What do I know already? (Do not look up the answers to these questions. The purpose is to assess your current level of knowledge on these topics.)

A. If one in five cars on the road today are painted red, how many cars out of 500 randomly chosen cars would you expect to be painted red? Since 1 in every 5 cars should be red, one fifth of the 500 cars should be red, so the answer should be 100.

B. One requirement of a binomial probability experiment is that the random variable of interest (x) counts the number of successes that occur. What are the four other requirements?

1) There are a fixed number of trials (n)

2) There are only two possible outcomes. These are usually labeled success and failure.

3) The individual trials are independent of each other.

4) The probability of success remains constant from trial-to-trial.

C. For a binomial experiment with 8 trials, we are asked to calculate the probability that we observe at least 3 successes. If x represents the number of successes, this can be calculated as follows: $P(x \geq 3) = P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$. Provide another approach to find this probability that relies on the idea of complements: Since the sum of all of the probabilities for all possible outcomes equals 1.0, we can subtract the probabilities for the events we do not want from 1.0 to find our answer. Specifically, we can do the following: $P(x \geq 3) = 1 - P(x \leq 2) = 1 - P(2) - P(1) - P(0)$. This saves time because we only need to find three probabilities to solve the problem. The other approach requires that we calculate 6 probabilities.

D. A life insurance policy has an expected value of \$125 for the insurance company. Explain what this means for the insurance company: Over the very long run, the more policies the company sells the closer the average profit per policy (pp) for the company will get to \$125. In other words, if millions of policies are sold, the average profit will be close to \$125 (pp). If billions of policies are sold, the average profit will be closer to \$125 (pp), and if trillions of policies are sold, the average profit will come closer still to \$125 (pp).

Learning Objectives: (Click the learning objectives below for a short clip on the topic.)

Know the Formula for the Mean of a Discrete Probability Distribution (1)
Calculate the Expected Value of a Discrete Random Variable (3)
Interpret the Expected Value of a Discrete Random Variable (2)
Know the Formula for the Variance and Standard Deviation of a Discrete Random Variable (1)
Calculate the Variance and Standard Deviation of a Discrete Random Variable (3)
Determine if an Event is Unusual Using the Mean and Standard Deviation of a Random Variable (2)
Identify the Five Characteristics of a Binomial Experiment (1)
Recognize the Binomial Probability Formula (1)
Calculate the Probability of X Successes in a Binomial Experiment (3)
Calculate the Probability of a Cumulative Set of Events for a Binomial Experiment (3)
Recognize the Formula for the Mean of a Binomial Probability Distribution (1)
Calculate the Mean of a Binomial Random Variable (3)
Recognize the Formula for the Standard Deviation of a Binomial Probability Distribution (1)
Calculate Standard Deviation of a Binomial Random Variable (3)

Exercises:

1. Use the data below from a study of mammograms conducted on women in their forties to estimate the probability that a randomly selected woman, in her forties, has cancer given that she has a negative exam result.

Cancer?	Positive Mammogram	Negative Mammogram	Totals
Cancer	32,608	10,870	43,478
No Cancer	267,298	2,689,224	2,956,522
Totals	299,906	2,700,094	3,000,000

$$P(C\mid neg)=\frac{number\text{ of women in the negative exam category with cancer}}{Total\text{ number of woman in the negative exam result category}}=\frac{10,870}{2,700,094}\approx 0.004$$

2. The Statistics department needs to form a two person technology committee from the thirteen full-time members of the department. These two committee members will have equal rights, titles, and responsibilities. How many unique committees of two people can be formed from the thirteen full-time faculty?

$${}_nC_r = {}_{13}C_2 = 78$$

3. Which of the following are discrete random variables? (select all that apply)
   A. X is a variable representing the number of emails received each weekday by randomly selected members of FIU's faculty.
   B. Y is a variable representing the weight of garbage thrown out by the Chili's restaurant on campus for randomly selected days of the year.
   C. Z is a variable representing the number of homework problems completed at midterm for randomly selected Calculus students at FIU.
   D. W is a variable that records the height for randomly selected Statistics students.
   E. None of the above

4. Among a group of ten students, four of them have completed all of their assigned homework. If their professor randomly selects three of these ten students and checks their progress on the homework, what is the probability that at least one of the selected students will have completed all of their homework?

   $$P(\text{at least one completed the homework}) = 1 - P(\textit{none}\text{ completed the homework}) =$$
   $$1-\left(\frac{\#\text{did not complete hw}}{\textit{total}}\cdot\frac{\#\text{did not complete hw } \textit{left}}{\textit{total}\text{ left}}\cdot\frac{\#\text{did not complete hw } \textit{left}}{\textit{total}\text{ left}}\right)=$$
   $$1-\left(\frac{6}{10}\cdot\frac{5}{9}\cdot\frac{4}{8}\right)=1-\frac{120}{720}\approx 0.833$$

5. Valeria is going to randomly grab two eggs from her refrigerator to make scrambled eggs. She has twelve eggs in her refrigerator, but half of them are spoiled. What is the probability that both of the eggs she selects are unspoiled (good)?

   $$P(\textit{both}\text{ unspoiled}) = P(1\textit{st}\text{ is unspoiled})\cdot P(2\textit{nd}\text{ is unspoiled} \mid 1\textit{st}\text{ was unspoiled}) =$$
   $$\frac{\#\textit{unspoiled}}{\textit{total}}\cdot\frac{\#\textit{unspoiled}\text{ left}}{\textit{total}\text{ left}}=\frac{6}{12}\cdot\frac{5}{11}=\frac{30}{132}\approx 0.227$$

6. Seventy-six percent of people who attend a high school in Florida graduate. If five students attending Florida high schools are randomly selected for tracking, what is the probability that three of them will graduate?
   n = 5, x = 3, p = 0.76, q =1 - 0.76 = 0.24
   $$P(x=3) = {}_5C_3(0.76)^3(0.24)^2 \approx 0.253$$

7. True or false: The sum of probabilities for any discrete probability distribution must be greater than 1.00. The sum must be equal to one, not greater than one.

