

What do I know already? (Do not look up the answers to these questions. The purpose is to assess your current level of knowledge on these topics.)

A. The variable below (X) is a continuous random variable. Are the two probabilities given below always equal, or is one of the probabilities always larger than the other? If one is always larger, which one is larger? $P(a < X < b)$ compared to $P(a \leq X \leq b)$

These probabilities are always the same, because the probability that X is equal to a or b is always zero when X is a continuous random variable.

B. IQ scores are scaled to ensure that the mean score is 100, and the standard deviation is 15. What is the z score for an IQ score of 122 points?

$$z = \frac{(122 - 100)}{15} = \frac{22}{15} \approx 1.47$$

C. What is the probability that a standard normal random variable (Z) is greater than 0? This probability is 0.5000 ($1/2$) since zero is the mean of the z distribution. Since the z curve is symmetric about its mean, half of the total area is above zero, and half of the total area is under zero.

D. There is not just one normal curve. There is a "family" of normal curves. Name as many things as you can that are the same for all normal curves, and name the two things that can make one normal curve different from another.

All normal curves have a total area equal to one. All normal curves are symmetric around their mean. All normal curves are bell shaped.

The two things that can make normal curves different from each other are the mean and the standard deviation. Two normal curves are different from each other if they have a different mean and/or a different standard deviation.

Learning Objectives: (Click the learning objectives below for a short clip on the topic.)

[Describe Continuous Random Variables \(2\)](#)

[Contrast Probability for Continuous Random Variables with that of Discrete Random Variables \(2\)](#)

Know that the Area Under a Continuous Probability Function is Always One (1)

Know that the Normal Distribution is Bell Shaped (1)

Know that the Normal Distribution is Symmetrical about the Mean (1)

Identify the Mean and Standard Deviation of a Standard Normal Distribution (1)

[Understand that there is a Family of Normal Distributions \(1\)](#)

Know the Formula to Find a Z Value (1)

[Use a Z table to Find Areas Under the Standard Normal Curve between the Mean and a Value \(3\)](#)

[Calculate Areas Under the Standard Normal Curve Inside an Interval Surrounding the Mean \(3\)](#)

[Calculate Areas Under the Standard Normal Curve between a Positive Z Value and Infinity \(3\)](#)

[Calculate Areas Under the Standard Normal Curve between Two Values on the Same Side of the Mean \(3\)](#)

[Calculate Areas Under the Standard Normal Curve between a Negative Z Value and Infinity \(3\)](#)

[Calculate the Probability a Non-Standard Normal Random Variable is Greater Than an Above-Average Value \(3\)](#)

[Calculate the Probability a Non-Standard Normal Random Variable is Less Than an Above-Average Value \(3\)](#)

[Calculate the Probability a Non-Standard Normal Random Variable is Between Two Values that Surround the Mean \(3\)](#)

[Calculate the Probability a Non-Standard Normal Random Variable is Between Two Values that Are on the Same Side of the Curve \(3\)](#)

Exercises:

1. **True** or false: The standard normal probability distribution (also called the z distribution) has a mean equal to zero and a standard deviation of 1.
2. Which of the following are continuous random variables? (select all that apply)
 - A. **X** is a variable representing the **volume** of liquids consumed by randomly selected marathon runners during the course of a race.
 - B. **Y** is a variable representing the **time** spent waiting in line for service at a local deli for randomly selected customers.
 - C. **Z** is a variable representing the number of houses visited while trick-or-treating for randomly selected children on Halloween night.
 - D. **W** is a variable that records the **distance** walked on campus for randomly selected Statistics faculty members.
 - E. None of the above

