Probability

3.1 Sample Spaces and Tree Diagrams

To complete this section of homework watch Chapter Three, Lecture Examples: 28 and 29.

- 1. List the possible outcomes (C: correct or I: incorrect) when a person takes 3 guesses on a set of three multiple choice questions. For example, the 3 guesses could result in an outcome of C, I, C.
- 2. List the possible answers that can be given for a pair of quiz questions where the first question is true/false and the second question is a multiple-choice question with options: A, B, C, D, and E.
- 3. If you toss a coin and then spin a spinner that has the integers 1 6 on it, what are all the possible outcomes for the two events? For example, one outcome is: H, 3.
- 4. Can you guess why there are 8 outcomes in the sample space for problem 1 above, 10 outcomes for problem 2, and 12 outcomes for problem 3?

3.1 Answers:

- 1. CCC, CCI, CIC, CII, ICC, ICI, IIC, and III.
- 2. TA, TB, TC, TD, TE, FA, FB, FC, FD, and FE.
- 3. H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, and T6.
- 4. The number of total outcomes in the sample space is determined by the product of the number of possible outcomes for each of the individual events forming the experiment. For example, for problem 1, we are supposed to imagine three guesses taken on an exam. Each guess can result in either a correct answer or an incorrect answer. That means there are two possible outcomes for each guess. (2 outcomes possible for guess 1) X (2 outcomes for guess 2) X (2 outcomes for guess 3) = 2 X 2 X 2 = 8 total possible outcomes.

Need more exercises?

3.2 Basic Probability

To complete this section of homework watch Chapter Three, Lecture Examples: <u>30</u>, <u>31</u>, and <u>32</u>.

- 5. Which of the following cannot be probabilities? 0.00143, 0.45, 1.02, 0, 1, 3, 17/18, 31/30, $\sqrt{3}$
- 6. If an experiment has 5 equally likely outcomes, what is the probability of each of the five outcomes?
- 7. In 2008, US colleges and universities awarded 3,081,000 degrees of which 620,000 were master's degrees. Use the data to find the probability that a randomly selected 2008 graduate from a US college or university earned his/her master's degree.
- 9. A box contains 365 small strips of paper. Each strip of paper has a unique month and day on it from the calendar year. For example, one (and only one) of the strips reads, "December 3rd." Find the probability that a randomly selected strip of paper from the box says January 1st (ignore leap year).
- 10. In 2007, 31,228,000 US citizens traveled overseas. Among those travelers, 12,304,000 visited Europe. Assume there were 301,290,000 citizens in the US in 2007. Use this information to estimate the probability that a randomly selected US citizen visited Europe in 2007.
- 11. In 2011, there were 21,297,000 teens aged between fifteen and nineteen in the USA. In 2011, the Census Bureau estimated the number of teens (aged 15-19) living alone to be 143,000. Use this information to estimate the probability a randomly selected teen (aged 15-19) in 2011 did <u>not</u> live on his/her own?

When rolling a pair of fair dice, there are 36 possible outcomes. Use the table below, which lists all 36 outcomes to answer the three questions that follow:

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

- 13. What is the probability that the person rolls a total of 7? 🔠 VS
- 14. What is the probability that the person rolls a total of 4?

🖺 : indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com

15. What is the probability that the person rolls a total of 8?

8.2 Answers
. 1.02, 3, 31/30, $\sqrt{3}$
. 1/5
. 0.201
. 0.568
. 1/365 = 0.00274
0. 0.0408
1. 0.993
2. 0.975
3. 1/6
4. 1/12
5. 5/36

Need more exercises?

3.3 Fundamental Counting Rule and Combinations

To complete this section of homework watch Chapter Three, Lecture Examples <u>36</u>, <u>37</u>, <u>37.5</u>, <u>39</u>, <u>39.5</u>, and the concept videos for <u>fundamental counting rule</u>, <u>factorial</u>, <u>combinations</u> and <u>permutations</u> in section 3.3.

