PHZ 3113 – PROBLEM SET 5

1) **Text Problem 4.4 –** note that moving the origin makes the function *symmetric*.

2) **Text Problem 4.5 –** note that the function is *antisymmetric* and that the series shown in the problem is just the Fourier series for a particular value of x.

3) Consider the function $f(x) = x^2$, defined over the interval $-\pi < x \leq \pi$.

a) Find the Fourier series for this function, noting that it is *symmetric*.

b) By inserting a particular value for x into the Fourier series derived in part a, evaluate the infinite sum given by $\sum (-1)^{n+1}/n^2$.

4) Consider the *antisymmetric* parabolic function f(x) defined by the relations

$$f(x) = 4x(1+x) \quad \text{for} \quad -1 \leq x \leq 0$$
$$= 4x(1-x) \quad \text{for} \quad 0 \leq x \leq 1$$

a) Find the Fourier series for this function

b) By substituting an appropriate value for x into the Fourier series obtained in part a, evaluate the sum $\sum (-1)^{(n-1)/2}/n^3$ over *odd* integers n>0.

5) **Text Problem 4.20 –** note that the function is *symmetric* over the range given.

6) **Text Problem 4.14 –** note that the function is *symmetric* over the range given. In the second part, you need to show that the constant of integration should be set equal to zero (consider what happens when x=0).

7) The Dirac δ-function (not actually a function) is defined so $\int f(x)\delta(x)\,dx=f(0)$, provided the integration range includes the point $x=0$.

a) Show that the Fourier series for $\delta(x)$ over the range $-\pi \le x \le \pi$ is given by

$$\delta(x) = 1/(2\pi) + (1/\pi) \sum \cos(nx)$$

where the sum is over integers $n>0$.

b) Integrate the result of part a using the Fourier series for $f(x)=x$ (problem 2) to obtain a Fourier series for the integrated δ-function.

c) Show that the result of part b is just the Fourier series for the unit step function given by $f(x)=0$ for $-\pi \le x \le 0$ and $f(x)=1$ for $0 \le x \le \pi$ if the integration constant in part b is chosen equal to the average value of the step function over the range $-\pi \le x \le \pi$.

8) **Text Problem 4.21** – recall that the complex Fourier series has terms with both positive and negative n. To get the final result, you will need to split the sum into three parts: terms with $n>0$, terms with $n<0$, and the $n=0$ term.

9) **Text Problem 4.19**

10) Consider the triangle function $f(x) = (2h/L)(L/2-|x|)$ defined over the range $-L/2 \le x \le L/2$.

a) Using the fact that $f(x)$ is a *symmetric* function of x, derive its Fourier series representation. Note that aside from the constant term, your result should contain only *odd* values of the integer n.

b) Evaluate the integral of $[f(x)]^2$ over the range $-L/2 \le x \le L/2$.

c) Using the result of part b and Parseval's theorem for Fourier series, show that

$$\sum 1/n^4 = \pi^4/96$$

where the sum is over *odd* integers n.