8. A Pew Research Center poll indicated that 86% of college graduates report that they are at least somewhat satisfied with their current job. Find the mean for the number of college graduates that report that they are at least somewhat satisfied with their current job when taking random samples of 40 college graduates.
   n = 40, p = 0.86

$$\mu = n \cdot p = 40(0.86) = 34.4$$

9. The following table is a probability distribution for X, where X represents the number of adults who have completed a bachelor's degree or higher out of a random sample of five adults from a community in California. What is the probability that at most three randomly selected adults of this community have a bachelor's degree or higher?
   At most 3 means: 3 or less, so add the probabilities for x = 0, 1, 2, and 3.
   In other words, 0.168 + 0.360 + 0.309 + 0.132 = 0.969

X	P(X)
0	0.168
1	0.360
2	0.309
3	0.132
4	0.028
5	0.003

10. A baseball card collector decides to buy and resell a collection of valuable cards. He paid $2,000 for the whole set. With a little luck, he will resell the cards one-by-one for an overall profit. However, if he is unlucky, he could lose money. Assume there is a ten percent chance the cards sell for $4,000, a thirty percent chance the cards sell for $2,500, a forty percent chance they sell for $2,000, and a twenty percent chance they sell for just $1,800. Calculate the collector's expected value for this set of cards. Note: he paid **$2,000** for the cards
    The expected value is $310.

Event	X	P(X)	X*P(X)
Sells for $4000	$4000 **- $2000** = 2000	0.10	200
Sells for $2500	500	0.30	150
Sells for $2000	0	0.40	0
Sells for $1800	-200	0.20	-40
Total:		1.00	$310

11. Half (50%) of the homeless veterans in the U.S. suffer from serious mental illness. If 15 homeless veterans are randomly selected, what is the probability that ten of them will suffer from serious mental ability?

$$P\left(x=10\right)={}_{15}C_{10}\left(0.50\right)^{10}\left(0.50\right)^{5}\approx 0.092$$

12. A recent survey found that 4.1% of the 12th graders in the U.S. have used anabolic steroids. Calculate the standard deviation for the number of 12th graders that have used anabolic steroids out of a random sample of 1,000 12th graders.

$$\sigma=\sqrt{n\cdot p\cdot\left(1-p\right)}=\sqrt{1000\left(0.041\right)\left(1-0.041\right)}\approx 6.270$$

13. This year, 48% of registered voters identify as Democrats, compared with 44% who identify as Republican. That is nearly identical to the balance of party identification in 2012. In a random selection of 10 registered voters, what is the probability that two or more will identify as Republican?

$$P(x\geq 2)=P\left(2\right)+P\left(3\right)+P\left(4\right)+P\left(5\right)+P\left(6\right)+P\left(7\right)+P\left(8\right)+P\left(9\right)+P\left(10\right)=$$

It's easier to use the idea of complements:

$$P\left(x\geq 2\right)=1-P(x\leq 1)=1-P\left(1\right)-P\left(0\right)=1-\left({}_{10}C_{1}\cdot 0.44^{1}\cdot 0.56^{9}\right)-0.56^{10}\approx 0.973$$

14. Which of the following meet the requirements to qualify as a binomial experiment? For those that do not qualify, explain why:
A. A six-sided die will be tossed 25,000 times, and a state investigator will record the outcome (which face of the die turned up) for each toss. Not binomial because there are six outcomes, not two.
B. A fair coin will be tossed. The number of required tosses to achieve ten tails will be counted. Not binomial because there is not a fixed number of trials.
C. A bag of marbles contains 25 red marbles and 15 blue marbles. Five marbles will be randomly taken without replacement. The number of red marbles selected will be counted. Not binomial because the trials are not independent, and the probability of success will not be constant from trial-to-trial.
D. A variety of watermelon seeds have a probability of 0.65 of germinating. An agriculturalist plans to sow 100 of these seeds and to count the number that germinate. Binomial, n = 100, p = 0.65

15. An illegal lottery requires players to choose 6 numbers from the values 1 to 12. If a participant matches all 6 numbers (in any order), he/she wins $300. If 5 values match, the user wins $5, and if four values match, the player wins $1. The respective probabilities for these events are: 0.0011, 0.0390, and 0.2435. What is the expected value for a single ticket purchased for $1? -$0.23

Event	X	P(X)	X*P(X)
Match all 6	$300 - $1 = $299	0.0011	0.3289
Match 5	$5 - $1 = $4	0.0390	0.1560
Match 4	$1 - $1 = $0	0.2435	0
Match 3 or less	-$1	0.7164	-0.7164
Totals		1.0000	-0.2315

16. If 4.1% of the 12th graders in the U.S. have used anabolic steroids, calculate the probability that 50 out of 1,000 randomly selected 12th graders have used anabolic steroids.

$$P(x=50) = {}_{1000}C_{50}\,(0.041)^{50}\,(0.959)^{950} \approx 0.022$$