- 16. For \$20 per person, P.F. Chang's offers a fixed-price menu that allows you to choose a cup of soup, a starter, an entrée, and a dessert. There are two soups, six starters, 14 entrees, and eight desserts to choose from to form your meal. How many possible unique meals can you create using these options? <u>VS</u> E.
- 17. An old fashioned "combination" bike lock has three numbered wheels side-by-side on a steel cylinder. Each wheel has the digits 0 9 on them. The "combination" consists of a three-digit integer such as 123. How many "combinations" are possible by choosing a digit from each wheel (Note: it is okay to repeat digits, such as 111)? <u>VS</u>

- 18. A girl wants to wear a sundress out to dinner. She has three dresses to choose from, four pairs of matching sandals, and two different pieces of jewelry. How many different outfits will she have to try on if she wants to see what each possible outfit looks like?
- 19. A softball coach has 8 girls that he can rotate in the batting line-up in any order he chooses. How many ways can he arrange the batting order of the 8 girls?
- 20. TV stations use four-letter call signs. For example, one possible call sign is WLRN. If no letter can be repeated in the four-letter call sign, how many different call signs are possible?
- 21. Some older car keys were made from a blank that would typically have 5 different cuts made into it. Each cut might have been made at any of 6 different depths. How many different keys can be made this way? What did that mean if there were 500,000 cars of that model on the road at any given time?
- 22. A state all-star cheerleading squad will select four girls out of the 240 high school cheerleaders in Broward County. How many different groups of four Broward cheerleaders can be chosen to join the state all-star squad?
- 23. For a road trip, you plan to select 2 movies out of a set of 50 movies you own. How many different pairs of movies can you bring?
- 24. The Stats department at FIU needs to fill a committee with three professors. If the department has 12 full-time faculty members that can serve on the committee, how many different committees are possible?
- 25. Explain why ${}_{50}C_{49} = {}_{50}C_1$.

3.3 Answers:

- 16. 1344
- 17. 1000
- 18. 24

19. 8*7*6*5*4*3*2*1=40,320

- 20. 26*25*24*23=358,800
- 21. 6*6*6*6*6=7,776; It means each key could open 64 separate cars of the same make.
- 22. $_{240}C_4 = 134,810,340$
- 23. $_{50}C_2 = 1,225$

3.3 Answers:

24. $_{12}C_3 = 220$

25. Fifty choose forty-nine means we need to form a subset by choosing forty-nine items out of fifty. Of course, this means we are choosing all but one item out of fifty, so we could accomplish the same task by choosing the one item not to include out of fifty. Hence, fifty choose one.

Need more exercises?

3.4 Additive Rule of Probability

To complete this section of homework watch Chapter Three, Lecture Examples <u>43</u>, <u>44</u>, <u>45</u>, and <u>46</u>.

The table below lists the marital status for a random selection of government employees along with their job status. Use the table to answer the next four questions.

	Marital status				
Job grade	Single	Married	Divorced	Widowed	Total
Low-skilled	58	874	15	8	955
Blue collar	222	3927	70	20	4269
White collar	50	2396	34	10	2490
Supervisor	7	533	7	4	551
Total	337	7730	126	42	8235

- 26. If one of the employees is randomly selected, find the probability that the employee is low-skilled or single.
- 27. If one of the employees is randomly selected, find the probability that the employee is a supervisor or single.
- 28. If one of the employees is randomly selected, find the probability that the employee is white collar or divorced.
- 29. If one of the employees is randomly selected, find the probability that the employee is blue collar or married.

The table below has data from a 2009 Canadian Journal of Human Sexuality study. The data shows gender differences in response to partner influence and social expectation questions among students who had ever had sexual intercourse. The sample for this study included only students who had ever had sexual intercourse (30.7% of the total sample) yielding 2145 respondents after corrections. The age range for the study sample was 13 to 21 years old (mean = 15.8, standard deviation = 1.19). The vast majority were 14 to 17 years old (93%) and 45% were 16 years old. Less than 2% were aged either 13 or 19 to 21. Use the results to answer the following two questions.

Did you use a condom the last time you had intercourse?	Male	Female	Total
Yes	700	744	1444
No	244	457	701
Total	944	1201	2145

- 30. If one of the students is randomly selected, find the probability that the student is female or did use a condom.
- 31. If one of the students is randomly selected, find the probability that the student is male or did not use a condom.

The table below has data from a 2009 Canadian Journal of Human Sexuality study. The data shows gender differences in response to partner influence and social expectation questions among students who had ever had sexual intercourse. The sample for this study included only students who had ever had sexual intercourse (30.7% of the total sample) yielding 2145 respondents after corrections. The age range for the study sample was 13 to 21 years old (mean = 15.8, standard deviation = 1.19). The vast majority were 14 to 17 years old (93%) and 45% were 16 years old. Less than 2% were aged either 13 or 19 to 21. Use the results to answer the next three questions.

