

What do I know already? (Do not look up the answers to these questions. The purpose is to assess your current level of knowledge on these topics.)

A. State the decision rule that is used when testing a hypothesis using a p-value? If the p-value is less than the significance level, reject the null hypothesis.

B. Will lowering the significance level raise or lower the likelihood of a type I error during a hypothesis test? Lowering the significance level will reduce the likelihood of a type I error.

C. Regarding the probability of a type II error, state the general effect of using a very small significance level during a hypothesis test. If all things are otherwise equal, a hypothesis test that uses a smaller significance level than an otherwise identical hypothesis test will have a greater probability of committing a type II error. In short, lowering the significance level will decrease the likelihood of a type I error, but increase the likelihood of the type II error.

D. If we want to use a small significance level, what can we do to counter the rise of the likelihood of a type II error? The effect can be countered by increasing the sample size.

Learning Objectives: (Click the learning objectives below for a short clip on the topic.)

[Discuss the Method for Finding the P-value for a Left-Tailed Test \(2\)](#)

[Discuss the Method for Finding the P-value for a Right-Tailed Test \(2\)](#)

[Discuss the Method for Finding the P-value for a Two-Tailed Test \(2\)](#)

[Explain the Decision Rule when Using the P-value Approach to Hypothesis Testing \(2\)](#)

Calculate the P-value for a One-Tailed Hypothesis Test (3)

Calculate the P-value for a Two-Tailed Hypothesis Test (3)

Use a Given P-value and Significance Level to Form a Conclusion (3)

Test a Hypothesis Using the P-value Approach (3)

List the Ways to Reduce the Likelihood of a Type I Error (1)

Determine the Consequences of Increasing or Decreasing the Significance Level (2)

Carry Out the Procedure to Find the Critical Value for a One-Tailed Test when n is Small (3)

Carry Out the Procedure to Find the Critical Values for a Two-Tailed Test when n is Small (3)

Conduct a Test of a Hypothesis about the Population Mean when n is Small (4)

Exercises:

1. If a left-tailed hypothesis test is conducted at a 1% significance level using a random sample of 36 measurements, which of the following changes would lower the likelihood of a type I error?
 - A. Increase the significance level to 5%
 - B. Lower the sample size to 25
 - C. Increase the sample size to 100
 - D. None of these

2. Consumer Reports claims the average fuel consumption for the 2016 Toyota Corolla is more than 36 mpg. State the null and alternative hypotheses for a test of Consumer Reports' claim. What is the type I error for this test?

$$H_0 : \mu \leq 36$$

$$H_A : \mu > 36$$

The type I error for this test is the error of rejecting the hypothesis that the mean is less than or equal to 36, when in fact the mean is less than or equal to 36.

3. An organic farmer claims that the average weight of his melons is at least 3 kg. If a random sample of 50 of these melons has a mean weight of 3.2 kg and a standard deviation of 0.5 kg, what is the critical value and rejection region for a test of the farmer's claim at the 10% significance level?

Critical value: -1.282

Rejection Region: Reject the null hypothesis if the test statistic is less than -1.282.

4. A car dealer claims the average amount of money households spend maintaining their automobiles each year is more than \$500. State the null and alternative hypotheses necessary to test the claim. If we were to decide to reject the null hypothesis, how would we state the final conclusion?

$$H_0 : \mu \leq \$500$$

$$H_A : \mu > \$500$$

The sample data allow us to support the claim that the mean is greater than \$500.

5. Use the given information to find the p-value:

Claim: $\mu < 150$ Test Stat: $z = -2.03$

0.0212

6. Use the given information to find the p-value:

Claim: $\mu \neq 75$ Test Stat: $z = 2.05$

0.0404 (two-tail test means you must multiply by two)

7. Use the given information to find the p-value:

Claim: $\mu \geq 75$ Test Stat: $z = -1.67$

0.0475

8. Use the given information to find the p-value:

Claim: $\mu \geq 2.50$ Test Stat: $z = 1.88$

0.9699 (This is a left-tailed test, so we find the area to the left of the test stat)

9. The owner of a coffee chain claims that a cup of their coffee has an average of 150 mg of caffeine. A study of 50 randomly selected cups of their coffee was conducted. The mean caffeine amount for the sample was 199.0 mg, and the standard deviation was 26.2 mg. Use a 1% significance level to test the claim.

Claim: $\mu = 150$

$$H_0 : \mu = 150$$

$$H_A : \mu \neq 150$$

Test stat:

$$z = \frac{(199.0 - 150)}{\frac{26.2}{\sqrt{50}}} \approx 13.22$$

Critical Values: ± 2.576 Rejection Region: reject if the test stat is less than -2.576 or more than 2.576.

Reject the null, support the alternative

The sample data allow us to reject the claim that the mean is equal to 150.

