What do I know already? (Do not look up the answers to these questions. The purpose is to assess your current level of knowledge on these topics.)

A. Fill in the missing boxes below with the following possible outcomes for a hypothesis test: correct decision, type I error, or type II error.

Conclusions	**Reality**	
	The Null Is True	**The Null Is False**
We Reject Null	Type I Error	Correct Decision
We Fail to Reject Null	Correct Decision	Type II Error

B. In your own words, state the definition of a type I and type II error:

Type I error: rejecting a true null hypothesis. Type II error: failing to reject a false null hypothesis.

C. For a one-tailed hypothesis test conducted at a significance level of α, what is the probability of committing a type I error? For a one-tailed hypothesis test, the probability of committing the type I error is at most α.

D. In your own words, define the p-value for a hypothesis test. Under the assumption that the null hypothesis is true, the p-value is the likelihood of observing a test statistic that is as extreme or even more extreme than the one calculated from the sample data.

Learning Objectives: (Click the learning objectives below for a short clip on the topic.)

Discuss the Four Possible Outcomes for a Hypothesis Test (2)
Describe a Type I Error (2)
Describe a Type II Error (2)
Classify an Error as Either Type I or Type II (2)
Discuss the Probability of Committing a Type I Error for a One-Tailed Test (2)
Discuss the Probability of Committing a Type I Error for a Two-Tailed Test (2)
Define the Critical Value (1)
Find the Critical Value for a One-Tailed Test when n is Large (3)
Find the Critical Values for a Two-Tailed Test when n is Large (3)
Determine a Decision Rule Based on Critical Values (2)
Use the Classical Approach of Hypothesis Testing to Determine the Initial Conclusion (3)
Express the Final Conclusion of the Hypothesis Test Based on the Initial Conclusion (3)
Use the Classical Approach to Conduct a Hypothesis Test of the Population Mean when the Sample Size is Large (3)
Define the P-value (1)

Exercises:

1. A sociologists claims that Americans watch an average of 500 minutes of media each day. A study of a random sample of 100 US residents found the average amount of time spent viewing media per day is 490 minutes with a standard deviation of 75 minutes. Calculate the test statistic for a test of this hypothesis.
   $$z = \frac{\overline{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{490 - 500}{\frac{75}{\sqrt{100}}} \approx -1.33$$

2. An organic farmer claims that the average weight of his apples is greater than 85 grams. State the null and alternative hypothesis that would be used to test the farmer's claim.
   $H_0 : \mu \leq 85$
   $H_A : \mu > 85$

3. Researchers created a confidence interval for the true proportion of computers infected with malware. Interpret the interval: 0.30 ± 0.05. If the FBI claims that only 20% of computers are infected with malware, does the provided interval contradict the FBI's claim?
   Since the interval ranges from 25% to 35%, we believe the percent of computers infected with malware is between these two values and is therefore higher than 20%. Thus, the interval contradicts the FBI's claim.

4. The head of a bank that provides auto loans claims that US households have an average amount of auto loan debt that is less than $29,000. A study of 50 randomly selected US households is conducted. The mean auto loan debt for the sample is $27,865, and the standard deviation is $7,500. Use a 5% significance level to test the claim.
   Claim: μ < $29,000
   $H_0: \mu \geq 29,000$
   $H_A: \mu < 29,000$
   Test Stat:
   $$z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{27,865-29,000}{\frac{7,500}{\sqrt{50}}} \approx -1.07$$
   Critical value and rejection region: reject if z is less than -1.645
   Initial conclusion: Do not reject Ho
   Final Conclusion: The sample data does not allow us to support the claim that the mean is less than $29,000.

5. A doctor claims that the average woman is not more than 63 inches tall. A sample of 47 women has a mean of 64.1 inches and a standard deviation of 2.3 inches. Calculate the test statistic for a test of this hypothesis.
   $$z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{64.1-63}{\frac{2.3}{\sqrt{47}}} \approx 3.28$$

6. A researcher claims that college students sleep for an average of 7 hours per night. A survey of 36 randomly selected college students was conducted. The sample mean amount of time the students spent asleep was 6.1 hours per night, and the standard deviation was 1.2 hours per night. Use a 5% significance level to test the researcher's claim.
   Claim: μ =7
   $H_0: \mu = 7$
   $H_A: \mu \neq 7$
   Test Stat:
   $$z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{6.1-7}{\frac{1.2}{\sqrt{36}}} = -4.50$$
   Critical values and rejection region: reject if z is less than -1.96 or if z is greater than 1.96.
   Initial conclusion: Reject Ho
   Final Conclusion: The sample data allows rejection of the claim that the mean is 7 hours.

