Normal Random Variables

5.1 Using the Z-table

To complete this section of homework watch Chapter five, Lecture Examples: 75, 76, 77, 78, and 79.

1. Male heights are normally distributed with a mean of 69 inches and a standard deviation of 2.8 inches. What is the probability that a randomly selected male is 72 inches (6 ft) tall?

2. The average length of time a Wilson light bulb can stay on before burning out is 10,000 hours with a standard deviation of 150 hours. What is the probability that a randomly chosen Wilson light bulb will work for 9,200 hours before burning out?

In problems 3 – 17, use the Z-table to find the indicated probabilities:

3. $P(0 < z < 1.16)$ VS
4. $P(-1.99 < z < 0.89)$ VS
5. $P(z > 2.58)$ VS
6. $P(0.56 < z < 3.03)$ VS
7. $P(z > -1.17)$ VS
8. $P(-0.99 < z < 0)$
9. $P(-1.33 < z < 1.24)$
10. $P(z < -2.25)$
11. $P(-3.07 < z < -2.07)$
12. $P(z < 2.61)$ VS
13. $P(z < -2.72 \text{ or } z > 2.72)$ VS
14. $P(z < -2.58 \text{ or } z > 2.58)$
15. $P(-0.50 < z < 0.50)$
16. $P(z > 6.73)$
17. $P(z < -8.01)$ VS

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

5.1 Answers:

1. 0, because the probability distribution is continuous.
2. 0, because the probability distribution is continuous.
3. $P(0 < z < 1.16)$ = 0.3770
4. $P(-1.99 < z < 0.89)$ = 0.7900
5. $P(z > 2.58)$ = 0.0049
6. $P(0.56 < z < 3.03)$ = 0.2865
7. $P(z > -1.17)$ = 0.8790
8. $P(-0.99 < z < 0)$ = 0.3389
9. $P(-1.33 < z < 1.24)$ = 0.8007
10. $P(z < -2.25)$ = 0.0122
11. $P(-3.07 < z < -2.07)$ = 0.0181
12. $P(z < 2.61)$ = 0.9955
13. $P(z < -2.72 \text{ or } z > 2.72)$ = 0.0066
14. $P(z < -2.58 \text{ or } z > 2.58)$ =0.0098
15. $P(-0.50 < z < 0.50)$ = 0.3830
16. $P(z > 6.73)$ = Approximately 0
17. $P(z < -8.01)$ = 0

Need more exercises?

5.2 Probability Using the Normal Distribution

To complete this section of homework watch Chapter Five, Lecture Examples: 84, 85, 86, and 86.5.

18. Body fat percentages are approximately normally distributed. The average guy in his 20's (ages 20 – 29) has a body fat percentage of 15.75%. The standard deviation for this group is 0.99%. Find the probability that a randomly chosen male in his 20's has a body fat percentage below 13%.

19. Finishing times for the second exam in my STA 3123 class are normally distributed. The second exam for STA 3123 has historically taken 86.56 minutes on average for students to complete. The standard deviation is 29.03 minutes. What percent of the class will finish before the 120-minute time limit? VS

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

20. A recent trial conducted in the Department of Kinesiology and Nutrition at the University of Illinois at Chicago investigated the consequences of alternate day fasting (ADF) in a group of obese men and women. The study design was quite complex and involved three dietary phases: the first part was a 2-week baseline period that established the typical diet of the participants; the second portion of the study involved the use of a carefully controlled diet which yielded 75% fewer calories on alternate days for 4 weeks; the final phase allowed the volunteers to select their own food while still maintaining a 75% calorie deficit every other day. After 8 weeks of treatment, participants had an average 12.5 lbs reduction in body weight with a standard deviation of 3.6 pounds (note: lean body mass of the participants remained relatively constant). Assuming the amount of weight loss has a normal distribution, find the probability that the weight loss for a randomly selected dieter on this plan is between 5 and 10 pounds.

21. The subjects involved in the ADF diet described above also had an average decrease of 4 cm in waist circumference with a standard deviation of 1.68 cm. Assume the decrease in waist circumference has a normal distribution and find the probability that while on the diet you will have a 2.54 cm or greater decrease in waist circumference.

22. Pregnancy lengths are normally distributed with a mean of 280 days and a standard deviation of 20 days. Nine months is approximately 273.6 days, what is the probability a woman is pregnant for at least 273? VS

23. Fuel economies for Chevy vehicles are normally distributed. The 2010 Chevy Malibu had an average EPA-estimated fuel economy of 30 mpg and a standard deviation of 0.67 mpg. If one Malibu is randomly selected, find the probability that its fuel economy is between 29.5 and 30.5 mpg. VS

24. Women's heights are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If a women's clothing manufacturer decided to make pants in sizes that will fit women who are between 60.5 inches and 71.3 inches tall, what percentage of women will not be able to fit into this manufacturer's clothing?