10. If a hypothesis test is conducted at a 5% significance level using a random sample of 35 measurements, which of the following changes would lower the likelihood of a type I error?
- A. Increase the significance level to 10%
 - B. decrease the significance level to 1%
 - C. Lower the sample size to 19
 - D. None of these
11. If a hypothesis test is conducted at a 5% significance level using a random sample of 35 measurements, what effect would the following change cause: raising the significance level to 10%.
- A. It will increase the likelihood of a type I error, but lower the likelihood of a type II error.
 - B. It will decrease the likelihood of a type I error, but increase the likelihood of a type II error.
 - C. It will lower the likelihood of both types of errors.
 - D. It will increase the likelihood of both types of errors.
12. A consumer protection group claims that FPL makes customers wait longer than 30 minutes to reach a customer service representative by phone. A test of this claim was conducted at a 5% significance level, and the p-value for the test turned out to be 0.2147. Form the appropriate final conclusion based on this p-value.
- Since the p-value is greater than the significance level, do not reject the null. The sample data does not allow us to support the claim that customers wait longer than 30 minutes for a customer service representative by phone.
13. If you are testing the claim: $\mu = 187$ and your significance level is 10%, what is the probability that you commit the type one error?
- The probability of a type I error is equal to 10%.
14. In an effort to determine the average amount of beer purchased by customers at a pub, the owner records the volume in ounces for a random sample of 24 customers. These volumes had an average of 25.1 ounces and a standard deviation of 4.2 ounces. Use the sample data to form a 98% confidence interval for the volume of beer consumed by customers visiting the pub. You may assume the volumes follow a normal distribution.
- $$t_{\alpha/2} = 2.50$$
- $$E = 2.50 * \frac{4.2}{\sqrt{24}} \approx 2.1433$$
- We are 98% confident that the true mean amount is between 22.96 to 27.24 ounces.

15. An organic farmer claims that the average weight of his apples is greater than 85 grams. It is safe to assume these apple weights follow a normal distribution. A random sample of 15 of the farmer's apples has a mean weight of 86.1 grams and a standard deviation of 5 grams. Test the farmer's claim at the 5% significance level.

Claim: $\mu > 85$

$H_0 : \mu \leq 85$

$H_A : \mu > 85$

Test stat:

$$t = \frac{(86.1 - 85)}{\frac{5}{\sqrt{15}}} \approx 0.852$$

Critical Value: 1.761 Rejection region: reject if the test stat is more than 1.761.

Do not reject the null, do not support the alternative

The sample data does not allow us to support the claim that the mean is more than 85.

16. An ounce of potato chips may vary in calories. Lays claims one of their products has 160 calories per ounce. A test of this claim was conducted at a 2% significance level, and the p-value for the test turned out to be 0.0041. Form the appropriate final conclusion based on this p-value.

Since the p-value is less than alpha, we reject the null.

The sample data allow us to reject the claim that the mean is 160 per ounce.

17. Lays claims that a one ounce bag of their potato chips has an average of 160 calories. It is safe to assume these calorie amounts follow a normal distribution. A random sample of 27 one ounce bags of chips has a mean calorie count of 169.1 calories and a standard deviation of 22 calories. Test Lay's claim at the 5% significance level.

Claim: $\mu = 160$

$H_0 : \mu = 160$

$H_A : \mu \neq 160$

Test stat:

$$t = \frac{(169.1 - 160)}{\frac{22}{\sqrt{27}}} \approx 2.149$$

Critical Values: ± 2.06 Rejection region: reject if the test stat is less than -2.06 or more than 2.06.

Reject the null, support the alternative

The sample data allow us to reject the claim that the mean is 160.

18. Find the appropriate critical value for a test of the following claim at the 1% significance level. The data used to test the claim involved a random sample of 25 measurements. You may assume the measurements follow a bell-shaped distribution. Claim: $\mu < 50$

Critical value: -2.492

19. If you are testing the claim: $\mu > 187$ and your significance level is 5%, what is the probability that you commit the type one error?

The probability of the type I error is **at most 5%**

20. A car dealer wants to know how much money households spend maintaining their automobiles each year. From a previous study, the standard deviation for these spending amounts appears to be \$100. If the researchers want to create a 95% confidence interval to estimate the true mean to within \$10, how many households must be randomly sampled?

$$n = \left[\frac{z_{\alpha/2} * \sigma}{E} \right]^2 = \left[\frac{1.96 * 100}{10} \right]^2 = 384.16 \text{ Round up to } 385$$

21. Find the appropriate critical value for a test of the following claim at the 10% significance level. The data used to test the claim involved a random sample of 18 measurements. You may assume the measurements follow a bell-shaped distribution. Claim: $\mu = 185$

Critical values: ± 1.74