PHZ 3113 – PROBLEM SET 2

1) In cylindrical coordinates, the position vector of a moving particle is given by

$$\mathbf{r} = \rho\ \boldsymbol{\rho}\textbf{-hat} + z\ \mathbf{z}\textbf{-hat}$$

a) Derive an expression for the velocity vector of the particle, $\mathbf{v} = d\mathbf{r}/dt$, in cylindrical coordinates. Keep in mind that the unit vectors in cylindrical coordinates are *not* constants, so that their time derivatives will not generally be zero.

b) Show that the acceleration vector of the particle, $\mathbf{a} = d^2\mathbf{r}/dt^2$, has components

$$a_\rho = d^2\rho/dt^2 - \rho(d\phi/dt)^2$$
$$a_\phi = \rho d^2\phi/dt^2 + 2(d\rho/dt)(d\phi/dt)$$
$$a_z = d^2z/dt^2$$

2) The electric potential from an elementary electric dipole located at the origin is given by the expression

$$\phi(\mathbf{r}) = \mathbf{p}\cdot\mathbf{r}/(4\pi\varepsilon_o r^3)$$

where $\mathbf{p}$ is the electric dipole moment vector. Show that the corresponding electric field is given by the expression

$$\mathbf{E} = -\nabla\phi = (3\ \mathbf{p}\cdot\mathbf{r}\textbf{-hat}\ \mathbf{r}\textbf{-hat} - \mathbf{p})/(4\pi\varepsilon_o r^3)$$

where **r-hat** is the unit vector in the direction of the vector **r.**

3) Verify the vector identity $(\mathbf{A}\cdot\boldsymbol{\nabla})\mathbf{r} = \mathbf{A}$, where $\mathbf{r}$ is the position vector and $\mathbf{A}$ is a constant vector, using

a) Cartesian coordinates

b) spherical coordinates. Note that the spherical unit vectors are *not* constant. To evaluate their derivatives, you will need to express them as linear combinations of Cartesian unit vectors.

4) The angular velocity vector of a rigid object rotating about the z-axis is given by **ω**=ω **z-hat**. At any point in the rotating object, the linear velocity vector is given by **v** = **ω** X **r**, where **r** is the position vector to that point.

a) Assuming that ω is constant, evaluate **v** and **∇** X **v** in cylindrical coordinates.

b) Evaluate **v** in spherical coordinates.

c) Evaluate the curl of **v** in spherical coordinates and show that the resulting expression is equivalent to that given for **∇** X **v** in part a.

5) **Text Problem 2.11 –** in part a, express your result in terms of x,y,z, and r. In part b, first obtain your result in spherical coordinates and then show that the result is the same result that you obtained in part a.

6) A force is given in spherical coordinates by the expression

$$\mathbf{F}(\mathbf{r}) = P/r^3\,(2\cos\theta\ \textbf{r-hat} + \sin\theta\ \boldsymbol{\theta}\textbf{-hat})$$

where P is a constant.

a) Show that the curl of the force is zero.

b) Since ∇X**F**=0, there exists a scalar potential ϕ such that **F**=-∇ϕ. Find ϕ.

7) **Text Problem 3.23**

8) **Text Problem 3.26 –** note in the first part that the closed surface has *four* parts: the inner and outer curved surfaces and the two end caps.

9) A vector field is given in Cartesian coordinates by the expression

$$\mathbf{A}(\mathbf{r}) = (-y\ \textbf{x-hat} + x\ \textbf{y-hat})/\rho \quad \text{with} \quad \rho^2 = x^2+y^2$$

a) Evaluate the curl of this field.

b) Find the cylindrical components of **A** (expressed in cylindrical coordinates). Then show that the two line integrals $\int \mathbf{A}\cdot d\mathbf{r}$ evaluated over semicircles in the *upper* half plane (with $y>0$ and azimuthal angle ϕ ranging from 0 to π) and the *lower* half plane (with $y<0$ and ϕ ranging from 0 to $-\pi$) are just the negatives of each other.

c) By rewriting the integrand in the first of the integrals above in Cartesian coordinates, show that $\int dx/(1-x^2)^{1/2}=\pi$, where the integral runs from $x=-1$ to $x=+1$.