25. The fastest finishing times for the NY marathon are normally distributed with a mean of 147.08 minutes and a standard deviation of 1.48 minutes. Find the probability that a randomly chosen race year had a fastest time between 145 and 146 minutes.

26. My commute to FIU from my home in Northeast Broward County takes on average 51.6 minutes with a standard deviation of 4.67 minutes. Assuming these travel times are normally distributed, what is the probability that on a randomly selected day my commute takes me less than 60 minutes? VS

27. A car manufacturer makes a certain transmission gear with a mean circumference of 11 mm. This is the size that allows the gear to fit precisely into the transmission to avoid unnecessary wear and tear on the transmission's other moving parts. The manufacturing process is not perfect, so there is some variation in the circumference of the gear. The circumference measurements are normally distributed, and the standard deviation of the gear's circumference is 0.05 mm. Find the probability that a randomly selected gear produced by this company has a circumference between 10.9 mm and 11.1 mm.

28. Men have hip widths that are normally distributed with a mean of 14.4 inches and a standard deviation of 1.0 inch. An engineer wants to know what percent of men sitting in a movie theater seat are uncomfortable because they have wider than expected hips. What is the probability that a randomly chosen male has a hip width measurement greater than 17.3 inches?

29. IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. Find the probability that a randomly selected person has an IQ score between 110 and 120.

30. Women's weights are normally distributed. The average woman weighs 152.2 pounds with a standard deviation of 26.1 pounds. What is the probability that a randomly selected woman weighs at most 120 pounds? VS

31. Men's weights are normally distributed with a mean of 189.8 pounds and a standard deviation of 24.9 pounds. What is the probability that a randomly selected man weighs between 155 pounds and 165 pounds?

32. Chest to waist ratios for men are normally distributed with a mean of 1.17 and a standard deviation of 0.08. What is the probability that a randomly selected male has a ratio higher than 1.32? What is the probability that a randomly selected male has a ratio higher than the ideal of 1.41? VS

5.2 Answers:

18. 0.0027
19. 0.8749
20. 0.2263
21. 0.8078
22. 0.6368
23. 0.5468
24. 1 - 0.8915 = 0.1085
25. 0.1534
26. 0.9641
27. 0.9544
28. 0.0019
29. 0.1596
30. 0.1093
31. 0.0779
32. 0.0301, 0.0013

Need more exercises?

5.3 Finding Percentiles of the Normal Curve (Using the Table in Reverse)

To complete this section of homework watch Chapter Five, Lecture Examples: 87, 88, and 88.5.

33. Body fat percentages are approximately normally distributed. The average man in his 20's (ages 20 – 29) has a body fat percentage of 15.75%. The standard deviation for this group is 0.99%. Find the body fat percentage separating the bottom 5% from the rest of the men in their 20's. This is the body fat that divides the leanest 5% of men from the rest.

34. Finishing times for the second exam in my STA 3123 class are normally distributed. The second exam for STA 3123 has historically taken 86.56 minutes on average for students to complete. The standard deviation is 29.03 minutes. How long will it take the slowest ten percent of the class to finish? VS

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

35. A recent trial conducted in the Department of Kinesiology and Nutrition at the University of Illinois at Chicago investigated the consequences of alternate day fasting (ADF) in a group of obese men and women. The study design was quite complex and involved three dietary phases: the first part was a 2-week baseline period that established the typical diet of the participants; the second portion of the study involved the use of a carefully controlled diet which yielded 75% fewer calories on alternate days for 4 weeks; the final phase allowed the volunteers to select their own food while still maintaining a 75% calorie deficit every other day. After 8 weeks of treatment, participants had an average 12.5 lbs reduction in body weight with a standard deviation of 3.6 pounds. Assuming the amount of weight loss has a normal distribution, find the weight loss achieved by the top two percent of dieters. VS

36. The subjects involved in the ADF diet described above also had an average decrease of 4 cm in waist circumference with a standard deviation of 1.68 cm. Assume the decrease in waist circumference has a normal distribution and find the reduction in waist circumference achieved by the top 2% of dieters. VS

37. Pregnancy lengths are normally distributed with a mean of 280 days and a standard deviation of 20 days. Nine months is approximately 273.6 days, what is the pregnancy length corresponding to the shortest one percent of pregnancies? VS

38. Fuel economies for Chevy vehicles are normally distributed. The 2010 Chevy Malibu had an average EPA-estimated fuel economy of 30 mpg and a standard deviation of 0.67 mpg. If one Malibu is randomly selected, find the fuel economy for the poorest performing 3% of vehicles.

