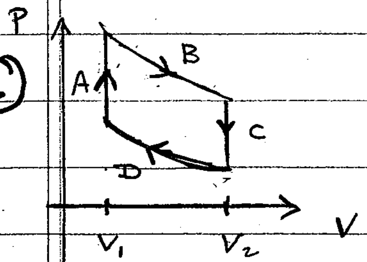
PHY 3513 — Solutions —

Test II – 21 Oct 2016

(1)



$V_1 = 2.5\ m^3 \quad V_2 = 10.0\ m^3 \quad n = 100$ moles

$T_1 = 200\ K \quad T_2 = 500\ K$

monatomic gas $\Rightarrow C_V = 3/2 R$

6/ (a) $Q_A = nC_V(\Delta T) = 374\ kJ \qquad Q_C = nC_V(\Delta T) = -374\ kJ$

$Q_B = -W_B = nRT_2 \ln(V_2/V_1) = 576\ kJ$

$Q_D = -W_D = nRT_1 \ln(V_1/V_2) = -231\ kJ$

4/ (b) $W_A = W_C = 0$ (isochoric)

$W_B + W_D = -(Q_B + Q_D)$ (isothermal) $\Rightarrow \boxed{W_{done} = -(W_B + W_D) = 346\ kJ}$

6/ (c) $\Delta S_A = \int \frac{dQ}{T} = nC_V \int_{T_1}^{T_2} \frac{dT}{T} = nC_V \ln\left(\frac{T_2}{T_1}\right) \rightarrow \boxed{\Delta S_A = 1143\ \frac{J}{K}}$

$\boxed{\Delta S_B = Q_B / T_2 = 1153\ J/K} \qquad \Delta S_C = nC_V \ln(T_1/T_2) = -\Delta S_A$

$\Delta S_D = Q_D / T_1 = -\Delta S_C \rightarrow \boxed{\Delta S_{total} = 0}$

4/ (d) $Q_{in} = Q_A + Q_B = 950\ kJ \Rightarrow \boxed{e = \frac{W_{done}}{Q_{in}} = 0.364}$

(2) 5/ (a) $F = -a(n^2/V) - nRT \ln(V - bn)$

$\rightarrow P = -\frac{\partial F}{\partial V} \Rightarrow \boxed{P = -a\frac{n^2}{V^2} + \frac{nRT}{V - bn}}$

5/ (b) $S = -\frac{\partial F}{\partial T} \Rightarrow \boxed{S = nR \ln(V - bn)}$

5/ (c) $F = U - TS \Rightarrow \boxed{U = F + TS = -a(n^2/V)}$

5/ (d) $G = U + PV - TS = F + PV$

$\Rightarrow \boxed{G = -2a\left(\frac{n^2}{V}\right) + nRT\left[\frac{V}{V - bn} - \ln(V - bn)\right]}$

(3) (a) At the triple point, the sublimation and vapor. pressures are equal, as well as the temperatures
6/
$\Rightarrow 0.04 - 6/T_{TP} = 0.03 - 4/T_{TP} \Rightarrow 2/T_{TP} = 0.01 \Rightarrow \boxed{T_{TP} = 200\text{ K}}$

(b) Clausius-Clapeyron $\rightarrow dP/dT = nL/T(\Delta V)$
8/
$V_{solid}, V_{liquid} \ll V_{gas} \Rightarrow \Delta V \cong V_{gas} = nRT/P \Rightarrow dP/dT \cong PL/RT^2$
$\Rightarrow L \cong (RT^2/P)\, dP/dT$

sublimation $\rightarrow (1/P)dP/dT = d(\ln P)/dT = 6/T^2 \Rightarrow \boxed{L_{sub} = 6R}$

vaporization $\rightarrow \frac{1}{P}\frac{dP}{dT} = \frac{d(\ln P)}{dT} = \frac{4}{T^2} \Rightarrow \boxed{L_{vap} = 4R}$

(c) At the triple, solid, liquid, and gas are all in equil.
6/
$\rightarrow$ consider a reversible loop at the triple point where 1 mole solid $\rightarrow$ 1 mole liquid $\rightarrow$ 1 mole gas $\rightarrow$ 1 mole solid
$\rightarrow$ negligible work $\Rightarrow$ no net heat transfer $\Rightarrow Q_{fus} + Q_{vap} - Q_{sub} = 0$
$\Rightarrow L_{fus} + L_{vap} - L_{sub} = 0 \Rightarrow \boxed{L_{fus} = L_{sub} - L_{vap} = 2R}$

(4) (a) $T_{high} = 400\text{ K}$ and $T_{low} = 250\text{ K}$, $Q_{in} = 1200\text{ J}$
5/
Carnot engine $\rightarrow \boxed{e = 1 - T_{low}/T_{high} = 0.375}$
$\boxed{W_{done} = e Q_{in} = 450\text{ J}}$

(b) Coeff of performance $\rightarrow \boxed{C = T_{low}/(T_{high} - T_{low}) = 1.67}$
5/
$\boxed{C = Q_{in}/W \Rightarrow W = Q_{in}/C = 720\text{ J}}$

(c)
5/
$\boxed{\text{Engine} \rightarrow Q_{out} = Q_{in} - W = 750\text{ J}}$
$\boxed{\text{Refrigerator} \rightarrow Q_{out} = Q_{in} + W = 1920\text{ J}}$

(d) $Q_{in}' = 1.5\, Q_{in} \Rightarrow C' = 1.5\, C = 2.5$
5/
$$\Rightarrow \frac{T_{low}}{T_{high}' - T_{low}} = 2.5 \Rightarrow \boxed{T_{high}' = \frac{3.5}{2.5} T_{low} = 350\text{ K}}$$