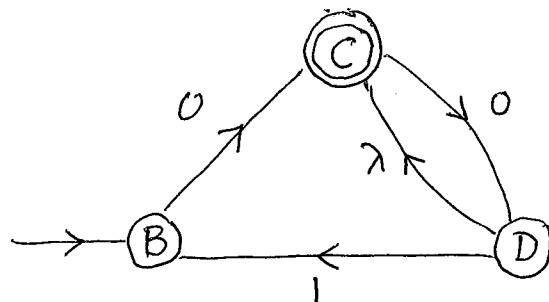


TEST #1 - FALL 2012TIME: 75 min.

Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH QUESTION ON A SEPARATE PAGE.

- (15) 1. (a) Define what is the extended transition function of a DFA.

- (b) Convert the NFA on the right into an equivalent DFA.



- (15) 2. Find regular expressions which describe the languages below.

- (a) $L_1 = \{\alpha \in \{0,1\}^*: \alpha \text{ contains both } 010 \text{ & } 001 \text{ as substrings}\}$
 (b) $L_2 = \{\beta \in \{0,1\}^*: \beta \text{ contains exactly two occurrences of } 11\}$

- (20) 3. (a) Define what is an inaccessible state of a DFA M.

- (b) Partition the states of the DFA below into blocks of indistinguishable states & then find the reduced machine.

	A	$\rightarrow B$	C	D	E	F	G
0	<i>f</i>	<i>f</i>	A	C	A	A	B
1	A	D	D	B	G	B	E

- (15) 4. (a) Let $f(\omega) = [2n_a(\omega) - n_b(\omega) - 3] \pmod{4}$. Find a DFA which accepts the language $L_4 = \{\omega \in \{a,b\}^*: f(\omega) \text{ is odd}\}$.

- (b) Check your DFA with baab as input.

- (20) 5. (a) Find a context-free grammar which generates the language $L_5 = \{a^k b^{2n}: k > 2n\} \cup \{c^k d^n: k < 3n+2\}$.

- (b) Using your CFG, find derivations for $a^4 b^2$ and $c^3 d^1$.

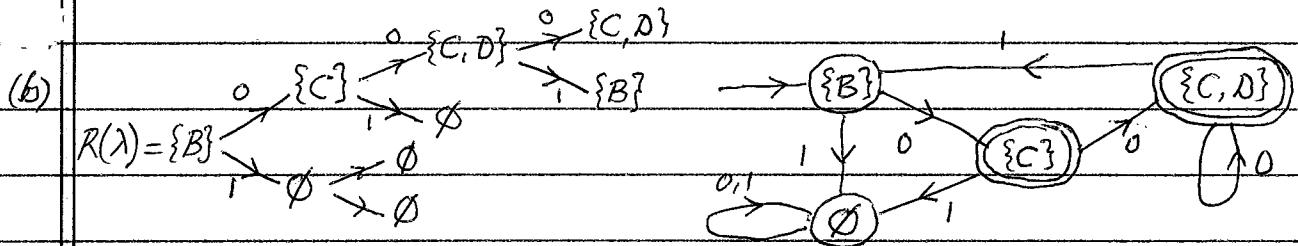
- (15) 6. Let A, B, and C be languages based on the alphabet $\{0,1\}$.

- (a) Is it always true that $(A^* \cdot B) \cup (A^* \cdot C) \subseteq A^* \cdot (B \cup C)$?

- (b) Is it always true that $(A^* \cdot B) \cap (A^* \cdot C) \subseteq A^* \cdot (B \cap C)$?

Justify your answers completely.

- 1(a) The extended transition function, $\delta^*: Q \times \Sigma^* \rightarrow Q$ of a DFA is defined recursively as follows. (i) $\delta^*(q, \lambda) = q$; and (ii) $\delta^*(q, \varphi a) = \delta(\delta^*(q, \varphi), a)$ for any $a \in \Sigma$ & $\varphi \in \Sigma^*$.



- 2 (a) ...010--001..., ..001--010..., --01001..., --0010---

$$(0+1)^* \cdot (010 \cdot (0+1)^* 001 + 001 \cdot (0+1)^* 010 + 01001 + 0010) \cdot (0+1)^*$$

- (b) ...11...11..., ---111---

$$(0+10)^* \cdot 110 \cdot (0+10)^* 11 \cdot (0+01)^* + (0+10)^* 111 \cdot (0+01)^*$$

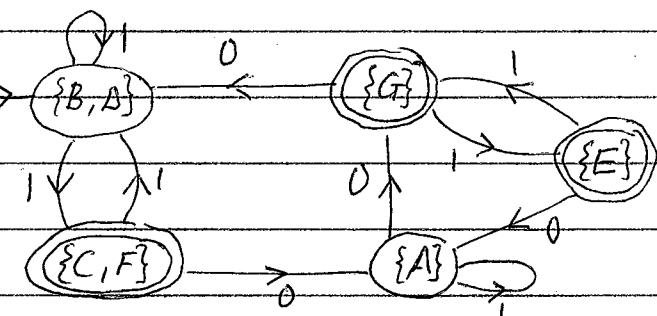
- 3(a) The state q_0 of a DFA M is inaccessible if there is no string $w \in \Sigma^*$ such that $\delta(q_0, w) = q$.

