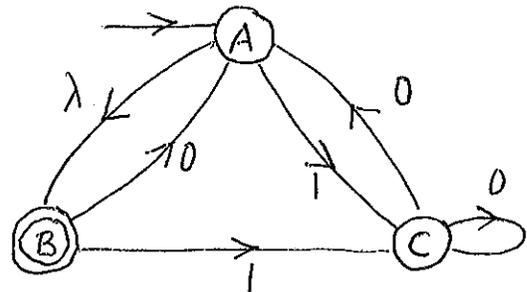


Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH QUESTION ON A SEPARATE PAGE.

- (15) 1. (a) Define what is a regular expression over the alphabet $\{0,2\}$.
 (B) Convert the NFA on the right into an equivalent DFA.

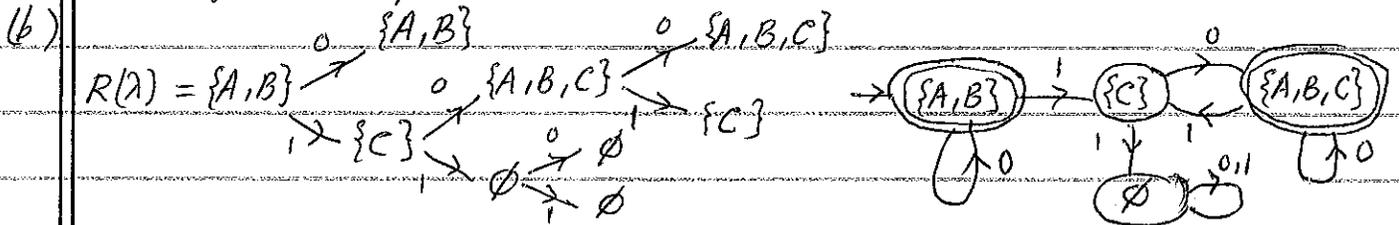


- (15) 2. Find regular expressions which describe the languages below.
 (a) $L_1 = \{\alpha \in \{0,1\}^* : \alpha \text{ contains both } 001 \text{ \& } 100 \text{ as substrings}\}$
 (b) $L_2 = \{\beta \in \{0,1\}^* : \beta \text{ has at most one occurrence of } 11\}$
- (20) 3. (a) Define when two states in a DFA are indistinguishable.
 (b) Partition the states of the DFA below into blocks of indistinguishable states & then find the reduced machine.

	A	B	(C)	→D	(E)	(F)	(G)
0	D	B	D	A	G	A	E
1	F	G	B	C	B	B	A

- (15) 4. (a) Let $f(\omega) = [1 + 2n_b(\omega) - 3n_a(\omega)] \pmod{4}$. Find a DFA which accepts the language $L_4 = \{\omega \in \{a,b\}^* : f(\omega) > 1\}$.
 (b) Check your DFA with abaab as input.
- (20) 5. (a) Find a context-free grammar which generates the language $L_5 = \{a^k b^{2n} : k > n\} \cup \{c^k d^n : k < 2n + 3\}$.
 (b) Using your CFG, find derivations for $a^4 b^4$ and $c^3 d^1$.
- (15) 6. Let A, B, and C be languages based on the alphabet $\{0,1\}$.
 (a) Is it always true that $A.(B \cap C)^* \subseteq (A.B^*) \cap (A.C^*)$?
 (b) Is it always true that $A.(B - C) \subseteq (A.B) - (A.C)$?
 Justify your answers completely.

1(a) A regular expression over $\{0,2\}$ is defined recursively as follows.
 (i) $0, 2, \lambda$ and \emptyset are regular expressions. (ii) If E and F are regular expressions, then so are $(E+F)$, $(E.F)$, & (E^*) .

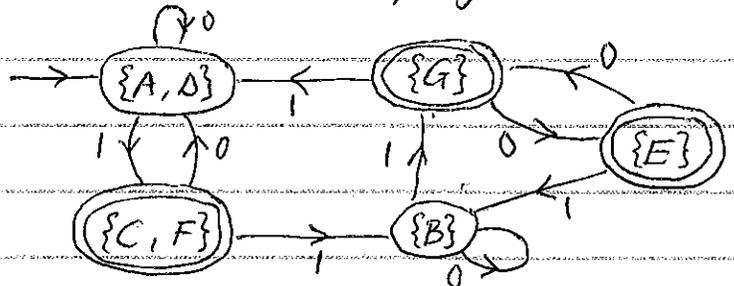


2(a) $\dots 001\dots 100\dots, \dots 100\dots 001\dots, \dots 00100\dots, \dots 1001\dots, \dots 10001\dots$
 $(0+1)^*(001(0+1)^*100 + 100(0+1)^*001 + 00100 + 1001 + 10001)(0+1)^*$

(b) no 11's: $(0+10)^*(\lambda+1)$, one 11's: $(0+10)^*.11.(0+01)^*$

3(a) Two states A & B of a DFA M are indistinguishable if for each $\varphi \in \Sigma^*$, $S^*(A, \varphi) \in A(M) \iff S^*(B, \varphi) \in A(M)$.
 [Here Σ = input alphabet of M & $A(M)$ = set of accepting states of M]

- (b) $P_0: \{A, B, D\} \{C, E, F, G\}$
- $P_1: \{A, B, D\} \{C, F\} \{E, G\}$
- $P_2: \{A, D\} \{B\} \{C, F\} \{E, G\}$
- $P_3: \{A, D\} \{B\} \{C, F\} \{E\} \{G\}$
- $P_4: \{A, D\} \{B\} \{C, F\} \{E\} \{G\}$.