39. Women's heights are normally distributed with a mean of 63.6 and a standard deviation of 2.5. If a women's clothing manufacturer decides, due to costs and demand, that they do not want to produce sizes that fit the very shortest or the very tallest 2.5% of women, what cutoff sizes will separate the shortest and tallest 2.5% of women's sizes from the rest? VS

40. The fastest finishing times for the NY marathon are normally distributed with a mean of 147.08 minutes and a standard deviation of 1.48 minutes. Find the finishing time for the slowest 4% of winning NY marathon runners.

41. My commute to FIU from my home in Northeast Broward County takes on average 51.6 minutes with a standard deviation of 4.67 minutes. Assume these travel times are normally distributed, what is the commute time that separates the longest 3% of commutes from the rest?

42. Toyota Corolla weights are normally distributed. The average Corolla weighs 2,931 pounds with a standard deviation of 51.3 pounds. What is the weight that separates the lightest 1% of Corollas from the rest?

43. Men's weights are normally distributed with a mean of 189.8 pounds and a standard deviation of 24.9 pounds. What is the weight that separates the heaviest 1% of men from the lightest 99% of men? VS

44. Chest to waist ratios for men are normally distributed with a mean of 1.17 and a standard deviation of 0.08. What is the ratio separating the highest (best) 5% from the rest? VS

5.3 Answers:

33. 14.12
34. 123.72
35. 19.88
36. 7.44
37. 233.4
38. 28.7
39. 58.7, 68.5
40. 149.67
41. 60.4
42. X = -2.33*51.3+2931 = **2,811.5 pounds**
43. 247.8
44. 1.30

Need more exercises?

5.4 Normal as Approximation to Binomial (optional)

To complete this section of homework watch Chapter Five, Lecture Example 91.5.

45. Use the normal approximation to calculate the probability that $P(x \geq 17)$ for $n = 30$, $p = 0.50$. VS
46. Use the normal approximation to calculate the probability that $P(x \geq 21)$ for $n = 39$, $p = 0.50$. VS
47. Use the normal approximation to calculate the probability that $P(x \geq 19)$ for $n = 32$, $p = 0.50$. VS
48. If you flip a fair coin 30 times, what is the probability that you get more than 15 heads?

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

49. What is the probability that when making random guesses on a fifty-question multiple choice exam, with four answer choices for each question, that you miss at least thirty-five of the questions?

50. A car dealership sells extended warranties on 50% of its vehicles. What is the probability that among 30 cars sold, at most 10 purchase the extended warranty?

5.4 Answers:

45. $P(z \geq 0.55) = 0.2912$
46. $P(z \geq 0.32) = 0.3745$
47. $P(z \geq 0.88) = 0.1894$
48. 0.4286
49. 0.8365
50. 0.0505

Need more exercises?

Chapter 5 Mixed Review

51. P(-2.17 < Z < -0.79)

52. The weights of males who play soccer professionally are normally distributed with a mean of 165.9 pounds and a standard deviation of 11.1 pounds. If a male professional soccer player is randomly selected, what is the probability that he will weigh between 170 and 175 pounds?

53. P(Z < 1.66)

54. Male weights are normally distributed with a mean of 176 pounds and a standard deviation of 29 pounds. Find the weight separating the lightest 35% of men from the heaviest 65% of men.

55. The length of time for students to complete this set of exercises is normally distributed with a mean of 30 minutes and a standard deviation of 4.9 minutes. Find the probability that a student takes more than 42 minutes to complete this set of exercises.

56. The weights of females who play soccer professionally are normally distributed with a mean of 125.7 pounds and a standard deviation of 8.2 pounds. If a female professional soccer player is randomly selected, what is the probability that she will weigh between 115 and 135 pounds?

57. Several FIU students choose to live off campus. The lengths of the commutes for students who live off campus are normally distributed. The mean of these commute distances is 5.4 miles, and the standard deviation is 2.3 miles. Find the probability that a randomly chosen commute is less than 10 miles.

58. P(Z > 2.09)

59. The lengths of the commutes for students who live off campus are normally distributed. The mean of these commute distances is 5.4 miles, and the standard deviation is 2.3 miles. Find the commute length that separates the longest 9% of commutes from the rest.

Chapter 5 Mixed Review Answers

51. 0.1998
52. 0.1496
53. 0.9515
54. 164.69 pounds
55. 0.0071
56. 0.7740
57. 0.9772
58. 0.0183
59. 8.482 miles