Number of partners	Male	Female	Total
1	428	640	1068
2	163	226	389
3	95	131	226
4+	258	204	462
total	944	1201	2145

- 32. If one of the students is randomly selected, find the probability that the student has had sex with one partner or four or more partners.
- 33. If one of the students is randomly selected, find the probability that the student has had sex with two or three partners.
- 34. If one of the students is randomly selected, find the probability that the student has had sex with one, two or three partners.

	3.4 Answers:
26.	$\frac{955+337-58}{8235} = 0.150$
27.	$\frac{551+337-7}{8235} = 0.107$
28.	$\frac{2490}{8235} + \frac{126}{8235} - \frac{34}{8235} = 0.314$
29.	$\frac{4269 + 7730 - 3927}{8235} = 0.980$
30.	$\frac{1201 + 1444 - 744}{2145} = 0.886$
31.	$\frac{944 + 701 - 244}{2145} = 0.653$
32.	$\frac{1068 + 462}{2145} = 0.7133$
33.	$\frac{389 + 226}{2145} = 0.2867$
34.	$\frac{1068 + 389 + 226}{2145} = 0.785$

3.5 Conditional Rule of Probability

To complete this section of homework watch Chapter Three, Lecture Examples <u>47</u>, <u>48</u>, <u>49</u>, <u>49.5</u>, and <u>51</u>.

The data summarized below shows the on-time performance of five different airlines based on a random sample of 1.6 million flights (Source: Bureau of Transportation Statistics). Use the results to answer the next three questions.

	On-time?		
Airline	Yes	No	Total
Delta	357,520	52,480	410,000
United	402,000	78,000	480,000
American	432,180	92,820	525,000
JetBlue	78,033	16,967	95,000
Spirit	77,949	12,051	90,000
Total	1,347,682	252,318	1,600,000

- 35. If one of the flights is randomly selected, find the probability that the flight is on time given that the flight was operated by Spirit.
- 36. If one of the flights is randomly selected, find the probability that the flight is <u>not</u> on time given that the flight is operated by Delta.
- 37. If one of the flights is randomly selected, find the probability that the flight is operated by United given that the flight is on time.

Use the table below, which is from a study on vitamin C intake and the common cold, to answer the following three questions.

Status	Vitamin C Group	Placebo Group	Total
Children free of colds	21	11	32
Children developing colds	36	35	71
Total	57	46	103

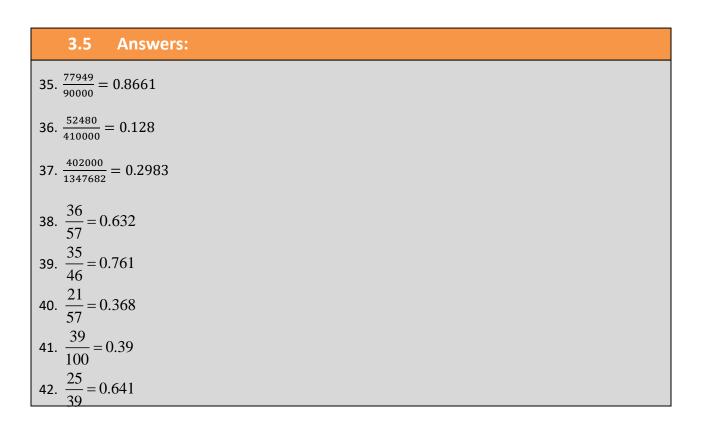
38. Find the probability that a randomly selected child from the study developed a cold during the study given that they were taking vitamin C.

- 39. Find the probability that a randomly selected child from the study developed a cold during the study given that they were taking a placebo.
- 40. Find the probability that a randomly selected child from the study was free of colds during the study given that they were taking vitamin C.

Use the table below, which is from a study on heart disease and smoking, to answer the following:

Risk Factor	Heart Disease	No Heart Disease	Total
Smoker	25	10	35
Nonsmoker	14	51	65
Total	39	61	100

- 41. Find the probability that a randomly selected member of the study has heart disease.
- 42. Find the probability that a randomly selected member of the study was a smoker given that they have heart disease.



3.6 Multiplicative Rule of Probability

To complete this section of homework watch Chapter Three, Lecture Examples <u>52</u>, <u>52.5</u>, <u>53</u>, and <u>53.5</u>.

Use the table below, which is from a study on heart disease and smoking, to answer the question below:

Risk Factor	Heart Disease	No Heart Disease	Total
Smoker	25	10	35
Nonsmoker	14	51	65
Total	39	61	100

43. Using the table above to form an estimate of the heart disease rate in North America, find the probability that 3 randomly selected North Americans all have heart disease.