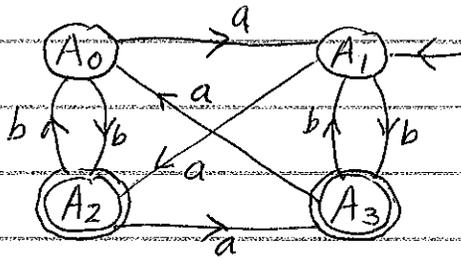
4(a) Let A_i ($i=0,1,2,3$) keep track of the fact that $f(w) \equiv i \pmod{4}$. Since $f(\lambda) = 1 + 2n_b(\lambda) - 3n_a(\lambda) = 1 + 0 - 0 \equiv 1 \pmod{4}$, A_1 will be the initial state. Also since $2 > 1$ and $3 > 1$, A_2 and A_3 will be the accepting states. Now we will express $f(wa)$ & $f(wb)$ in terms of $f(w)$ to see what an extra a or b does.

$$4(a) \quad f(wa) = 1 + 2n_b(wa) - 3n_a(wa) = 1 + 2n_b(w) - 3n_a(w) - 3$$

$$= f(w) - 3 = f(w) + 1 \pmod{4}$$

$$\& \quad f(wb) = 1 + 2n_b(wb) - 3n_a(wb) = 1 + 2n_b(w) + 2 - 3n_a(w)$$

$$= 1 + 2n_b(w) - 3n_a(w) + 2 = f(w) + 2$$



(b) input : a b a a b
states : A₁ A₂ A₀ A₁ A₂ A₀

$$f(abaab) = 1 + 2(2) - 3(3) = -4 \equiv 0 \pmod{4}$$

5(a) $S \rightarrow E|F;$

This gives the union

$E \rightarrow aEbb|A, A \rightarrow aA|a;$ gives $\{a^k b^{2n} : k \geq n+1\}$

$F \rightarrow CCFD|CC, C \rightarrow c|\lambda, D \rightarrow d.$ gives $\{c^k b^n : k \leq 2n+2\}$

$S \Rightarrow E \Rightarrow aEbb \Rightarrow \dots \Rightarrow a^n E b^{2n} \Rightarrow a^n A b^{2n} \Rightarrow a^n a A b^{2n} \Rightarrow \dots \Rightarrow a^{n+l} A b^{2n} \Rightarrow a^{n+l+1} b^{2n}$

$S \Rightarrow F \Rightarrow CCFD \Rightarrow \dots \Rightarrow C^{2n} F D^n \Rightarrow C^{2n} C C D^n = C^{2n+2} D^n \Rightarrow \dots \Rightarrow c^k d^n : k \leq 2n+2$

(b) $S \Rightarrow E \Rightarrow aEbb \Rightarrow aaEbbbb \Rightarrow a^2 A b^4 \Rightarrow a^2 a A b^4 \Rightarrow a^2 a a b^4 = a^4 b^4$

$S \Rightarrow F \Rightarrow CCFD \Rightarrow CCCC D \Rightarrow \lambda C C C d = c^3 d$

6(a) YES. Let $\varphi \in A.(B \cap C)^*$. Then $\varphi = \alpha \beta_1 \beta_2 \dots \beta_k$ where $\alpha \in A$ and $\beta_1, \beta_2, \dots, \beta_k \in B \cap C$. Since $\beta_i \in B \cap C$, $\beta_i \in B$ and $\beta_i \in C$ for each i . So $\varphi \in A.B^*$ because $\alpha \in A$ & $\beta_1, \dots, \beta_k \in B$; and $\varphi \in A.C^*$ because $\alpha \in A$ & $\beta_1, \dots, \beta_k \in C$. Hence $\varphi \in A.B^* \cap A.C^*$. Thus $A.(B \cap C)^* \subseteq (A.B^*) \cap (A.C^*)$.

(b) NO. Let $A = \{\lambda, 0\}$, $B = \{1\}$ and $C = \{01\}$. Then

$A.(B \cap C) = \{\lambda, 0\}.(\{1\} \cap \{01\}) = \{\lambda, 0\}. \{1\} = \{\lambda, 0\}$, and

$A.B \cap A.C = \{\lambda, 0\}. \{1\} \cap \{\lambda, 0\}. \{01\} = \{\lambda, 0\}. \{1\} \cap \{\lambda, 0\}. \{01\} = \{\lambda, 0\}. \{1\} = \{\lambda, 0\}$

\therefore it is not always true that $A.(B \cap C) \subseteq (A.B) \cap (A.C)$.