(b) $P_0: \{A, B, D\} \quad \{C, E, F, G\}$

$P_1: \{A, B, D\} \quad \{C, F\} \{E, G\}$

$P_2: \{A\} \{B, D\} \{C, F\} \{E\} \{G\}$

$P_3: \{A\} \{B, D\} \{C, F\} \{E\} \{G\}$



- 4(a) Let A_i ($i=0, 1, 2, 3$) keep track of the fact that $f(w) \equiv i \pmod{4}$.

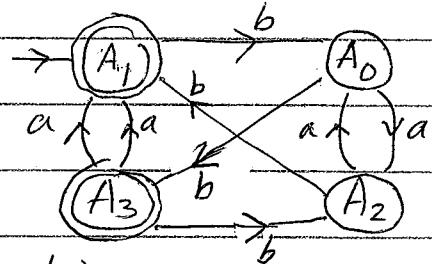
$f(\lambda) = 2n_a(\lambda) - n_b(\lambda) - 3 \equiv 0 - 0 - 3 \equiv 1 \pmod{4}$, so A_1 will be the initial state. The accepting states will be A_1 & A_3 because 1 & 3 are odd. To see what an extra "a" or an extra "b" does, express $f(aw)$ & $f(wb)$ in terms of $f(w)$. Now

$$\begin{aligned}
 4(a) \quad f(wa) &= 2n_a(wa) - n_b(wa) - 3 = 2n_a(w) + 2 - n_b(w) - 3 \\
 &= [2n_a(w) - n_b(w) - 3] + 2 = f(w) + 2 \pmod{4} \\
 f(wb) &= 2n_a(wb) - n_b(wb) - 3 = 2n_a(w) - n_b(w) - 3 - 1 \\
 &= f(w) - 1 \equiv f(w) + 3 \pmod{4}
 \end{aligned}$$

(b) input: $b \ a \ a \ b$

states: $A_1 \ A_0 \ A_2 \ A_0 \ A_3$

$$f(baab) = 2(2) - 2 - 3 = -1 \equiv 3 \pmod{4}.$$



$$5(a) \quad S \rightarrow E/F \quad \text{This gives the union}$$

$$E \rightarrow aaEbba/A, A \rightarrow aA/a \quad \text{This gives } \{a^k b^{2n} : k \geq 2n+1\}$$

$$F \rightarrow CCCFd/C, C \rightarrow c/\lambda \quad \text{This gives } \{c^k d^n : k \leq 3n+1\}$$

$$(b) \quad S \rightarrow E \Rightarrow aaEbba \Rightarrow aaAbba \Rightarrow a^2aAb^2 \Rightarrow a^2aab^2 = a^4b^2.$$

$$S \rightarrow F \Rightarrow CCCFd \Rightarrow \lambda CCCd \Rightarrow \lambda cCCd \Rightarrow ccCd \Rightarrow c^3d.$$

6(a) YES. Let $\varphi \in (A^* \cdot B) \cup (A^* \cdot C)$. Then $\varphi \in A^* \cdot B$ or $\varphi \in A^* \cdot C$.

Now if $\varphi \in A^* \cdot B$, then $\varphi = \alpha \cdot \beta$ with $\alpha \in A^*$ and $\beta \in B$.

Since $\beta \in B \cup C$, $\varphi = \alpha \cdot \beta$ with $\alpha \in A^*$ and $\beta \in B \cup C$.

Hence $\varphi \in A^* \cdot (B \cup C)$.

And if $\varphi \in A^* \cdot C$, then $\varphi = \alpha' \cdot \gamma$ with $\alpha' \in A^*$ and $\gamma \in C$.

Since $\gamma \in B \cup C$, $\varphi = \alpha' \cdot \gamma$ with $\alpha' \in A^*$ and $\gamma \in B \cup C$.

Hence $\varphi \in A^* \cdot (B \cup C)$. So in both cases $\varphi \in A^* \cdot (B \cup C)$

Hence $(A^* \cdot B) \cup (A^* \cdot C) \subseteq A^* \cdot (B \cup C)$.

(b) NO. Take $A = \{\$0\}$, $B = \{\$1\}$, and $C = \{\$0\}$. Then $B \cap C = \emptyset$

Also $A^* = \{\$, 0, 00, 000, \dots\}$. So $A^* \cdot (B \cap C) = \emptyset$ and

$$A^* \cdot B = \{\$, 0, 00, 000, \dots\} \cdot \{\$\}\} = \{\$, 0, 00, 000, \dots\}$$

$$A^* \cdot C = \{\$, 0, 00, 000, \dots\} \cdot \{\$0\} = \{0, 00, 000, \dots\}$$

$$\therefore (A^* \cdot B) \cap (A^* \cdot C) = \{0, 00, 000, \dots\} \neq \emptyset = A^* \cdot (B \cap C)$$