The following report is from a researcher at the University of Chicago. Use it to answer the questions below:

Laumann and co-researcher Dr. Amy Derick, of the University of Chicago, found that year of birth was a predictive factor for tattoos: 36 percent of people aged 18 to 29; 24 percent of those aged 30 to 40; and only 15 percent of those aged 40 to 50 had tattoos. Sixteen percent had obtained their first tattoo before the age of 18. People of lower educational status were more likely to have a tattoo and more likely to have more than one tattoo than those of higher educational status. Drinking alcohol and using recreational drugs were related to having tattoos. Over a third of ex-drinkers and a fourth of current drinkers had tattoos, as did almost 40 percent of those who have ever used recreational drugs and 60 percent of those who have been in jail for more than three days. Tattoos were seen in all ethnic groups but were more common among those with Hispanic ancestry than among all other ethnic groups combined...

- 44. What is the probability that four randomly selected people aged 18 to 29 all have tattoos? \mathbf{E}_{VS}
- 45. What is the probability that two randomly selected people with tattoos got their first tattoo before turning 18 years old?
- 46. The following excerpt is from an MSNBC article on the prevalence of genital herpes in the population: "Despite the decline, blood tests of more than 11,000 people found 11 percent of men and 23 percent of women carry the genital herpes, or type 2, virus. Among people in their 20s, the infection rate was almost 11 percent." Use the information from the article to answer the following question. What is the probability that among 13 randomly selected women none of them are infected with genital herpes?

- 47. A batch of 95 batteries has 3 defective batteries. If two batteries are to be chosen for use in a toy, what is the probability that both batteries are defective?
- 48. Among 58 applications to the department of statistics, 11 have errors. What is the probability that when a member of the admissions committee randomly chooses three applications to review that none of the applications contains an error?
- 49. A bag contains 5 green marbles, 3 white marbles, and 4 blue marbles. Find the probability that when drawing two marbles without replacement they are both white.



3.7 Probability of At Least One

To complete this section of homework watch Chapter Three, Lecture Examples <u>54</u>, <u>56</u>, and <u>57</u>.

- 50. Some components in a car are critically important. If one or more of these critically important components fails, the car will not run. Let's imagine that there are only 10 such critical components in a car. If each critical component has a 0.95 (95%) probability of working properly, calculate the probability of the car not running due to a failure of one or more of these critical components.
- 51. Laumann and co-researcher Dr. Amy Derick, of the University of Chicago, found that year of birth was a predictive factor for tattoos: 36 percent of people aged 18 to 29; 24 percent of those aged 30 to 40; and only 15 percent of those aged 40 to 50 had tattoos. Sixteen percent had obtained their first tattoo before the age of 18. What is the probability that among 10 randomly selected people aged 30 to 40 at least one of them has a tattoo? Sixteen Percent VS

- 52. The following excerpt is from an MSNBC article on the prevalence of genital herpes in the population: "Despite the decline, blood tests of more than 11,000 people found 11 percent of men and 23 percent of women carry the genital herpes, or type 2, virus. Among people in their 20s, the infection rate was almost 11 percent." Use the information from the article to find the probability that among 13 randomly selected women at least one of them has genital herpes?
- 53. A company that produces aluminum soda cans for Pepsi has discovered that a defect occurs in 1.1% of its soda cans. These defects seem to occur randomly and independently from one another. What is the probability that among 12 randomly selected cans at least one has this defect?
- 54. The total number of Hispanic or White, male HIV cases that were attributed to heterosexual activity in the US is around 12,274. If the total number of sexually active, heterosexual Hispanic or White males in the US is currently 75,328,725, what is the probability that one randomly selected heterosexual, Hispanic or White male has HIV?
- 55. Assuming the rate of HIV infection among sexually active heterosexual Hispanic or White males is
 0.000163. What is the probability that among seventy randomly selected Hispanic or white males that at least one has HIV?
- 56. Arsenic is dangerously high in about 1.5% of soil samples tested. An environmental agency is analyzing soil samples for arsenic contamination. To save money, the agency combines 5 samples (from five different sites) and tests the combined mix. If the combined mix is found to have safe levels of arsenic, the agency knows the five sites have acceptable levels of arsenic in the soil. However, if the combined mix tests positive a sample from each of the five sources must be tested. What is the probability the combined mix will test positive for dangerous levels of arsenic?

