5 \times 10^7\ K$, $\boxed{P_{rad} = 1.28 \times 10^{13}\ Pa}$

$\rho = 10^3\ kg/m^3 \Rightarrow N/V = \rho/m_p = 5.988 \times 10^{31}\ m^{-3}$

$\Rightarrow P_{gas} = (N/V) k_B T = 1.240 \times 10^{16}\ Pa \Rightarrow \boxed{P_{rad}/P_{gas} = 1.03 \times 10^{-3}}$

⑥ (a) Define $x = h\nu/k_B T$

$$\rightarrow u(\nu) = \frac{8\pi h \nu}{c^3}\frac{\nu^3}{e^x - 1} = \frac{8\pi h}{c^3}\left(\frac{k_B T}{h}\right)^3 \nu \frac{x^3}{e^x - 1} \Rightarrow \boxed{u = 8\pi h \nu \left(\frac{k_B T}{hc}\right)^3 \frac{x^3}{e^x - 1}}$$

(b) peak value at $du/dx = 0$

$$\Rightarrow 3x^2/(e^x - 1) - x^3 e^x/(e^x - 1)^2 = \left[3x^2(e^x - 1) - x^3 e^x\right]/(e^x - 1)^2 = 0$$

$$\Rightarrow \boxed{3(e^x - 1) = x e^x}$$

For $x = 2.82$, both sides $\cong 4.73$

(c) $\nu = 3.55 \times 10^{14}$ Hz $\Rightarrow h\nu = 2.352 \times 10^{-19}$ J

$$\Rightarrow k_B T = h\nu/x_{peak} = 8.341 \times 10^{-20} \text{ J} \Rightarrow \boxed{T = 6040 \text{ K}}$$

[Text 7.54] ⑦ (a) Energy peaks $\rightarrow \mathcal{E}_{sirius} = 2.4$ eV and $\mathcal{E}_{sun} = 1.41$ eV

Peak occurs at $x = \mathcal{E}/k_B T = 2.82$

$$\Rightarrow \mathcal{E}_{sirius}/T_{sirius} = \mathcal{E}_{sun}/T_{sun} \Rightarrow T_{sirius}/T_{sun} = \mathcal{E}_{sirius}/\mathcal{E}_{sun} = 1.702$$

Now $P = (\sigma T^4)(4\pi R^2) \Rightarrow P_{sirius}/P_{sun} = (T_{sirius}/T_{sun})^4 (R_{sirius}/R_{sun})^2$

$$\Rightarrow R_{sirius}/R_{sun} = (P_{sirius}/P_{sun})^{1/2}/(T_{sirius}/T_{sun})^2$$

$P_{sirius} \cong 24\, P_{sun} \Rightarrow \boxed{R_{sirius}/R_{sun} = 1.69}$

(b) $P_B/P_{sun} = 0.03$ and $\mathcal{E}_B = 7$ eV $\Rightarrow T_B/T_{sun} \cong 4.964$

$$\rightarrow \boxed{R_B/R_{sun} = 7.0 \times 10^{-3}}$$

(c) $P_{Betel}/P_{sun} = 10^4$ and $\mathcal{E}_{Betel} = 0.8$ eV $\Rightarrow T_{Betel}/T_{sun} = 0.567$

$$\rightarrow \boxed{R_{Betel}/R_{sun} = 311}$$

$\mathcal{E} = hc/\lambda \Rightarrow \underline{\lambda = hc/\mathcal{E} = 1.55\ \mu m}$ at energy peaks

$\rightarrow$ <u>infrared</u>