7. Consumer Reports claims the average time for a Chrysler Concorde to accelerate from 0 to 60 mph is 9 seconds. State the null and alternative hypotheses for a test of Consumer Reports' claim at the 5% significance level. What is the type I error for this test, and what is the probability that the type I error occurs?

   $H_0 : \mu = 9$

   $H_A : \mu \neq 9$

   Type I error: the error of rejecting the claim that the mean is 9 seconds when it is in fact true that the mean is 9 seconds.
   There is a 5% chance of committing the type I error in this problem.

8. An organic farmer claims that the average weight of his apples is greater than 85 grams. If a random sample of 100 apples has a mean weight of 86.1 grams and a standard deviation of 5 grams, what is the critical value and rejection region for a test of the farmer's claim at the 5% significance level?
   C.V. = 1.645
   Rejection region: reject the null if z > 1.645

9. Energizer claims that two of their D batteries will power a flashlight for more than 20 hours. State the null and alternative hypotheses for a test of Energizer's claim at the 2.5% significance level. What is the type I error for this test, and what is the probability that the type I error occurs?

   $H_0 : \mu \leq 20$

   $H_A : \mu > 20$

   Type I error: the error of rejecting the hypothesis that the mean is less than or equal to 20 hours when it is in fact true that the mean is less than or equal to 20 hours.
   There is an **at most** 2.5% chance of committing the type I error in this problem.

10. A survey of 1024 randomly selected adults found that 66% of those surveyed prefer daylight saving time over standard time. Use the sample data to construct a 95% confidence interval for the true proportion of adults that prefer daylight saving time.

    $\left(0.66 - 1.960\sqrt{\frac{0.66 * 0.34}{1024}}, 0.66 + 1.960\sqrt{\frac{0.66 * 0.34}{1024}}\right) = (0.631, 0.689)$ We are 95% confident the true proportion is between 0.631 and 0.689.

11. An economist claims that the average US mortgage is less than $180,000. A random sample of 40 US mortgages had an average value of $172,341 and a standard deviation of $106,312. State the null and alternative hypotheses necessary to test the economist's claim. If we were to decide to reject the null hypothesis, how would we state the final conclusion?

    $H_0: \mu \geq \$180,000$

    $H_A: \mu < \$180,000$

    If we rejected the null hypothesis, we would be supporting the alternative hypothesis, which was our original claim here, so we would say the sample data support the claim that the average mortgage is less than $180,000.

12. Rayovac makes low cost batteries that last a long time. A Rayovac scientist claims that two of their D batteries will power a standard flashlight for 24 hours. If a random sample of 45 of their D batteries powers a flashlight for an average of 24.9 hours, what is the critical value(s) and rejection region for a test of the scientist's claim? Assume a 1% significance level.

    C.V.s = ±2.576
    Rejection region: reject the null if z < -2.576 or z > 2.576

13. A random sample of 100 US households had an average amount of auto loan debt of $27,865 and a standard deviation of $8,300. Use the sample of data to construct a 90% confidence interval to estimate the true mean amount of auto loan debt held by US households.

    $\left(27,865 - 1.645\frac{8,300}{\sqrt{100}}, 27,865 + 1.645\frac{8,300}{\sqrt{100}}\right) = (\$26,499.65, \$29,230.35)$ We are 90% confident the true mean is between $26,499.65 and $29,230.35.

14. How much does the average newborn weigh? A doctor claims that the average weight of newborn children is greater than 7.0 pounds. A random sample of 32 newborns had a mean weight of 7.5 pounds and a standard deviation of 1.1 pounds. Use a 1% significance level to test the doctor's claim.

    Claim: μ > 7

    $H_0: \mu \leq 7$

    $H_A: \mu > 7$

    Test Stat:

    $$z = \frac{\overline{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{7.5 - 7}{\frac{1.1}{\sqrt{32}}} \approx 2.57$$

    Critical values and rejection region: reject if z is greater than 2.326
    Initial conclusion: Reject Ho
    Final Conclusion: The sample data allow us to support the claim that the mean is greater than 7 pounds.

15. State the type I error for a test of the following hypothesis. Also assume the significance level for the test is 5%, and state the likelihood that the type I error occurs.
An organic farmer claims that the average weight of his apples is greater than 85 grams.

    Type I error: rejecting the hypothesis that the mean weight is less than or equal to 85 grams when it actually is less than or equal to 85 grams.

    There is at most a 5% chance of this error occurring.

16. A 95% confidence interval for the true mean amount of snowfall in Boston each year was constructed. Interpret the interval: 43.8 inches ± 7.1 inches. Does the interval contradict the claim that the true mean amount of snowfall each year in Boston is greater than 35 inches?
Since the interval ranges from 36.7 inches to 50.9 inches, the interval supports the claim that the mean snowfall is greater than 35 inches. Therefore, the interval does not contradict the claim.