3.7 Answers:

50. $1 - 0.95^{10} = 0.401$ 51. $1 - .76^{10} = 0.936$ 52. 1 - 0.0334 = .966653. $1 - .989^{12} = 0.124$ 54. 0.00016355. $1 - .999837^{70} = 0.0113$ 56. $1 - 0.985^5 = 0.0728$

Need more exercises?

3.8 Advanced Problems

To complete this section of homework watch Chapter Three, Lecture Examples <u>58</u>, <u>58.5</u>, and <u>59</u>.

- 57. A bag has 20 female student numbers and 25 male student numbers in it. Two student numbers are to be randomly selected without replacement, what is the probability that both numbers will be from members of the same sex?
- 58. A bag has 30 identical poker chips in it consisting of: 8 red, 12 blue, and 10 green. What is the probability that when drawing two chips from the bag without replacement they are both the same color?
- 59. A woman asks three men separately which dress they like the best out of five she tries on for them. They all pick the same dress as their favorite. She is concerned that they didn't really look at the dresses but just chose randomly. What is the probability that they would all pick the same dress if they were simply making random guesses? VS
- 60. Three students showed up late to the exam with the excuse that they were on the way to class together and the car they were traveling in got a flat tire. The professor takes them aside separately and asks, "Which tire was it that went flat?" If the students are lying and haven't carefully worked out their story, what is the probability that they will all randomly select the same tire as the one that went flat?
- 61. A bag contains 5 green marbles, 3 white marbles, and 4 blue marbles. Find the probability that when drawing two marbles without replacement there is one green and one white marble.

3.8 Answers:

Chapter 3 Mixed Review

- 62. During the production of a certain phone 0.8% are defective. A sample of ten of these phones is randomly selected without replacement from a set of 20,000 and tested. What is the probability that at least one of those ten tested phones is defective?
- 63. A bag contains 50 names in it. A professor plans to draw three names from the bag to win a free graphing calculator. How many different sets of three winners can be drawn from the bag?
- 64. A social media company produces an app that is very popular. The company reports that 48% of the population uses their app. If three people are randomly selected, what is the probability that all three will **not** use the app produced by this social media company?
- 65. Use the data below from a children's hospital to estimate the probability that a randomly selected child is less than 130 pounds.

		Weight in Pounds			
Age	<100	100 -129	130 - 149	150 - 169	Totals
Under 5 yrs	25	2	1	0	28
6 - 9 yrs	65	23	7	2	97
10 - 12 yrs	55	39	15	8	117
Totals	145	64	23	10	242

- 66. A campus lab has 10 desktop computers which are infected with a virus and 3 which are virus free. If two friends randomly sit at two of the computers in the lab, what is the probability that both computers are virus free?
- 67. Use the data below from a children's hospital to estimate the probability that a randomly selected child weighs between 150 and 169 pounds or is 6 to 9 years old.

Age	<100	100 -129	130 - 149	150 - 169	Totals
Under 5 yrs	25	2	1	0	28
6 - 9 yrs	65	23	7	2	97
10 - 12 yrs	55	39	15	8	117
Totals	145	64	23	10	242

- 68. When working on a very difficult probability problem, Jay realized that he made a mistake somewhere because his answer turned out to be 107/103. How did Jay know his answer was incorrect?
- 69. At a recent soccer match, 39,318 people were there to root for the home team, and 6,550 people were there to root for the visitors. If one of the fans is randomly selected from the crowd, what is the probability that the fan is rooting for the home team?
- 70. A bag contains 15 yellow marbles, 10 red marbles, and 5 white marbles. The marbles are identical except for their colors. What is the probability that when three marbles are selected without replacement that one of them is white?
- 71. A DJ plans to form a playlist of four songs. He will pick these songs from a set of ten songs. The order of the chosen songs will be important because a different ordering will create a different listening experience. How many unique playlists can be formed by picking four songs from the set of ten songs?
- 72. Use the data below from a children's hospital to estimate the probability that a randomly selected child weighs between 150 and 169 pounds given the child is 6 to 9 years old.

Age	<100	100 -129	130 - 149	150 - 169	Totals
Under 5 yrs	25	2	1	0	28
6 - 9 yrs	65	23	7	2	97
10 - 12 yrs	55	39	15	8	117
Totals	145	64	23	10	242

Chapter 3 Mixed Review Answers
62. 0.0772
63. 19,600
64. 0.141
65. 0.864
66. 0.0385
67. 0.434
68. 107/103 is greater than 1.00. Probability cannot be larger than 1.00.
69. 0.857
70. 0.369

71. 5,040 72. 0.0206