3. Dutch men are now the tallest men in the world. The average height for Dutch males is 71.9 inches (That's almost 6 feet tall!). The standard deviation for their heights is 2.6 inches. Assuming that these heights have a bell-shaped distribution, what is the probability that a randomly selected Dutch man is under 69 inches tall? (i.e. - shorter than the average American male)
 $z = (69 - 71.9) / 2.6 = -1.12$ Table value: 0.3686, Solution: $0.5 - 0.3686 = \mathbf{0.1314}$
4. The variable Z has a standard normal probability distribution. Find $P(1.09 < Z < 2.58)$ Solution: $0.4951 - 0.3621 = \mathbf{0.1330}$
5. The amount of calories people consume when eating a large bag of popcorn while watching a movie follows a normal distribution. The average number of calories consumed is 1250, and the standard deviation for the number of calories consumed is 210. If one person who is eating from a bag of large popcorn during a movie is randomly selected, what is the probability that the person consumes between 1300 and 1500 calories? $z = 1.19, z = 0.24, 0.3830 - 0.0948 = \mathbf{0.2882}$
6. True or false: The normal probability distribution has a bell-shaped curve that is symmetric about its mean, so precisely half of its area is below the mean, and half of its area is above the mean.
7. Find $P(Z > -1.34) = 0.4099 + 0.5000 = \mathbf{0.9099}$
8. This year, thirty-eight percent of people graduating from a higher education program (either at a college or at a university) took at least one online course. If ten recent graduates from higher ed programs are randomly selected, what is the probability that five of them took at least one online course? This is a binomial probability problem $n = 10, x = 5, p = 0.38, P(5) = {}_{10}C_5 (0.38)^5 (0.62)^5 \approx 0.183$
9. True or false: The sum of probability (i.e. - the total area under the probability density curve) for any continuous probability distribution must be equal to 1.00.
10. A one-year life insurance policy will pay \$25,000 to the purchaser's family if he/she dies within one year of the start of the policy. The policy sells for \$150. If the likelihood of a person surviving the year is 0.9999, what is the expected value of the policy for the insurance company that sells this policy?

Event	x	P(x)	x*P(x)
Life	+150	0.9999	149.985
Death	150-25,000 = -24,850	0.0001	-2.4850
		Mean =	\$147.50

11. Find $P(-1.62 < Z < 1.75) = 0.4474 + 0.4599 = \mathbf{0.9073}$

12. A study in 2014 indicated that the average height for women in Guatemala was 58.7 inches (a little under 4ft 11in). The standard deviation for these heights is 1.9 inches. It is safe to assume these heights are normally distributed. What is the probability that a randomly selected Guatemalan woman is between 60 inches tall and 62 inches tall? $z = 1.74, z = 0.68, 0.4591 - 0.2517 = \mathbf{0.2074}$

13. Find $P(0 < Z < 0.77) = \mathbf{0.2794}$

14. The lengths of time it takes Spirit airlines to board flights traveling from FLL in Florida to BOS in Boston are normally distributed with a mean of 25 minutes and a standard deviation of 4.2 minutes. What is the probability that a randomly chosen Spirit flight from FLL to BOS takes longer than 20 minutes to board? $z = -1.19, 0.3830 + 0.5000 = \mathbf{0.8830}$

15. Find $P(-2.27 < Z < -0.39) = 0.4884 - 0.1517 = \mathbf{0.3367}$

16. The average weight of women in the United States is 164.7 pounds. The standard deviation for these weights is 37.5. If one woman is randomly selected from the US population, what is the probability that she weighs exactly 165.1 pounds? Weight is measured on a continuous scale, so $P(x = 165.1) = 0$

17. Find $P(Z > 0.31) = 0.5000 - 0.1217 = \mathbf{0.3783}$

18. A research firm reports that 33.7% of laptops fail in the first year of ownership. Assuming this failure rate is accurate, find the standard deviation for the number of laptops that will fail in the first year among a random sample of 140 laptops. Using the formula for the standard deviation of a binomial random variable: $n = 140, p = 0.337, q = 0.663, \sigma = \sqrt{140 \cdot 0.337 \cdot 0.663} \approx 5.59$